APPLIED
THERMODYNAMICS

A TEXTBOOK COVERING THE SYLLABUSES OF
THE B'Sc , (Eng.), A.M.Inst.C.E., AND A.M.I.Mech.E.
EXAMINATIONS IN THIS SUBJECT

BY

WILLIAM ROBINSON, M.E.

MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS
MEMBER OF THE INSTITUTION OF MECHANICAL ENGINEERS
EMERITUS PROFESSOR OF ENGINEERING AND TECHNOLOGY
UNIVERSITY COLLEGE, NOTTINGHAM

PREFACE

THE aim of this book is to aid engineering students and engineers studying heat engines and the scientific principles which condition their performance and thermal efficiency. The scope of the work is that indicated by the syllabus of Applied Thermodynamics for the B.Sc. degree (Engineering) Examination of the University of London, and includes a portion of the work suitable for engineers taking the Associate Membership examinations of the Institutions of Civil and Mechanical Engineers, as well as for the advanced National Diploma and higher certificate awarded jointly by the Mechanical Institution and the Board of Education. As a further guide, by the kind permission of the Senate of the University of London, and the University of London Press, Ltd, and of the Councils of the Institutions of Civil and Mechanical Engineers, questions have been selected from papers set at their recent examinations. However, experimental work and accurate measurements in the laboratory are all-important, and the engineering student must still keep in touch with modern practice and the rapid advances of research on the different branches of this subject.

Experience in teaching has encouraged me to endeavour by simple treatment from first principles to make the work useful as a textbook. A sketch, curve or diagram represents relations more clearly and is more easily understood by the average student than a description or general symbols. Accordingly, at the outset, the laws of curves, indicators for tracing the diagrams of the pressure in the engine cylinder, and a few measuring instruments, are briefly dealt with.

Mr. J. M. Dickson, B.Sc., kindly collaborated with me by writing the four chapters on the steam engine and steam turbines; and by drawing the temperature-entropy chart for water and steam.

I have much pleasure in thanking Mr. William Reavell,

of Ipswich, who very kindly supplied me with data on air compressors ; and Mr. John L. Hodgson, B.Sc , with his assistant, Mr. R. M. Maling, B.Sc., for data on the measurement of compressed air ; and Mr. J. Bedford, B.Sc., for the preparation of some of the drawings.

I am also indebted to Professor W. E. Dalby, Professor A. H. Gibson, Mr. Harry R. Ricardo, and other friends and firms who helped me with data, for which acknowledgment is made in the foot-notes.

Although care has been taken in correcting proofs and checking numerical examples, errors may remain, and I shall be grateful to the reader for any corrections

<div align="right">WILLIAM ROBINSON.</div>

CONTENTS

CHAPTER IV

CHAPTER VIII

CHAPTER IX

CHAPTER X

CHAPTER XI

CHAPTER XII

APPLIED
THERMODYNAMICS

CHAPTER 1

INTRODUCTION

A **Heat Engine** is a machine which converts heat into mechanical work. Heat is usually generated by the combustion of a fuel: either coal, oil, or gas. The heat, supplied to the working substance at a high temperature, is reduced in temperature by expansion within the engine, and, in the process, part of the heat is converted into work and part rejected at a lower temperature. As the heat energy in the working substance is gradually changed into mechanical energy, the temperature falls, and the greater the temperature drop of the working substance within the engine, the larger will be the proportion of heat utilized or turned into work.

In the reverse process, work is spent in driving a **Refrigerating Machine** or heat pump, which takes in heat from bodies at a low temperature and discharges heat at a higher temperature, generally by changes of state of the working substance. In this reversed heat engine, the smaller the difference between the high and low temperatures, the less work will be required to extract a given quantity of heat from the cold body. The working substance may be air, or a mixture of liquid and vapour, and may change its state in various ways.

Applied Thermodynamics deals with the relation of heat to mechanical work, and the various physical properties of the working substance. The working substances chiefly used in heat engines are fluids in the gaseous and liquid states : air and other gases or vapours in the *internal combustion engine* ; water vapour in the *steam engine* or *turbine* ; also ammonia and carbon dioxide, commonly called carbonic acid, mostly in *refrigerating machines*.

Ideal Cycle of Operations. In the action of a heat engine, when a given mass of the working substance receives heat at a high temperature, it undergoes a series of changes of pressure, volume, temperature and physical state, rejects some heat at a lower temperature, and is brought back to its initial state or condition in every respect. It has thus passed through a complete cycle of operations.

Systems of Units. In order to reduce and compare the experimental results from various investigations and the results of modern

1

práctice, it is necessary to consider the different systems of units commonly adopted by engineers: (1) British—Foot-pound and Centigrade or Fahrenheit; (2) Metric—Metre, kilogramme and Centigrade; and (3) the C.G.S.—Centimetre-gramme-second system, universally adopted for scientific purposes, especially in Chemistry and Experimental Physics.

It is desirable for the engineering student to be familiar, at least, with the first two systems in order to compare the performance of engines given in either of them. Most British engineers still use the Foot-pound-Fahrenheit system, although modern research is expressed in values on the Centigrade scale, which is convenient for comparison with the metric system. Thus quantities of heat are represented by the same figures when given either as lb.-degree Centigrade per lb., or kilo-calorie per kilogramme.

The engineering Institutions of this country are making laudable efforts to promote international agreement on the rating and performance of the various types of heat engines.

UNITS OF MEASUREMENT

Units of Length. One metre = 100 centimetres = 1,000 millimetres = 39 3702 inches; and one-third of the standard yard = 1 foot = 0 3048 metre, or 1 inch = 2·54 centimetres.

Units of Force. British engineers take the gravitational unit of force as the weight of the standard *pound* (1 lb) avoirdupois of 16 ounces; and Continental engineers the weight of the international standard *kilogramme*, which is equal to 2 20462 lb.; or 1 lb. = 0·4535924 kg. = 453·5924 grammes.

A body of weight W lb. has a mass $m = \dfrac{W}{g}$, since force = mass × acceleration. The *weight* of a body means the force of attraction of the earth on the mass of the body. The acceleration of gravity, g, is slightly less at the equator than towards the poles, and the weight of 1 lb. is greater in London than in New York by 1 part in 1,000. The intensity of gravity also varies along a given parallel of latitude, so that, for scientific precision, the locality should be given. American and Continental engineers generally take the acceleration of gravity at the sea-level in Lat. 45°, $g = 980·665$ C.G S. units. In London it is 981·16 C.G S. or 32·19 ft. per sec. per sec. For nearly all engineering purposes, $g = 32·2$, and the difference is negligible.

The C.G.S. unit of force is 1 *dyne*, or that force which, acting on a gramme mass for a second, gives it a velocity of a centimetre per second, i e. unit acceleration. This absolute unit of force is independent of gravity, and is the same everywhere.

The unit or measure of *volume* is the British legal* Imperial

* See *Weights and Measures Acts* of 1878, 1889, 1892 and 1893; and *Our Weights and Measures*, by H.J. Chaney.

standard *gallon*, containing 10 lb. weight of distilled water at the temperature of 62° F. and barometer 30 inches. The Imperial gallon contains 8 *pints*, so that, in Great Britain, 1 pint of pure water weighs 1¼ lb. An American gallon of pure water weighs only 8·331 lb., thus 6 American gallons are nearly equal to 5 Imperial gallons. A standard oil barrel of 42 American gallons contains only 35 British gallons.

Specific volume, 1 cub. metre per kg. = 16·0186 cub. ft. per lb.

Pressure is the weight or force per unit of area, and is expressed in pounds weight per square inch and kilogrammes per square centimetre : 1 lb. per sq. in = 0·070307 kg. per sq. cm. (in London) ; or 1 kg. per sq. cm. = 14·223 lb. per sq. in.

The standard *atmospheric pressure* is that of a column of mercury 760 mm. high at 0 C. in Lat. 45°, nearly 14·689 lb. per sq. in., or 1·0327 kg. per sq. cm. (London), and is practically 30 in. of mercury column at 62° F in London ; and 30 in. of mercury at 62° F. = 14·7 lb. per sq. in., so that 1 in. mercury column is equal to 0·49 lb per sq in. Thus the barometer reading in inches multiplied by 0·49 gives the atmospheric pressure in lb. per sq. in. 1 cm. of mercury column = 0·1933 lb. per sq in. = 0·01359 kg. per sq. cm. The absolute (C.G.S.) unit of pressure is 1 dyne per sq. cm., and 1 atmosphere is 1·0133 × 10⁶ dynes per sq. cm , which is independent of gravity

Absolute pressure, reckoned from zero, is obtained by adding the atmospheric pressure to the gauge or indicator pressure.

The *actual* or *absolute pressure* in a condenser is found by subtracting the reading of the vacuum gauge from that of the barometer, both readings being usually expressed either in inches of mercury or pounds per square inch.

The Bourdon pressure gauge is used to measure the pressure of steam in a boiler, also the pressure of air and steam in a condenser.

Pressure gauges and indicator springs are tested or calibrated by comparison with a mercury column subjected to steam pressure at the same temperature as when in use ; or by balancing the pressure against a column of fluid of known vertical height and specific gravity. Also, in the *dead weight tester*, a standard weight acts on a piston of hardened steel, accurately fitting into a steel cylinder, and pressing on oil, the free surface of which is acted on by the pressure to be measured. This tester has been developed by the National Physical Laboratory. The piston and cylinder can be changed, so that a comparatively small weight is used, even at the highest pressures. This instrument will show 1 lb increase in a pressure of 20,000 lb. per sq. in.

A *water gauge* manometer is employed to measure the slight variation in the pressure of air and gas above or below atmospheric pressure. The difference in the heights of water, oil, or mercury in a glass U-shaped tube is shown in inches, one limb of the tube being

open to the atmosphere and the other connected to the gas supply, or to the furnace gas in the base of the chimney to measure the draught. The single tube and curved tube manometer (p. 151) are convenient to use; other special instruments of great precision in measuring air or gas pressures have been specially designed in various investigations.*

Pure water at 10° C. (50° F.) weighs 62·41 lb. per cub. ft., so that each foot of head of water is equal to a pressure of 62·4 lb. per sq. ft., or 0·4334 lb. per sq. in.; and 1 m. of water column = 0·036 lb. per sq. in. Mercury is 13·596 times heavier than water at standard temperature and pressure.

The engineer's units of work are the foot-pound (ft.-lb.), and kilogramme-metre. 1 ft.-lb. is the work done by a force of 1 lb. weight acting through a distance of 1 ft. in its own direction. 1 ft.-lb. = 0·138255 kg.-metre (London), or 1 kg.-metre = 7·233 ft.-lb. The C.G.S. unit of work is an *erg*, being the work done by a dyne acting through a distance of a centimetre. The more convenient *practical* unit of work employed by electrical engineers is 1 joule = 10^7 ergs = 0·7372 ft.-lb.; and 1 ft.-lb. = $12 \times 2\cdot54$ cm. $\times 453\cdot6$ grammes $\times 981$ ergs = $1\cdot356 \times 10^7$ ergs = 1·356 joules.

Power is the *rate of doing work* = $\dfrac{\text{work done}}{\text{time taken}}$, or, work done = power $\times$ time.

In order to determine the *power* of an engine, we have to make two measurements, viz. a measurement of the work and of the time during which the work is done, then the power is the quotient of work divided by time. The *unit of power*, introduced by James Watt and commonly adopted by British engineers, is 1 horse-power = 33,000 ft.-lb. of work done per minute = 550 ft.-lb. per sec., or 1,980,000 ft.-lb. per hour.

The *metric* horse-power, or French *force-de-cheval*, is 75 kg.-metres per sec. = 542·475 ft.-lb. per sec. = 0·9863 British H.P.; or 1 British H.P. = 1·014 metric *force-de-cheval* (approx.).

The C.G.S. unit rate of working is 1 erg per sec., but this is an extremely small unit, and the *practical unit of electrical power is* 1 *joule per second*, and called 1 *watt*, which is equal to the work done per second by a current of 1 ampere in a circuit having 1 volt of electric pressure maintained between its terminals, and is equal to 1 ampere $\times$ 1 volt; 1 watt = 0·7372 ft -lb. per sec, or 44·23 ft.-lb. per min. 1 British H.P. = $\dfrac{550}{\cdot7372}$ = 746 watts, and 1 metric *force-de-cheval* is 736 watts; 1 kilowatt = 1·3405 H.P.

The *electrical horse-power* expended in a circuit and transformed into heat or mechanical energy is equal to the product of amperes of *current* and *volts* of electric pressure divided by 746. The *Board of*

---

* See, for instance *Proc. Inst. C.E.*, Vol. CCIV, Part 2 (1918), pp. 126 and 134.

and volume follow a law of the form $p \cdot v^n = a \ constant$, C, where p is the absolute pressure, and v the total volume, including clearance. To find a suitable value of n from the curve, take logarithms, and we have $\log p + n \log v = \log C$, which is the equation of a straight line.

Example 1. During the expansion of a given mass of air, the initial pressure was $p_1 = 80$ lb. per sq. in. when the volume, including clearance, $v_1 = 1$ cub. ft. At two other points measured on the curve, when the volume was 2 and 3 cub. ft., the indicated pressures were 33·36 and 20 lb. per sq. in. respectively.
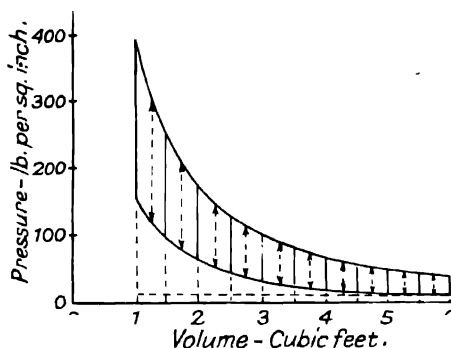
FIG. 2. *PV* DIAGRAM

If the air followed the law $p \cdot v^n = $ constant, $p_1 \cdot v_1{}^n = p_2 \cdot v_2{}^n = p_3 \cdot v_3{}^n = c$, that is, $80 \times 1^n = 20 \times 3^n = c$; hence $3^n = 4$, and, taking logarithms of each side, we have $n \log 3 = \log 4$, and $n = \dfrac{0 \cdot 60206}{0 \cdot 47712} = 1 \cdot 262$. Obviously, $c = 80$ when $v_1 = 1$, so that the law of expansion is $p \cdot v^{1 \cdot 262} = 80$

This rough calculation, only from two points on the curve, may be checked by substituting values of p and v, measured at the other point. Here, $p \cdot v^{1 \cdot 262} = 33 \cdot 36 \times 2^{1 \cdot 262} = C$, and

$$\log C = \log 33 \cdot 36 + 1 \cdot 262 \log 2 = 1 \cdot 52322 + 1 \cdot 262 \times \cdot 30103$$
$$= 1 \cdot 9031,$$

hence $C = 80$, as before.

It is necessary to take several points on a curve in order to correct errors of observation.

Example 2. The $p\,v$ diagram, Fig. 2, shows the compression of 6 cub. ft. of gas at 14·7 lb. per sq. in. to 1 cub. ft., and after receiving heat at constant volume, which raises the pressure to 400 lb. per sq. in., the gas expands to the original volume.

To find the *law of expansion*, read off and tabulate the pressures and corresponding volumes from the expansion curve, Fig. 3, plotted to an enlarged scale. We have—

v cub. ft. .	1	1·2	1·5	2	2·5	3	4	5	6
p lb. per sq. in.	400	318·4	241	168·2	127·3	101·3	70·7	53·5	42·6
log v	0	·08	·18	·30	·398	·477	·60	·699	·778
log p	2·6	2·5	2·38	2·23	2·105	2·005	1·85	1·73	1·63
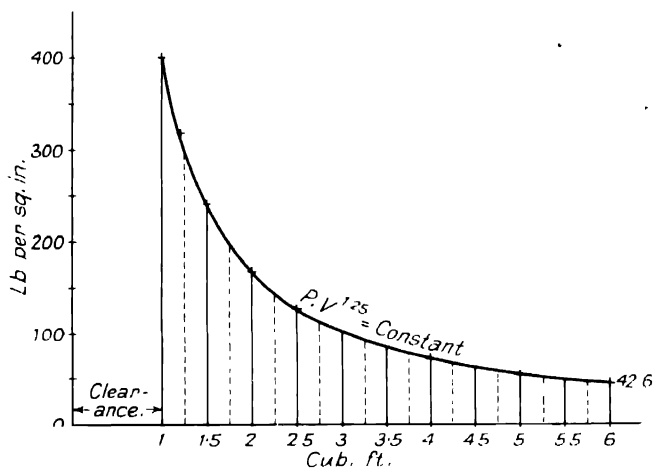
Fig. 3

Plotting the values of log p and log v, on squared paper, Fig. 4, the points lie very nearly in a straight line, found by means of a fine thread stretched along them. Take any two points on the line, one near each end, for the corrected values of each pair of logarithms ·

			Difference
log p	2·35	1·6	·75
log v	0·2	0·8	·6

Now $p_1 \cdot v_1{}^n = p_2 \cdot v_2{}^n$, taking logarithms,

$$\log p_1 + n \log v_1 = \log p_2 + n \log$$

$$\therefore n = \frac{\log p_2 - \log p_1}{\log v_1 - \log v_2} = \frac{0 \cdot 75}{0 \cdot 6} = 1 \cdot 25.$$

Or, substituting these values in the equation,

$$\log p + n . \log v = \log C.$$

gives $\qquad$ $1\cdot6 + 0\cdot8n \qquad = \log C.$

and $\qquad$ $2\cdot35 + 0\cdot2n \qquad = \log C.$

Subtracting, we have $0\cdot6n = 0\cdot75$, and $n = 1\cdot25$; also, when $v = 1$, $C = 400$. On this graph the index n gives the *slope* of the line, that is, the ratio of the decrease of $\log p$ to the change of $\log v$ between any two points on the line.

Hence the *law of expansion* is $p . v^{1\cdot25} = 400$.

In the same way the relation between p and v, on the *compression curve*, is determined and expressed by the equation $p . v^{1\cdot35} = 165$.
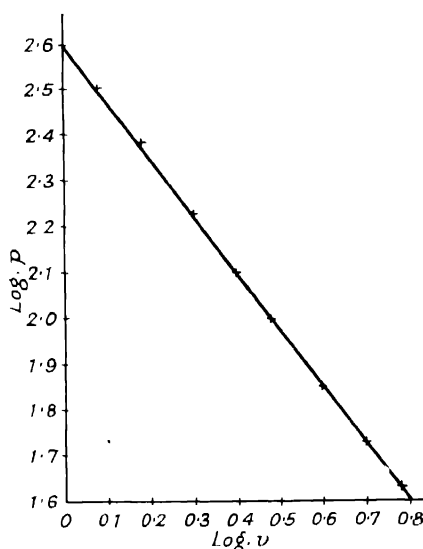
Fig. 4

These equations give information as to the behaviour of the charge in a gas engine cylinder during compression, explosion, and expansion.

Work done *by a fluid during expansion* according to the law

$$p . v^n = a \; constant = C.$$

The work done is measured by the area under the *expansion curve*, Fig. 1, between the given limits v_1 to v_2, and denoted by

$$W = \int_{v_1}^{v_2} p . dv.$$

We are given $p = \dfrac{C}{v^n} = C . v^{-n}$

In this case, by integration,

$$W = C \int_{v_1}^{v_2} v^n \cdot dv = \frac{C}{1-n} \left[v^{1-n} \right]_{v_1}^{v_2} = \frac{C}{1-n} [v_2^{1-n} - v_1^{1-n}],$$

and since $p_1 \cdot v_1^n = p_2 \cdot v_2^n$,

$$W = \frac{1}{1-n} (p_2 v_2 - p_1 v_1) = \frac{p_1 v_1 - p_2 v_2}{n-1} \qquad \qquad (1)$$

or

$$W = \frac{p_1 v_1}{n-1} \left\{ 1 - \left(\frac{v_1}{v_2} \right)^{n-1} \right\} \qquad \qquad (2)$$

also $\left(\dfrac{v_1}{v_2} \right)^n = \dfrac{p_2}{p_1}$, so that the expression becomes

$$W = \frac{p_1 v_1}{n-1} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right\} \qquad \qquad (3)$$

During *compression* the work done *by* the fluid is negative, since work is done *upon it* by the piston, then equation (1) becomes

$$W = \frac{p_2 v_2 - p_1 v_1}{n-1} \qquad \qquad (4)$$

and equation (3) is changed to

$$W = \frac{p_1 v_1}{n-1} \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} \qquad \qquad (5)$$

When the pressure is expressed in pounds per *square foot*, and the volume in cubic feet, the work done will be obtained in foot-pounds.

In the case when $n = 1$, the expansion or compression curve is expressed by the equation, $pv =$ constant, and $p_1 v_1 = p_2 v_2 = k$, say.

Then

$$W = \int_{v_1}^{v_2} p \cdot dv = \int_{v_1}^{v_2} k \frac{dv}{v} = k \int_{v_1}^{v_2} \frac{dv}{v} = k \left[\log_\varepsilon v \right]_{v_1}^{v_2}$$

and $W = k \cdot \log_\varepsilon \left(\dfrac{v_2}{v_1} \right) = p_1 v_1 \cdot \log_\varepsilon r \qquad \qquad (6)$

r is called the *ratio of expansion* or *compression*.

Multiply common logarithms of numbers (to the base 10) by 2·3026 to convert them into " Napierian " or " natural " logarithms, $\log_\varepsilon$, (to the base $\varepsilon = 2·71828$) used in equation (6).

Example 3. Given the results in Example 2, from the diagram, Fig. 2, calculate the work done during expansion and compression of the gas; also the net effective work done by the gas, and the mean effective pressure during the cycle.

During expansion of the gas, $p \cdot v^{1 \cdot 25} =$ constant, and when the volume is 1 cub. ft. the pressure is 400 lb. per sq. in., and

$$p_2 \cdot v_2^{1 \cdot 25} = p_1 \cdot v_1^{1 \cdot 25} \quad \therefore p_2 = p_1 \left(\frac{v_1}{v_2} \right)^{1 \cdot 25} = 400 \left(\frac{1}{6} \right)$$

Take logarithms, $\log p_2 = \log 400 - \dfrac{5}{4} \log 6$

$$= 2 \cdot 60206 - \frac{5}{4} \times 0 \cdot 77815 = 1 \cdot 6294$$

$\therefore p^2 = 42 \cdot 6$ lb. per sq. in., or $42 \cdot 6 \times 144$ lb. per sq. ft.

Work done during expansion *by* the gas, take equation (1):

$$W = \frac{p_1 v_1 - p_2 v_2}{n - 1}$$

and work done

$$\frac{144(400 \times 1 - 42 \cdot 6 \times 6)}{1 \cdot 25 - 1} \quad \frac{144}{\cdot 25}(400 - 255 \cdot 6)$$

$$= 576 \times 144 \cdot 4 = 83,175 \text{ ft -lb.} \qquad \textit{Answer.}$$

Equations (2) or (3) may be used to check this answer.

The work done may also be measured directly from the expansion curve, Fig. 3. Divide the area under the curve into ten strips by equidistant ordinates. Read off the mid-heights, and the average value of these gives the mean pressure during expansion $115 \cdot 52$ lb. per sq. in. Work done by the gas is pressure $\times$ change in volume

$$= 144 \times 115 \cdot 52 (6 - 1) = 83,175 \text{ ft.-lb.}$$

For work done *on* the gas during compression, we have the initial pressure $p_3 = 14 \cdot 7$ lb. per sq. in., and the *ratio of compression* $= 6$.

The law is $p_4 \cdot 1^{1 \cdot 35} = p_3 \cdot 6^{1 \cdot 35}$, or $p_4 = 14 \cdot 7 \times 6^{1 \cdot 35}$.

Taking logarithms, $\log p_4 = \log 14 \cdot 7 + 1 \cdot 35 \log 6$

$$= 1 \cdot 16732 + 1 \cdot 35 \times 0 \cdot 77815 = 2 \cdot 21782$$

and $\qquad\qquad \therefore p_4 = 165 \cdot 125$ lb. per sq. in.

Substituting these values in equation (4),

$$\text{Work done} = \frac{144(165 \cdot 125 \times 1 - 14 \cdot 7 \times 6)}{1 \cdot 35 - 1} = \frac{144}{0 \cdot 35}(165 \cdot 125 - 88 \cdot 2)$$

$$= \frac{144}{0 \cdot 35} \times 76 \cdot 925 = 31,650 \text{ ft.-lb.} \qquad \textit{Answer.}$$

To check this result by equation (5), we have

$$\text{Work done} = \frac{1 \cdot 4 \times 14 \cdot 7 \times 6}{1 \cdot 35 - 1} \left\{ \left(\frac{165 \cdot 125}{14 \cdot 7} \right)^{\frac{0 \cdot 35}{1 \cdot 35}} - 1 \right\}$$

$$= 36,288 (1 \cdot 8722 - 1) = 31,650 \text{ ft.-lb.}$$

$\therefore$ The effective work done *by* the gas during the two operations of compression and expansion is $83,175 - 31,650$, or $51,525$ ft.-lb.

Mean Pressure. Again, since the work done is equal to the mean effective pressure (M.E.P. or p_m), multiplied by the stroke volume, $p_m(v_2 - v_1) = 5 p_m = 51,525$ ft.-lb.; and

$$\therefore p_m = \frac{51,525}{5} = 10,305 \text{ lb. per sq. ft.} = 71 \cdot 56 \text{ lb. per sq. in.}$$

The *mean effective pressure* (M.E.P.) during compression and expansion, Fig. 2, is represented by the average width between the two curves, measured parallel to the pressure axis of the closed area of diagram. Then the work done is this mean effective pressure (lb. per sq. in.) multiplied by the effective area of the piston in square inches, and by the length of the piston stroke in feet.

The area may be divided into ten strips by equidistant lines at right angles to the atmospheric lines (Fig. 2). Measure the mid-length of each strip as shown by dotted lines, and the average value of these, multiplied by the pressure scale of the diagram, that is the scale number of the indicator spring used in tracing the diagram, gives the mean effective pressure.

The average width of the diagram is generally measured more accurately and quickly by means of the Amsler *planimeter*, thus –

Set the two points on the adjustable sliding bar to the *exact* length of the diagram, measured on the atmospheric line. Start with the tracer at a point about mid-length of the diagram and having the arms of the planimeter at right angles. Then move the tracing point clockwise around the diagram, and the reading divided by 40 gives the average width of the diagram. The accuracy of the instrument may be tested by using it to measure the known height of a carefully drawn rectangle of about the same area and length.

In order to determine the relation between the *absolute pressure* and volume of the fluid, as in Fig. 2, from an indicator diagram, the barometer reading is noted when the diagram is taken. The height of the mercury column in inches, multiplied by 0·49 (see p. 3), gives the atmospheric pressure in pounds per square inch. The line of zero pressure can then be drawn parallel to the atmospheric line, at a distance below it representing the atmospheric pressure to the appropriate scale of the diagram.

Clearance. The space between the cylinder cover and the piston at the end of the compression or in-stroke, including valve passages in which the working fluid may remain, is called the *clearance volume*. It is found directly in some engines by the measured quantity of water that fills it, taking care to avoid air being locked in the passages. The length, *Ol*, Fig. 2, is set off in the same ratio to the length of the diagram as the clearance volume is to the piston displacement.

Engine Indicators. The function of the engine indicator is to trace and register a pressure-volume diagram, showing the actual changes in pressure and volume of the working substance in the cylinder during a cycle of operations. The *indicator diagram* thus shows the events of the cycle, and by its shape indicates faulty setting of valves, and other irregularities in the action of the engine. From the area of the diagram, the mean effective pressure during the cycle and the rate at which the fluid is doing work upon the piston, called the *indicated horse-power*, may be determined.

The simple *piston and pencil indicator*, invented by James Watt, has been improved and designed to obviate the difficulties due to rapid variation in pressure and increased speed of engines by McNaught, Richards, and other inventors, who introduced the paper drum, light piston and moving parts, short and stiff springs
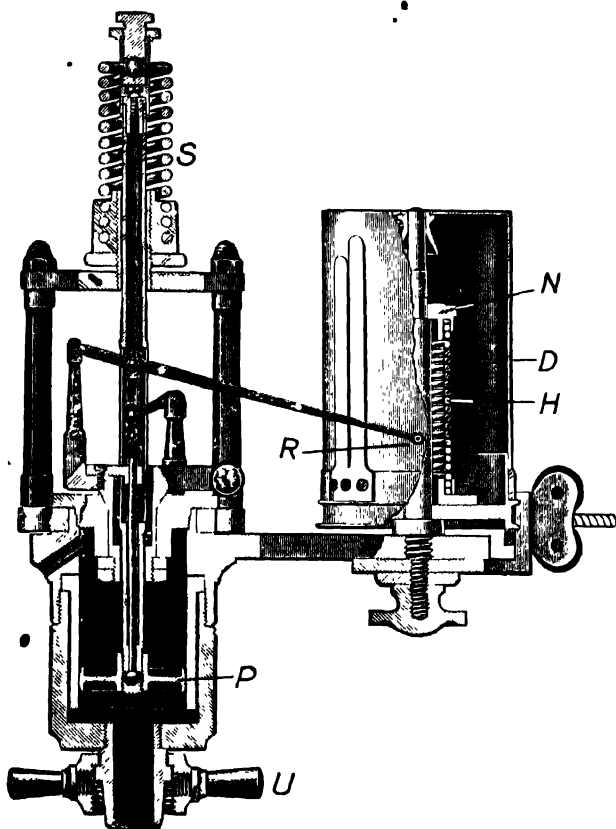
FIG. 5

THE CROSBY INDICATOR.
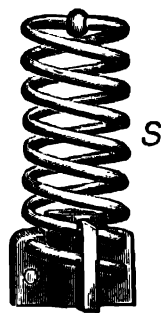
FIG. 6

CROSBY SPRING.

allowing the indicator piston only a short travel and magnifying this motion four- or six-fold by the parallel motion linkage. The other modern types– the *optical indicator* and the *electrical indicator* –are less portable and more especially adapted for scientific research purposes and work in the laboratory.

The Crosby Indicator, Fig. 5, manufactured by the Crosby Valve and Engineering Company, is one of the best known of the piston and pencil type, designed for steam, gas, or oil engines. The indicator is fixed by the union U on a cock screwed into the end of

the engine cylinder, and communicates with the clearance space. The piston P, of $\frac{1}{4}$ or $\frac{1}{2}$ sq. in. area, on which the fluid pressure acts when the indicator cock is open, is in the form of the central zone of a sphere, having line contact with cylinder in order to reduce friction. One end of the hollow piston-rod is pivoted at the centre of the piston, and the other end attached to the top of the controlling spring S by ball and socket joints. The helical spring is fixed externally so as not to be exposed to the high temperature of the gases, which would affect the elasticity of the steel. This spring, also shown in Fig. 6, is formed from a piece of steel wire, having a steel bead fixed in the middle, and made into a double spiral, the ends of which are passed through holes drilled helically in four radial wings of the metal head to hold the spring securely. Adjustment of the exact strength is made by screwing the ends of the wire into the head more or less. While the piston is moved rapidly upwards by pressure the spring is in tension. The cylinder of the indicator is held by a screwed flange at the top end only, and is free to expand. The piston needs to be kept clean and carefully lubricated. Its rise or fall is proportional to the change of pressure in the engine cylinder, and its travel is kept very short by the strong spring, but multiplied six times by the light linkage guiding the long, stiff arm carrying the pencil point R. which traces a vertical line on the paper drum D.

The length of the vertical lines traced by the pencil R upon the paper wrapped round the indicator drum, is proportional to the pressure to the scale marked on the spring. With a spring marked 200, a pressure of 200 lb. per sq. in. will give a height of 1 in. on the diagram, but, if the area of the indicator piston is $\frac{1}{2}$ sq. in., then 1 in. vertical height on the diagram will indicate a pressure of 400 lb. per sq. in. in the engine cylinder.

The total actual pressure on the indicator piston should be between 100 and 125 lb., which would be given by a pressure of 500 lb. per sq. in. on a piston area $\frac{1}{4}$ sq. in. The spring should then be chosen which would give a diagram not more than $1\frac{1}{2}$ in. high.

The spiral spring H, inside the paper drum D, keeps the cord connected to the *reducing gear* from the engine piston in tension, and can be adjusted by the nut N to suit the speed of the engine.

In reducing the reciprocating motion of the engine piston to the indicator drum, the tension in the cord has to overcome the inertia of the drum resisting acceleration and the varying pull of the spring H. The special cord used must have the least possible amount of yield or stretch, and steel wire is better, in order to avoid errors of stretching, especially at high speeds, otherwise the horizontal motion of rotation of the paper will not be an exact miniature of the piston's motion.

The whole pencil mechanism can be turned so as to bring the pencil into contact with the paper. The pencil must touch the

paper very lightly, since friction causes the pencil to lag behind its true position, so that the area of the indicator diagram is too large.* The fine point of a "Koh-i-noor" pencil on very smooth paper reduces friction and its effects.

Mr. Willans stated that for *steam* engines at 400 r.p.m., the Crosby indicator produced clear diagrams, and the inaccuracy did not exceed 1 per cent In the case of internal combustion engines, the shock of the explosion may produce errors of at least 2 to 3 per cent, or more at high speeds. If the explosion attains maximum pressure while the crankshaft moves through $15°$ or $\frac{1}{24}$ of a revolution, and the petrol engine speed is 2,100 r.p.m. or 1 revolution in $\frac{1}{35}$ of a second, then the high maxi-mum pressure is applied in $\frac{1}{840}$ of a second.

Obviously the piston and pencil type of indicator is not suitable for internal combustion engines running at speeds above about 400 r.p m. when the explosions are rapid and the pressures very high. *Errors in the diagram* are caused by the com-bined effects of inertia, friction,

FIG. 7. INDICATOR DIAGRAM DISTORTED: EFFECT OF BACK-LASH AND STRETCH OF CORD

vibration of the spring, backlash due to slackness in the pin-joints of linkage, and high temperatures in the indicator cylinder.

(1) The effect of *friction* of the piston and pencil, and *inertia* of moving parts, is to increase the mean pressure, because the pencil lags behind its true position, so that the indicated pressure is too low during compression and above the actual pressure during expansion. (2) The position of the drum may not correspond to motion of the engine piston due to *backlash* from slackness and wear in the pin-joints of the pencil mechanism, also to *stretch* of the indicator cord by sudden pulls. The latter effect is shown in the pencil indicator diagram, Fig. 7, in which the full line is the correct diagram and the broken line the boundary of the actual diagram traced by the indicator pencil. (3) Inertia effects are due to suddenly applied pressure, like blows, which produce *varying* accel-erations of the masses of moving parts—the indicator piston, pencil linkage, and the paper drum—tending at first to retard the motion of the tracing pencil, and finally to drive it too high, so that the registered maximum pressure of an explosion is much higher than

* "Errors of the Indicator," by Osborne Reynolds and H. W. Brightmore, *Proc. Inst.C.E.* (1885), Vol. 83, Part 1 ; and by J. G. Stewart, *Proc. Inst. Mech. E.*, January, 1913 ; also "Symposium of Papers on Engine Indicators," *Proc.Inst.Mech.E.*, No. 2 (1923).

centre on pivots of steel points, which are pressed upward against the hardened steel under-surface of the beam. The disk A is kept cool by water W circulating in the jacket below it, while the upper surface is exposed to atmospheric pressure. The pressure of the gases in the engine cylinder forces the disk and rod upwards, and tilts the mirror in the plane of the paper, so that light from an arc lamp is reflected upwards from M to a plane mirror, which is turned perpendicular to the paper and proportional to the motion of the piston.
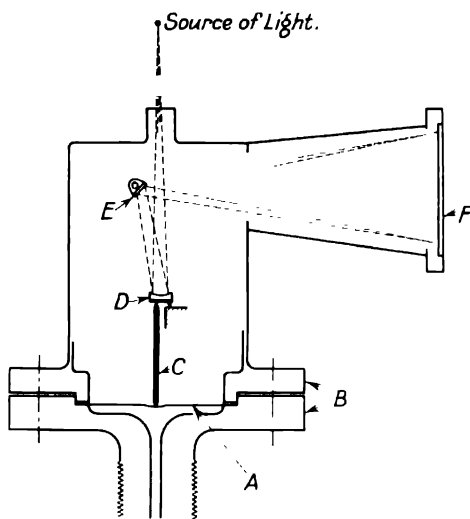
Fig. 9A. Diagram of the Dalby-Watson Optical Indicator

Dr. Watson employed this optical indicator in researches on air-petrol mixtures* in the engine, and in the study of ignition in the petrol motor.

Professor W. E. Dalby has designed this indicator in a commercial form, shown in the diagram Fig. 9A, and devised different types from time to time to meet the various conditions that have arisen, like that illustrated by the diagram, Fig. 9

The *Dalby-Watson Indicator* consists essentially of a disk A firmly supported between flanges B, and arranged for water circulation to keep the disk as cool as possible. The variation of the pressure is measured by the variation in the deflection of the disk. The thickness of the disk must be proportioned so that the deflection corresponding to the maximum pressure is small. The deflection of the disk A is transmitted by a rod C to a light mirror D, suitably mounted and constrained by a spring (not shown) to follow the deflection of the disk so that the angular displacement of this

mirror is proportional to the variation of the pressure in the engine cylinder, with practically negligible error.

A plane mirror E is mounted so that its axis is at right angles to the axis of the mirror D, and is so connected that it tilts proportionally to the displacement of the piston.

A beam of light, originating either in a " Point-o'-Lite " lamp or in an arc, is projected centrally on the mirror D, and reflected by the second mirror E, and finally brought to a focus on the camera screen F

When the engine is working, the simultaneous tilting of the two mirrors causes the spot of light to describe the pressure-volume diagram on the plate.

By means of an ingenious phase-changing device, designed by Professor Dalby, which is totally enclosed in an oil bath, the phase
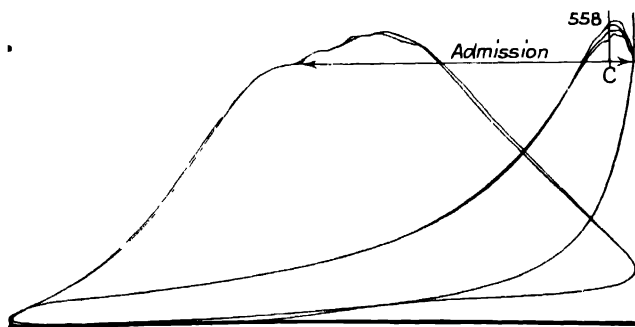
FIG. 10. DIAGRAMS TAKEN WITH A DALBY-WATSON OPTICAL INDICATOR

(From a Diesel Engine R.P.M. 240 ; Air Blast Pressure 715 lb. per sq. in. ; Mean Pressure 118 lb. per sq. in.)

of the piston displacement may be altered to produce the kind of diagram so useful for examining the dead points in the p.v. diagram. By *setting the phase* at 90° from normal, the period of ignition is at the centre of the diagram, on what is practically a time basis. The print, Fig. 10, from the original plate, taken by Professor Dalby with his optical indicator from a Diesel engine, gives four pressure-volume diagrams placed one over the other, and two superposed cycles differing in phase 90° from the normal cycle. The horizontal line marked " admission " indicates corresponding points on both diagrams. The compression curve in each of the p.v. cycles is coincident on one fine line, and clearly shows the precise point at which the mixture began to burn The small variations and differences of pressure which followed ignition in these cycles are exactly recorded, and there is very little difference in the expansion curves. The spot of light in the phase-changed diagram moved from left to right, showing clearly for two other cycles the time rate of rise of pressure in the cylinder before and after ignition.

large scale the pressures so averaged, and the smooth curve drawn through the points will represent the *mean* diagram.

The **mean effective pressure,** p lb. per sq. in., acting in the engine cylinder may also be measured rapidly and directly from the indicator diagrams by the use of a planimeter.

Fig. 13. Diagrams Taken on an Aeroplane in Flight
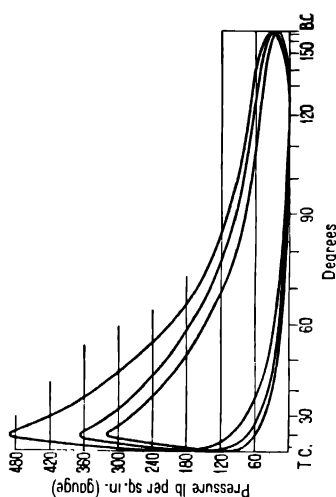
Fig. 14. Diagrams at Different Altitudes

Napier Lion Engine in Aeroplane

Height.	R.P.M.	I.M.E.P.
500 feet	2,150	148·0
5,000 feet	2,140	124·0
10,000 feet	2,100	108·0

The **indicated horse-power** of an engine is the average rate at which work is done by the gas or vapour on the piston, as calculated from the *mean* indicator diagram. Let a represent square inches of the cross-sectional area of the cylinder, l the length of piston stroke in feet, and n the number of explosion cycles or indicator diagrams traced per minute. Then for every diagram or p.v. cycle

pa lb. represents the total mean effective pressure or force driving the piston, and *pal* ft.-lb. the work done by the gas or vapour on the piston in one explosion or for every diagram. Hence the product *plan* ft.-lb. gives the work done on the piston per minute, and this rate of working is the

$$\text{Indicated horse-power} = \frac{plan}{33,000} \qquad . \qquad . \qquad . \qquad . \quad (7)$$

When working out the indicated horse-power from a large number of indicator diagrams taken during a trial, the calculation is made easier by finding the value of the *cylinder constant* which is the product of the constant factors $\frac{la}{33,000}$, and this, multiplied by $p \times n$, gives the indicated horse-power.

In an internal combustion engine, n represents the number of explosions per minute. Thus the *indicated power* is the area of the indicator diagram multiplied by the number of diagrams or explosions per minute, and by the appropriate constant for reducing to horse-power. In a single-acting steam engine there is one cycle per revolution, so that n is the number of revolutions per minute. In a double-acting steam engine there are two p.v. cycles or diagrams per revolution, so that in this case n is twice the number of revolutions per minute, that is, the number of piston-strokes per minute. Also, on one side of the piston the effective area is a, minus the sectional area of the piston-rod or tail-rod, and the *mean pressures* on each side of the piston will be p_1 and p_2 on effective piston areas a_1 and a_2 respectively, then the

$$\text{Indicated horse-power} = \frac{(p_1 a_1 + p_2 a_2)ln}{33,000} \qquad . \qquad . \quad (8)$$

where n is the number of revolutions per minute, and l the length of stroke in feet.

Example 4. Explain what is meant by the mean effective pressure of an engine, and show how its value is obtained from an indicator card. Average cards taken during a trial have an area of 4·2 sq. in., and a length of 3·0 in., and the scale of the spring used is 1/60. The engine indicated had a cylinder diameter of 15 in., a stroke of 30 in., and ran at 120 r.p.m. Estimate the average I.H.P. developed during the trial. (*A.M. Inst.C.E.*, 1923.)

The mean effective pressure on the piston is measured by the average width of the indicator diagram multiplied by the scale number of the spring used. In this case, M.E.P. $= \dfrac{4·2}{3} \times 60 = 84$ lb. per sq. in., and the effective piston area $= \dfrac{\pi}{4} \times 15^2 = 176·7$ sq. in.

∴ the average force on the piston $= 84 \times 176·7$ lb.

The work done on the piston per revolution $= 84 \times 176\cdot7 \times \dfrac{30}{12}$ ft.-lb.

$$\therefore \text{ the average indicated horse-power} = \frac{84 \times 176\cdot7 \times 30 \times 120}{33,000 \times 12}$$

$$= 134\cdot9 \qquad \textit{Answer}.$$

If the engine is double-acting, giving two diagrams per revolution, the indicated horse-power $= 270$, nearly.

Example 5. A double-acting steam engine has cylinder 18 in. diameter, piston stroke 30 in., piston-rod 6 sq. in. sectional area, speed 120 r.p.m. By planimeter measurement of the indicator diagram, the mean pressure on one side of the piston is 31, and on the other side 30 lb. per sq. in. Find the indicated horse-power.

The sectional area of cylinder is $\pi \times 9^2 = 254\cdot5$ sq. in., and the effective piston area on one side, found by deducting area of piston rod, is $248\cdot5$ sq. in. Substitute these values in equation (8), and we have

$$\text{Indicated horse-power} = \frac{(31 \times 254\cdot5 + 30 \times 248\cdot5) \times 2\cdot5 \times 120}{33,000}$$

$$= 139\cdot4 \qquad \textit{Answer}.$$

Again, if we take the average mean pressure $\frac{1}{2}(30 + 31)$, or $30\cdot5$ lb. per sq. in., and the sum of areas on the two sides of piston 503 sq. in., the work done by the steam on the piston per revolution is $30\cdot5 \times 503 \times 2\cdot5 = 38,354$ ft.-lb., and the

$$\text{Indicated horse-power} = \frac{38,354 \times 120}{33,000} = 139\cdot47$$

Also, in this case, neglecting the sectional area of the piston-rod which prevents the steam acting on that part of the piston, the average of the mean pressure gives the average force driving the piston $= 30\cdot5 \times 254\cdot5 = 7,762$ lb. ; and the work done by the steam per stroke $= pal = 7,762 \times 2\cdot5$ ft.-lb. $= 19,405$ ft.-lb. There are 2×120 strokes per minute, and we have

$$\text{Indicated horse-power} = \frac{19,405 \times 240}{33,000} = 141\cdot1$$

Obviously 140 H.P. is a close enough approximation, within the limits of error of the mean pressure obtained by the ordinary piston and pencil indicator, and may be used when the effective piston areas are nearly equal and the mean pressures differ only slightly.

The **brake power** of an engine is the effective power delivered at the crankshaft or brake wheel, and available for external work.

Mechanical efficiency is the *ratio* of the brake to indicated power.

The *difference* between the indicated and brake power is the *mechanical loss*, i.e. that part of the indicated power which is wasted in overcoming the internal resistances of the engine, including :

(1) *piston friction* due to the inertia of the piston and reciprocating parts, and friction of piston-rings ; (2) *friction* of the bearings, valves and other parts of the mechanism, caused by resistance to relative motion ; (3) *fluid* resistance in the *pumping strokes* of the piston of the internal combustion engine.

In this type of engine the total mechanical losses under good conditions at normal load usually vary from 10 to 20 per cent of the indicated power, or the mechanical efficiency is from 90 to 80 per cent.

Suppose an engine at full load and normal speed develops 100 I.H.P., and of this 10 H.P. is absorbed in internal friction, then the brake horse-power will be 90 and the mechanical efficiency 90 per cent. If the load is reduced to 40 I.H.P. and the mechanical losses remain the same 10 H.P., now the brake horse-power is 30, and mechanical efficiency becomes $\frac{30}{40}$, or 75 per cent. At full load the friction on the bearings may increase slightly, owing to increased pressure, even with good lubrication ; on the other hand, the fluid or pumping loss is then less than at no load. The results of careful tests give the total mechanical losses 12 per cent of the indicated power, and of this piston friction absorbs one half, or 6 per cent, bearing friction 3 per cent, and fluid friction 3 per cent. However, in some very high speed petrol engines, piston friction alone absorbs 50 to 60 per cent of the total mechanical losses.

When this type of engine is running at half load or less than full load, the mean effective pressure in successive cycles is variable. Hence the uncertainty of the correct value of the indicated power at less than full load, which could only be measured directly by an accurate integrating indicator, and the instruments of this kind available are unsuitable for high speed engines.

Without using the ordinary pencil indicator, the indicated horse-power and *mechanical efficiency* of an internal combustion engine may be estimated indirectly from the brake horse-power, which can be measured accurately.

(1) One method adopted by the Committee of the Institution of Civil Engineers in gas engine trials, depended on the accuracy of measurement of the gas supplied to the engine. Keep the speed constant and assume friction the same at different loads. Measure the brake horse-power and total gas used per hour at full load and half load. Plot the total gas as ordinates against brake horse-power as base line. Thus at full load point, *A*, the vertical *AB* represents total gas used ; at half load point, *C*, the vertical *CD* represents the total gas at half load. Produce the line through the points *B* and *D* to meet the vertical through no load point, *E*, at *F*. Draw *DL* parallel to base line *AE*. Then *BL* represents the gas required to change the brake power by the amount *DL*, and, provided the friction remains constant, this quantity of gas per hour would

change the indicated horse-power by the same amount. Hence the ratio *BL* to *DL* is the gas per indicated horse-power hour, and the

mechanical efficiency at full load = $\dfrac{\text{gas per I.H P. hour}}{\text{gas per B.H P. hour}}$.

(2) In high speed multicylinder petrol engines, the method devised by Mr. L. G. C. Morse to find the mechanical efficiency consists in measuring the brake power of the engine when all the cylinders are working at constant speed and full load. Then the ignition of one cylinder is stopped by cutting off the current, and the brake load reduced until the engine speed is brought up to its normal value. The reduction in the brake horse-power is approximately the indicated horse-power of the cylinder which has been cut out. Treat the other cylinders in the same way, and the sum of the results is approximately the indicated horse-power of the engine at constant speed. Mr. Morse points out that the value so obtained is too high, and he deducts the power between the compression and expansion in a cylinder not firing. Also the pumping and fluid losses are usually greater in a cylinder when not firing than when firing.

(3) Another method is to measure the power required to drive the engine at normal speed without external load ; then, assuming the mechanical losses constant at all loads, the indicated horse-power at any load is the brake horse-power added to the indicated power at no load. The engine is driven, without load, at normal speed by an electromotor, and the power absorbed is measured ; but the working conditions are different, as also the friction and fluid losses, when the engine is firing.

The power wasted in overcoming all the internal resistances of an engine may be expressed as the *equivalent torque* on the crankshaft at a given speed. Thus $\dfrac{T\omega}{550}$ = H.P., or $T = \dfrac{550 \text{ H.P.}}{\omega}$, where T is torque or resultant turning moment in lb.-ft., and ω the angular velocity in radians per second. Suppose the difference between the indicated and brake horse-power of an engine is 5 H.P., which is absorbed or wasted in the engine when the speed is 300 r.p.m. or $\dfrac{2\pi \times 300}{60}$ = 31·42 radians per second, then

the equivalent frictional torque $T = \dfrac{5 \times 550}{31\cdot42}$ = 87·53 lb.-ft.

The total frictional torque T on the crankshaft at a given speed may be made up of the torque T_0 when the engine is running light, and a part, T_1, proportional to the brake horse-power, then

$$T = T_0 + a \cdot T_1,$$

where a is a constant which can be found, as well as the frictional torque at no load, from two determinations of the indicated and brake horse-power of an engine at different speeds.

The *mechanical efficiency*, $m = \dfrac{\text{Brake horse-power}}{\text{Indicated horse-power}}$, that is,

brake horse-power $= m \times$ indicated horse-power.

From equation (7), page 25, we have

$$\text{brake horse-power} \quad \frac{m \times pa}{33,000} \times \frac{s}{12} \times \frac{n}{2}$$

where $n =$ revolutions per minute,

and $s =$ stroke in inches,

                $d =$ cylinder diameter in inches.

or, brake horse-power $= mp \times \dfrac{\pi}{4}d^2 \times \dfrac{s}{12} \times \dfrac{n}{2 \times 33,410}$

$$= \frac{mp \cdot d^2 \cdot sn}{1,008,410}$$

and, $mp = \dfrac{1,008,410 \times \text{brake horse-power}}{d^2 sn}$.   (9)

The product mp is called the "brake mean effective pressure" which may be determined from brake tests, and is found to increase with the bore of the cylinder in high-speed petrol engines.

Assume that, in the ordinary petrol engine, mp is 84 lb. per sq. in. at 1,200 r.p.m., then by equation (9), a rough estimate of the brake horse-power $= \dfrac{d^2 s}{10}$

The brake horse-power is measured by a brake or dynamometer applied to a wheel on the crankshaft. In friction-brake dynamometers all the power of the engine is absorbed or spent in friction producing heat

A simple and accurate friction brake, invented by Lord Kelvin, consists of a rope, or couple of ropes, carried once round a pulley fitted on the crankshaft, with weights hanging freely on one end to oppose the motion of the engine, and a spring balance gives the vertical pull at the slack end of the rope. The rim of the pulley should be flat and not rounded. The little wooden cross-pieces, with grooves to keep the ropes from slipping off, should be laced to the ropes and not fastened by nails to touch the rim of the wheel, and thereby become heated by friction so as to burn the rope.

When each block of wood is lubricated with tallow and plumbago, the coefficient of friction is steady between 0·14 and 0·25, depending on the load ; when the wood is covered with lubricated leather, the coefficient of friction for the different loads is nearly the same as with rope contact from 0·15 to 0·35, and the brake runs smoothly and steadily. The friction may be increased by fixing ferrodo on the wood.

'For any given load or resistance, W, the pull S, Fig. 15, in the spring balance at the slack end of the rope varies slightly, due to alterations in the coefficient of friction, μ, caused by heating of the wheel and drying of the rope. On this account, frequent readings of the spring balance must be taken to obtain a correct average value.

The work done by the engine in overcoming the frictional resistance of the brake is converted into heat at the surface of the wheel, warming up the rim rapidly, and consequently the brake pulley must be water-cooled to prevent overheating and keep the temperature uniform.

A brake pulley is usually made with flanges on the rim, forming a trough outside, which dispenses with the wooden blocks, and inside the rim the trough for cooling water prevents overheating. Water drips continuously into this trough, and centrifugal force, due to speed of rotation, keeps the water against the rim, where it evaporates ; or, for cooler running, any surplus water is scooped away by another pipe. The water supply should be turned off before the end of a test to allow time for the water in the rim trough to evaporate, otherwise, as the engine slows down, any water in the trough falls and splashes all around.
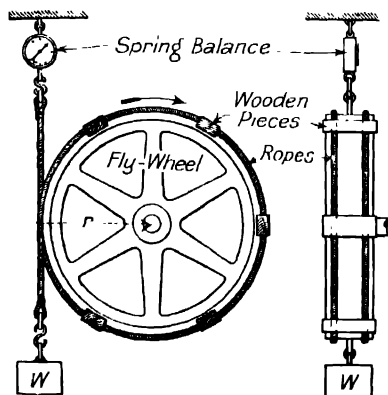
FIG. 15. ROPE BRAKE

If θ is the angle, in radians, subtended at the centre by the arc of contact between the pulley and rope, then $\dfrac{W}{S} = \varepsilon^{\mu\theta}$, where 2·71828, and μ is the coefficient of friction between rope and pulley

In order to keep S a small fraction of W, the angle θ is made large. Usually, with the rope once round, when $\theta = 2\pi$, the ratio $\dfrac{W}{S}$ may be constant at $\dfrac{1}{10}$ to $\dfrac{1}{6}$ for a dry rope.

The load or resistance is adjusted by the weight, W lb., and the pull, S lb., in the spring balance changes automatically.

The difference of the vertical pulls, $(W - S)$ lb., at the ends of the rope, multiplied by the effective radius, R, measures the total turning moment or mean *torque*, T, on the crankshaft, necessary to overcome the frictional resistance—

$$T = (W - S)R \text{ lb.-ft.}$$

R is the effective radius of the brake measured from the axis of

rotation to the centre of the rope where the vertical pulls are applied.

At the speed n r.p.m. of the wheel, $2\pi n$ radians is the angle turned through per minute; and the angular velocity $\omega = \dfrac{2\pi n}{60}$ radians per second. Then the work done per second is $T\omega$, and the

$$\text{brake horse-power} = \frac{T\omega}{550} = \frac{2\pi n R(W - S)}{33,000} \qquad (10)$$

where $2\pi R$ ft. is the circumference of the pulley added to that of the rope.

Alternative Method. The pull of the freely hanging weight, W lb., may be regarded as the load or resistance opposing the rotation of the brake wheel, while the vertical pull S, at the slack end of the rope, helps the engine, so that the difference of the vertical and opposite pulls on the two ends of the rope, $(W - S)$ lb., represents the net load or frictional resistance of the brake. The work done by the engine in overcoming this frictional resistance is $2\pi R(W - S)$ ft.-lb. per revolution, where $2\pi R$ ft. is the effective circumference of the brake, i.e. the sum of the circumference of the pulley and that of the rope, or the distance through

FIG. 16. ROPE BRAKE

which the resistance is overcome during one revolution. When the brake wheel is making n r.p.m., the rate of working or power is $2\pi R n(W - S)$ ft.-lb. per minute, and we have the

$$\text{brake horse-power} = \frac{2\pi R n(W - S)}{33,000}$$

Thus, *the difference of the pulls, in pounds weight, at the ends of a rope or belt on a pulley, multiplied by the rim speed, or velocity, in feet per minute, and divided by 33,000, gives the horse-power either absorbed or transmitted by the rope or belt.*

In the case of high speed motors up to 40 or 50 H.P., a spring-balance is commonly applied at each end of the rope, which is wound once round the water-cooled brake pulley, Fig. 16.

The load, W lb., opposing the motion of the engine can be readily adjusted by having one end of the rope carried over a pulley, while the pull on the spring-balance, S lb., at the tail end changes automatically, depending on the friction, and must be observed
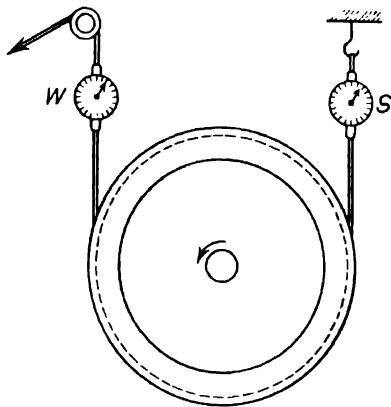
frequently, owing to changes of the frictional resistance between the rope and pulley. Care should be taken to keep a steady water supply to the brake wheel.

Example 6. In a brake test of an engine, the diameter of the water-cooled brake wheel fitted on the crankshaft is 24 in., and that of rope, wrapped round the wheel, is 1 in. The rope is kept at rest by a vertical pull of 315 lb. on a spring balance at one end opposing the rotation of the shaft, and a vertical pull of 85 lb. on the tail end, when the speed is 1,200 r.p.m. Calculate the mean torque on the shaft, and the brake horse-power.

The effective radius of the brake wheel is $\dfrac{25}{2 \times 12}$ ft., and the torque exerted by the shaft is $(315 - 85)\dfrac{25}{24} = 239.58$ lb.-ft.

The angular velocity $\omega = \dfrac{1200}{60} \times 2\pi = 125.66$ radians per second, and therefore the

$$\text{brake horse-power} = \frac{T\omega}{550} = \frac{239.58 \times 125.66}{550} = \underline{54.74}$$

Answer.

Otherwise, the net load on the brake is $315 - 85 = 230$ lb., and the work done by the engine in overcoming this resistance through *one* revolution is $230 \times \dfrac{25\pi}{12}$ ft.-lb. Therefore, at 1,200 r.p.m., the

$$\text{brake horse-power} = 230 \times \frac{25 \times 3.1416}{12} \times \frac{1200}{33,000} = 54.74.$$
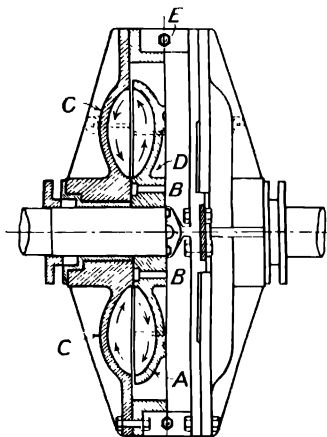
FIG. 17. FROUDE WATER BRAKE (HALF SECTION)

Water Brake. In 1877 William Froude invented the hydraulic brake to measure the power of large marine engines.* A central wheel A, Fig. 17, is keyed to the engine shaft, and consists of two similar parts bolted together at B, and is enclosed in the outside casing, C, like an annular box which is free to turn on the shaft. The ring of the central wheel, or rotor, is divided into semi-elliptical vanes or cups inclined at 45° to the vertical plane of r o t a t i o n. Facing these, on both sides, there are similar vanes or baffles around the casing, with clearance from the rotor. The water inlet is at D and the outlet at E. When the casing is full of water, and the engine turns the central wheel, A, the water is under the influence of centrifugal force, and the pairs of

* *Proc. Inst. Mech. Engrs.*, 1877.

opposite vanes, when they come together, form egg-shaped pockets, in which the centrifugal action produces vortices in a plane at 45° to the axis.   The vortices, on one side of the brake, whirl in the opposite direction to those on the other side, and so balance reactions parallel to the axis of the shaft.

The other component of the centrifugal pressure acts on the casing, and the turning moment of the shaft, producing the motion of the water, must balance the moment of resistance of the outer casing, which is measured by the torque applied by weights on a lever rigidly fitted to the casing.

The casing is always full of water, and the resistance is regulated by thin metal sluices between the vanes of the central wheel and casing.

The work done by the engine against hydraulic friction is converted into heat, which warms the water flowing away by the
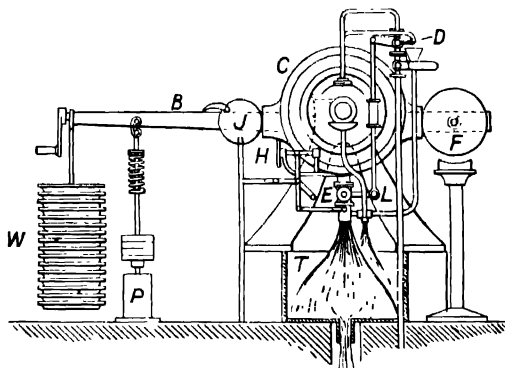
FIG. 18.   REYNOLDS -FROUDE HYDRAULIC BRAKE

outlet E.   The work done per minute is equal to the torque multiplied by 2π times the revolutions of the shaft per minute.

Reynolds-Froude Hydraulic Brake.   Professor Osborne Reynolds designed a modification of the Froude brake,* without the sluices, which has constant atmospheric pressure at the centre of the hollow vortices in the brake wheel, even when the 24 pockets on both faces of the rotor are only partly full of water.   The essential feature is that small holes, drilled in the walls of the casing buckets, allow any air brought into the wheel by the water to escape to the atmosphere. Also the resistance of the brake is regulated automatically by the quantity of water in the wheel by the water-cocks D and E, Fig. 18. A slight clockwise motion of the casing and lever, B, about the shaft, partly opens the outlet E, and, through the linkage L, partly closes the inlet D, thus reducing the quantity of water in the brake.   By

* *Proc. Inst. C E.*, Vol. 99, p. 167; and *Phil. Trans.*, Vol. 190 (1897), pp. 305-422.

adjusting the handle wheel, *H*, this regulation is made more sensitive. The water supply enters radially, passes through the holes in the vanes of wheels, and by centrifugal action from the central cavity to the vortex pockets, enters the outer chamber between the wheel and casing, which is always full when the wheel is running, and leaves the casing radially through *E* to the trough *T*. The weights, *W*, hang on a knife-edge, and the dash-pot piston, *P*, moves freely in oil to steady the motion of the lever and damp out oscillations set up by fluctuation in the speed of the engine. The jockey weight, *J*, on the lever *B*, is adjusted by a fine screw, while the balance weight, *F*, compensates the weight of the lever arm *B*, and the jockey weight, *J*, when not loaded.

Professor Reynolds and Mr. W. H. Moorby made a careful determination of the mechanical equivalent of heat by using this hydraulic brake driven by a 100 H.P. steam engine at 300 r.p.m. The brake gave the exact amount of work expended in raising the temperature of water between the two physically fixed points : the temperature of melting ice and that of water boiling, under standard atmospheric pressure, by the agitation of the water in the hydraulic brake. The heat so generated was discharged in the water which entered ice-cold and, after circulating through the brake, left at the boiling point ; then the water was cooled and allowed to flow into a tank on a weighing machine. Steam was prevented from forming by a pressure head of 11·3 ft. Before every trial the brake was balanced without load, but filled with water in the same conditions as during the trial, when the lever was always kept level. Standardized thermometers indicated the temperature of the water entering and leaving the brake. In heavy trials the torque or moment of resistance was 1,200 lb.-ft., and in the light trials 600 lb.-ft. Differences were taken for each pair of heavy and light loads to eliminate friction and other losses. The average corrected result of **44** reliable trial runs, each of 62 minutes' duration, was—

Trials.	Torque. (lb.-ft.)	Average Work. (ft.-lb. per trial)	Heat Given to Water. (lb.-degree Fahr.)
21 heavy	1200	134,337,403	171,510
23 light	600	61,355,503	77,710
Differences . .		72,981,900	93,800

This gives $\quad \dfrac{\text{Work done (ft. lb.)}}{\text{Heat generated (lb. deg. F.)}} = 778{\cdot}06 = J.$

After various small corrections, the mean value of the MECHANICAL EQUIVALENT OF HEAT, between the temperatures 32° F. and 212° F., is 777·91, taking the weights in air at Manchester. Reduced to

sea-level, Lat. 45° gives 777·6 ft.-lb., equivalent to the *mean British Thermal unit.* The unit so defined does not depend upon the scale of any particular thermometer. For all practical purposes, we may take the round number 778 ft.-lb. of work equal to the *mean British Thermal unit* (1 B.Th.U.) between 32° F. and 212° F. Thus 180 × 778 ft.-lb. of work expended,in friction raises the temperature of 1 lb. weight of water under standard pressure from 32° to 212° F.

Hence the *mean British Thermal unit* (1 B Th U.) is $\dfrac{1}{180}$ of the quantity of heat required to raise the temperature of 1 lb. weight of pure water from the freezing point to the boiling point, at standard atmospheric pressure, and is equivalent to 778 ft.-lb. of mechanical energy very nearly. The Centigrade Heat Unit (1 C.H.U.), or the *mean pound calorie* is the one-hundredth part of the quantity of heat required to raise 1 lb. weight of water from 0° to 100° C., and is equivalent to 1,400 ft.-lb. of mechanical energy.

The *thermal equivalent of* 1 *horse-power hour* of mechanical energy is therefore, $\dfrac{33,000 \times 60}{778} = 2,545$ B.Th.U. of heat energy.

or $\dfrac{33,000 \times 60}{1400} = 1,414$ C.H.U. of heat energy.

· Messrs. Heenan and Froude, Ltd., Worcester, make two types of the **"Froude" Hydraulic Dynamometer,** which are most commonly used for testing the brake power of engines.

In one type the hydraulic resistance is regulated by thin metal sluice gates between the central wheel (rotor) and the casing, by means of a hand wheel. When these sluice gates are moved towards the main shaft they cut off communication between the rotor and some of the cups in the casing, and so reduce the effective resistance. This type is used for high speed petrol engines having a very wide range of speed.

The standard type of the *Heenan and Froude Hydraulic Dynamometer,* shown in cross-sectional elevation, Fig. 19, consists of a central wheel or rotor, A, revolving inside a casing, C, through which water circulates to provide the hydraulic resistance, and at the same time to carry away the heat generated. Each face of the rotor and casing has oblique vanes of semi-elliptical cross-section, forming half of the egg-shaped pockets all round the annular space. The casing is mounted on bearings, BB, and anti-friction trunnions. The water supply enters by the channels DD, and through holes F, in the vanes of the casing. Another series of radial holes, H, are air passages from the centre of the cups to the atmosphere.

The water, entering the cups of the casing, is caught up by the rotor A, which flings it forward into the cups of the casing, and the water is returned again to the rotor, and finally escapes by the outlet E, on the top of the casing.

The torque exerted by the engine shaft on the rotor or turbine wheel re-acts on the casing, which tends to turn on its roller supports. This turning tendency is counteracted and measured by weights suspended on the lever arm fitted to the casing. Before making a test, the lever arm is adjusted by the weights and spiral spring with balance, to float freely in a horizontal position between the stops. The automatic valve for regulating the flow of water from the casing is at the outlet *E*. When the lever is horizontal and floating freely, the addition of a weight to the load on the lever moves the lever from the normal position, and this motion partly closes the outlet valve, so that more power will be absorbed until the additional load is balanced. Similarly a reduction of weight on the
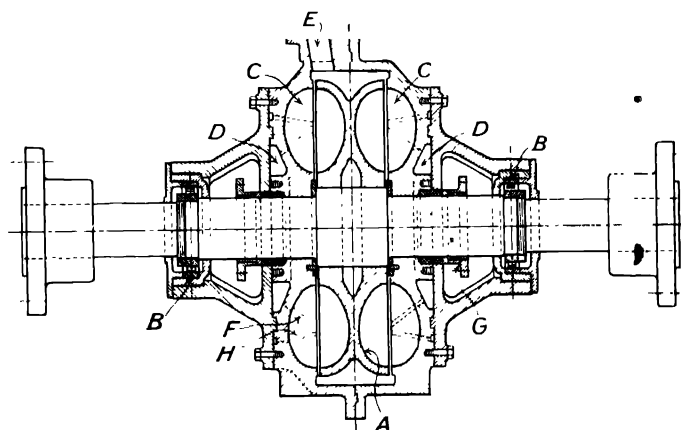
FIG. 19. HEENAN AND FROUDE HYDRAULIC DYNAMOMETER

lever causes the outlet valve to open slightly, allowing more water to flow out, and decreasing the power absorbed by the brake.

The brake horse-power of the engine coupled to the brake is $\dfrac{Wn}{K}$

where *W* is total net load of the weights hanging freely at the end of the lever arm, *n* the speed in revolutions per minute, and the constant *K* is stamped on the name-plate of the dynamometer.

The quantity of cooling water supplied to the brake should be sufficient to keep the temperature of the water at the outlet about 140° F., and not higher than 180° F. Each brake horse-power hour absorbed generates heat 2.545 B.Th.U., or 1,414 C.H.U. per hour. If the temperature of the inlet water is 50° F., and the outlet 150° F., giving a rise of 100° F., that is, 100 B.Th.U. is absorbed per pound weight of water ; then the water per hour would be $\dfrac{2545}{100}$, that is, more than 25 lb. weight, or 2·5 gallons per brake horse-power hour of the engine output.

Example 7. A rope brake on the flywheel of an engine carries a load of 250 lb. at one end, and a load of 10 lb. at the other end. The diameter of the flywheel is 5 ft., and the engine runs at 200 r.p.m. What brake horse-power is the engine developing ?

Assuming that 85 per cent of the heat generated by friction between the rope and the flywheel is absorbed by a stream of cooling water, in which the temperature rise is 10° C., calculate the amount of water required per minute.

.° (*A.M. Inst.Mech. 'E.*, 1924.)

Given effective circumference of flywheel $= 5\pi$ ft.

Net load on brake $= 250 - 10 \qquad = 240$ lb.

$$\therefore \text{Brake horse-power} = \frac{240 \times 5 \times 22 \times 200}{7 \times 33,000} = \frac{160}{7}, \text{ or } 22 \cdot 85 \text{ H.P.}$$

Again, the heat equivalent of 1 H.P. minute is

$$\frac{33,000}{1400} = \frac{165}{7} \text{ C.H.U. per minute.}$$

$$\therefore \text{Heat absorbed by cooling water} = \frac{85}{100} \times \frac{165}{7} \times \frac{160}{7}$$

$$= 458 \text{ C.H.U. per minute.}$$

We have, $10° \times W$ lb. $= 458$, and $W = 45 \cdot 8$ lb. per minute, or $4 \cdot 58$ gallons per minute. *Answer.*

Units of Heat. Quantities of heat are measured by the heat required to raise the temperature of unit weight of water through a given range, namely, between the two physically fixed points, the temperature of melting ice to that at which water boils under the constant pressure of one atmosphere. The Centigrade heat unit (C.H.U.) defined above and called the mean pound calorie (lb. cal.) $= 1 \cdot 8$ times the British Thermal unit (B.Th.U.).

The KILO-CALORIE is one-hundredth part of the quantity of heat required to raise the temperature of 1 kilogramme of water from 0° to 100° C.

Thus 1 kilo-calorie is the heat taken to raise 1 kilogramme of water 1° C.

$$= 2 \cdot 20462 \text{ lb.} \times 1 \cdot 8° \text{ F.} = 3 \cdot 9683 \text{ B.Th.U.}$$

The unit of heat commonly adopted in scientific work is the *gramme-calorie*, which is one-thousandth of the kilo-calorie, and is the quantity of heat required to raise the temperature of 1 gramme of water, under atmospheric pressure, by 1° C. measured on the scale of the standard hydrogen thermometer, either at 15° C. or, preferably, at 20° C.

Although Fahrenheit is still the most familiar scale of temperature, the Centigrade is being gradually adopted by British engineers because of its international use and convenience, since the same numbers express quantities of heat energy in the British, metric, and C.G.S. systems.

The Mechanical Equivalent of Heat. James Prescott Joule, of

Manchester, was the first* to prove that when heat is produced by mechanical, electrical, or other means, the quantity of heat generated by the friction of bodies is always proportional to the quantity of energy expended, and that the production of 1 unit of heat always requires a certain number of units of work to be spent. In other words, Joule demonstrated the *constant relation between the mechanical work spent in producing heat and the amount of heat produced.*

Also, in accordance with the **principle of the conservation of energy** in the conversion of heat into work, *when work is done by means of heat in the cylinder of any heat engine, a definite quantity of heat disappears for every unit of work done,* and, conversely, *when heat is generated by mechanical energy, the same definite quantity of work is spent which is the equivalent of every unit of heat generated.* The relation between heat and work is called Joule's equivalent (J). Whenever W units of mechanical energy are wholly expended in generating Q units of heat, $W = JQ$. This *law of equivalence* of heat and work is the **first law of thermodynamics.**

Joule showed by experiments, from 1842 to 1849, in which the potential energy of raised weights was spent in turning a paddle, which generated heat by churning water in a vessel, and observing the rise of temperature, that the quantity of heat produced by the friction between the metal paddles and water is always proportional to the mechanical energy expended. The mean result was that 772 ft.-lb. of work generated heat enough to raise the temperature of 1 lb. weight of water 1° F. on the mercury thermometer, between 55° and 60° F. Joule concluded that 772 ft -lb was equivalent to 1 B.Th.U.

Experiments by electrical heating, based on the value of the British Association unit of electrical resistance, gave the result $J = 782.5$, which did not agree with the value of J obtained from the friction of fluids. So, in 1878, Joule made another series of experiments. The method adopted was to revolve a paddle in a calorimeter containing water, and suspended from a bearing on the vertical axis of rotation, and to measure the heat produced.

The friction between the water, driven by the paddle, and the walls of the calorimeter produced a turning moment. The work done, W, measured by the product of the lifted weights w, to keep the calorimeter from turning, $2\pi R$ the effective circumference of the groove around the calorimeter, and n the number of revolutions of the paddle, was $W = 2\pi Rnw$. The result, reduced to the sea level at Greenwich, becomes 773.49 ft.-lb., equivalent to 1 B.Th.U. for the temperature 60° to 61° F. on the mercury in glass thermometer. Taking weights *in vacuo,* $J = 772.55$ ft.-lb. When afterwards reduced to the scale of an air thermometer standardized by Professor H. A. Rowland, at Baltimore, and for latitude 45°, brought the value J up to about 776 ft.-lb. per B.Th.U. at 60° F.

* *Joule's Scientific Papers,* Vol. I, p. 149.

In 1879 Rowland repeated Joule's experiment on a larger scale. The calorimeter was suspended by a torsion wire ; the water was churned in a spiral direction by curved paddles with perforations to reduce the jerky resistance, the vertical spindle passing through the bottom of the calorimeter being driven from a steam engine. The work was measured by the friction balance method employed by Joule, by the moment of resistance necessary to prevent the calorimeter from turning owing to the friction of the water while the paddle was rotated.

Rowland discovered that the capacity for heat of water diminishes as the temperature rises from 0° to about 30° C. over the range of temperature 5° to 35° C. He found the B.Th.U. equivalent to 776·8 ft.-lb., or the kilo-calorie equivalent to 426·2 kilogram-metres in latitude 45° at 20° C. on the air thermometer ; and the gram-caloric equivalent to 4·18 joules. This absolute value of J also depends on the scale of temperature, whether on the hydrogen, nitrogen, or mercury in glass thermometer.

Joule also measured the *heating effect of the electric current,* and found that the heat generated in an electric conductor is proportional to the work done, W, by the electromotive force or difference of potential at its terminals, e volts, in passing a given quantity of coulombs of electricity through the conductor, $w = eq = ect = c^2rt$; or the work in *joules* done by a steady electric current of c amperes in t seconds, in heating a conductor of r ohms resistance equals c^2rt.

Since the mean gramme calorie is equivalent to 4·1868 joules, it follows that the thermal equivalent of the work done by a steady current of c amperes flowing through a resistance of r ohms, generates in t seconds,

$$\text{heat in gramme calories} = 0\cdot239\ c^2rt. \text{ or } 0\cdot239\frac{e^2}{r}.t \quad . \quad (11)$$

Professor E. H. Griffiths adopted this method of heating water in a calorimeter by passing an electric current* through a thin platinum wire, coated with amber varnish, immersed in the water and having its ends kept at a constant difference of potential by a battery of Clark cells. The rise of temperature was measured by means of a platinum thermometer standardized by the air thermometer, and compared, under the same conditions as when used, with the nitrogen scale of the Bureau International, when the reading was found correct.

The value of J deduced from the electric standards used, subsequently corrected for an error in the electromotive force of the Clark standard cell, was 4·187 joules = the gramme calorie at 20° C.

Griffiths also confirmed Rowland's result that the capacity for heat of water decreases as the temperature rises, over the range 14° to 26° C.

* *Phil. Trans.,* Vol. 184 (1893) ; *ibid.* (1895).

Callendar and Barnes* heated a continuous steady stream of water flowing through a fine tube by passing an electric current through a central wire. The difference of temperature between the inlet and outlet water was given by a single reading of a pair of differential platinum thermometers. The flow-tube and thermometer pockets at each end were sealed in a vacuum jacket to reduce loss of heat. The water jacket, around the vacuum jacket of the calorimeter, was kept at any desired temperature.

The capacity for heat of water, measured by this method, shown in Fig. 20, agrees closely with the values found by Rowland. The result for the mechanical equivalent of the *mean* gramme calorie in absolute (C.G.S.) units between 0° and 100° C. is 4·1868 joules ; and
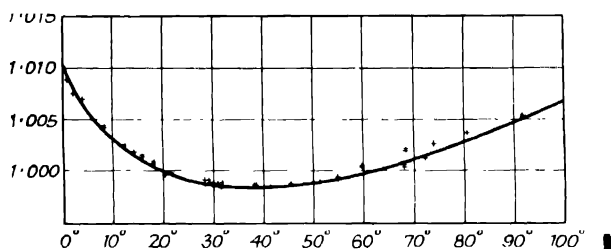
Fig. 20. Capacity for Heat of Water, (Callendar and Barnes).

the mean pound calorie (0° to 100° C.) is 1,400 ft.-lb. The corresponding value for the mechanical equivalent of the mean British Thermal unit is $1400 \times \frac{5}{9}$; or 777·78 ft -lb. (London), and that of the kilo-calorie = 426·7 kilogram-metres.　　　　　　　　　　**a**

By using J, the numerical equivalence of mechanical and heat energy, the heat obtained by the combustion of a fuel may be expressed either in thermal units or foot-pounds ; while the work done by a fluid on the piston in an engine cylinder may be calculated in the equivalent heat units. For example, when working at the rate of one I.H.P. for one hour, a heat engine converts

$$\frac{33,000 \times 60}{778} = 2545 \text{ B.Th.U., or } 1414 \text{ C.H.U.}$$

into work on the piston. Hence, the amount of heat 2,545 B.Th.U or 1,414 C.H.U., is the *thermal equivalent of one H.P. hour*. The engineer also uses one kilo-watt-hour, equivalent to 3,412 B.Th.U. or 1,896 C.H.U.

Conservation of Energy. Thus ENERGY, which means the *capacity for doing work*, may be changed from one kind into another ; and, in any transformation of energy, as for instance, from mechanical

* *B.A. Reports*, 1897 and 1899. *Phil. Trans.*, 1902, pp. 58 263 ; also 1913, pp. 1–32.

energy to electrical energy or this into heat energy, a quantity of one form of energy disappears and is converted into another kind, exactly equivalent. The function of a heat engine is to do mechanical work with the least possible expenditure of heat. The ratio of the quantity of heat converted into work in the cylinder to the total heat supplied to an engine. is called the absolute *thermal efficiency* of the engine.

In a complete cycle of operations, by the principle of the CON-SERVATION OF ENERGY, when the working fluid in any heat engine returns to the initial state,

Heat taken in = Heat rejected + Work done by the substance.

The **internal energy,** E, of a fluid may be defined as the quantity of energy in the fluid in a given state ; that is, its value depends only on the state of the fluid and not on how the fluid has reached that state. There is no means of measuring the stock of intrinsic or internal energy which a fluid contains in virtue of its pressure, volume, temperature, and state. Only *changes* of the whole internal energy can be measured when the substance receives heat, does work, or gives out heat. Internal energy is usually expressed in thermal units per unit weight, and, of course, cannot be calculated absolutely, but must be measured from an arbitrary zero. For instance, the internal energy of steam is generally reckoned as zero when the fluid is in the state of liquid water at 0° C. under its own vapour pressure.

If the working substance takes in heat without doing work, its stock of internal energy increases ; on the other hand, if the substance does work without receiving heat, it does the work at the expense of its own stock of heat energy, which is reduced by an amount equivalent to the work done.

The product pv has a definite value for any state of a fluid, and is therefore a function of the state. It may be defined as the work done in ft.-lb. per lb by forcing unit weight of volume v cub. ft. into a cylinder against a constant pressure p lb. per sq. ft. To convert this into thermal units we must divide by J. Hence, if we use the symbol A for $\frac{1}{J}$, the quantity becomes Apv heat units.

When heat is received by the substance while it is doing external work, we have,

Heat supplied = Increase of internal energy + External work done.

By this **energy equation,** the change of internal energy, due to any change of state, can be measured. The working substance or fluid is the vehicle by which the heat passes through the engine. The fluid has capacity for taking in heat at a high temperature, and by changes of pressure and volume work is done by it and the temperature lowered.

The total heat, H, of a fluid is defined by Callendar as the sum $E + Apv$. It is generally measured in thermal units per pound, and is an important function of the state of a fluid. (See pp. 43 and 177.)

It should be noticed that, by this definition, the total heat of a liquid at $0°$ C. is not zero, but equal to Ap_0v_0, where p_0 is the vapour pressure in lb. per sq. ft. at $0°$ C.; and v_0 is the specific volume of the liquid at $0°$ C., and pressure p_0. Water at $0°$ C. has a vapour pressure of 0.0892 lb. per sq. in., and a specific volume of 0.01602 cub. ft. per lb. Hence, the total heat of water at $0°$ C. is

$$H_0 = E_0 + A \cdot p_0v_0 = 0 + \frac{144}{1400} \times 0.0892 \times 0.01602$$
$$= 0.000147 \text{ C.H.U.}$$

This quantity is negligible and generally neglected in practice; thus Callendar* assumes that the total heat of water is zero at $0°$ C., and is 100 at $100°$ C.

For carbonic acid this quantity is about 1 C.H.U., and for ammonia much smaller at $0°$ C.

Total Energy of a Fluid. In some modern applications of thermodynamics, the kinetic energy of a fluid is an important factor. The kinetic energy per lb. weight of a body is $\dfrac{U^2}{2g}$ ft.-lb., where $U =$ velocity in feet per second and g is the acceleration due to gravity in feet per second per second.

Therefore the total energy per pound of a fluid is

$$E + Apv + A \cdot \frac{U^2}{2g} \text{ heat units.}$$

Equation of Steady Flow of a Fluid. Imagine an enclosure through which a fluid is passing with a *steady flow* : that is, the mass entering per second, which is equal to the mass leaving per second, remains constant. Let the fluid as it enters have specific volume, velocity, pressure, and internal energy equal to V_1, U_1, p_1, and E_1 ; also as it leaves the enclosure, V_2, U_2, p_2, and E_2 respectively. Now suppose a turbine or heat appliance within this enclosure that will do an amount of work, W, for every lb. weight of fluid passing through it ; also, let an amount of heat, Q, be lost to the enclosure and external bodies per pound of fluid passing through.

Now the work done *on* each pound of the fluid by that behind it in passing into the enclosure is p_1v_1, while the work done *by* the fluid on that in front of it in passing out is p_2v_2.

By the conservation of energy, the difference between these quantities, together with the change in internal and kinetic energy,

---

* Some authorities call what is here defined as total heat the total energy, and denote it by the letter I instead of H.

between entry and exit must be equal to the work done by the appliance and the heat energy lost to the enclosure. That is,

$$A(p_1v_1 - p_2v_2) + (E_1 - E_2) + A\left(\frac{U_1^2}{2g} \cdot \frac{U_2^2}{2g}\right) = AW + Q,$$

or,

$$(Ap_1v_1 + E_1) - (Ap_2v_2 + E_2) + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = AW + Q,$$

hence the change $= H_1 - H_2 + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = AW + Q.$

This relation is very useful when dealing with the steam turbine and various appliances in practical thermodynamics

Throttling.— In the case of STEADY FLOW of a fluid through a throttle valve or reducing valve, the total energy in heat units per pound in the initial state before entering the valve is

$$E_1 + Ap_1v_1 + AU_1^2/2g,$$

and, after throttling,

$$E_2 + Ap_2v_2 + AU_2^2/2g.$$

By the conservation of energy, the small difference between the total energy of each pound of fluid entering and leaving the valve is the sum of the loss of heat to external bodies and the external work done, and must be at the expense of its stock of internal energy.

When the valve merely reduces the pressure without external work being done, assuming neither gain nor loss of heat to the valve or external bodies, and the velocity of flow so small that the change in kinetic energy is negligible, as in the porous plug experiments by Joule and Thomson (p. 60), then it follows that the *total heat remains constant.*

Before entering the valve, the work done on one pound of the fluid by the pressure behind it is p_1v_1; when the flow has become steady after throttling, each pound does work, p_2v_2, upon the fluid in front of it. Any difference in the work done must reduce the stock of internal energy, and

$$Ap_2v_2 - Ap_1v_1 = E_1 - E_2,$$

that is,

$$E_2 + Ap_2v_2 = E_1 + Ap_1v_1$$

or

$$H_2 = H_1.$$

Thus, the *total heat of a fluid does not change in a throttling process,* as in a porous plug, expansion valve of a refrigerating machine, or in a reducing valve, during the ideal process.

Transfer of Heat at Constant Pressure. Let a quantity of heat, Q, be given to or taken away from one pound of a fluid at constant pressure p, the internal energy of the fluid changing from E_1 to E_2, while the volume changes from v_1 to v_2.

Then, by the first law of thermodynamics,

Heat supplied = *increase of internal energy* + *external work done*.

$$Q = \int_1^2 dE + A\int_1^2 p \cdot dv, \text{ where } p \text{ is constant}$$
$$= E_2 - E_1 + Ap(v_2 - v_1),$$
$$= E_2 + Apv_2 - (E_1 + Apv_1)$$
$$= H_2 - H_1, \text{ by definition of } H.$$

Hence, when heat is absorbed or rejected by a fluid at constant pressure, the quantity of heat so transferred is equivalent to the change in the total heat of the fluid.
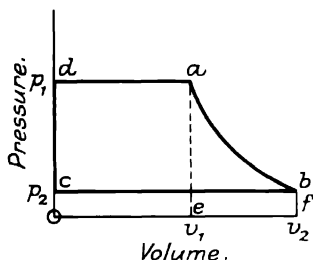
FIG. 21. ADIABATIC
EXPANSION

Change of Total Heat during Adiabatic Expansion of a Fluid. (Heat-drop.) Let ab, Fig. 21, represent the adiabatic expansion of unit weight of a fluid, from volume and pressure v_1 and p_1 to v_2 and p_2, when no heat enters or leaves the fluid, while it does work at the expense of its own stock of internal energy.

By the energy equation from first law of thermodynamics, for a small heat change,

$$dQ = dE + A \cdot p \cdot dv,$$

but in adiabatic expansion $dQ = 0$, and we have

$$dE = -A \cdot p \cdot dv.$$

Integrate, $E_1 - E_2 = \int_2^1 dE = A \int_2^1 p \cdot dv,$

which is represented by the area $abfe$ in work units, and is the difference between the internal energy of the fluid at a and b.

Now, by definition,

$$H_1 - H_2 = E_1 + Ap_1v_1 - (E_2 + Ap_2v_2)$$
$$= (E_1 - E_2) + A \cdot p_1v_1 - Ap_2v_2$$
$$= A \times (\text{area } abfe) + A(\text{area } daeo) - A(\text{area } cbfo)$$
$$= A \times \text{area } abcd.$$

This area represents the work done in admitting into the cylinder the volume v_1 of the fluid at pressure p_1, expanding adiabatically to pressure p_2 and volume v_2, and then discharging from the cylinder at pressure p_2.

The same result may be obtained from the total heat equation,

$$H = E + A \cdot pv, \text{ differentiate}$$
$$dH = dE + A \cdot pdv + Av \cdot dp$$

But, as above, $dE = -Ap \cdot dv,$

and, substituting this value of dE, gives

$$dH = - A \cdot pdv + A \cdot p \cdot dv + A \cdot v \cdot dp$$
$$= A \cdot v \cdot dp,$$

hence, integrating,

$$H_1 - H_2 = \int_2^1 dH = A \int_2^1 v \cdot dp.$$

This decrease in the total heat energy of the fluid during adiabatic expansion is called the adiabatic " Heat Drop," and is represented by the work area *abcd*. Therefore, the thermal equivalent of the whole work done in the cylinder is measured by the adiabatic Heat Drop.

By the **Second Law of Thermodynamics** only a fraction of the total heat energy supplied to an engine can be transformed into mechanical work done on the piston. If an ideal perfectly reversible Carnot heat engine (pp. 94 and 98) receives a quantity of heat, Q_1, at a high absolute temperature, T_1, and rejects heat at an absolute temperature, T_2, the greatest fraction of the heat received that it can convert into work is given by the relation,

$$\frac{\text{Heat converted into work}}{\text{Total heat supplied}} = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} \quad \text{or} \quad W = Q_1 \frac{(T_1 - T_2)}{T_1}.$$

This general law is independent of the physical properties of the working substance, is based upon reason, and follows from the fact of general experience that heat cannot of itself, without external agency, flow automatically from a cold body to a warmer body. No engine working between the same limits of temperature can be more efficient than this, and every actual engine is found to be less efficient.

The thermal efficiency of an actual heat engine relative to that of the ideal perfect engine working on the same cycle of operations is called the *Efficiency Ratio*, or relative efficiency.

Example 8. A perfect reversible heat engine, in which the working substance in the cylinder takes in the whole of the heat at the constant temperature $T_1 = 227°$ C. $+ 273 = 500°$ C. absolute, and rejects heat only at the temperature $T_2 = 27°$ C. $+ 273 = 300°$ C. absolute, could not possibly have a higher thermal efficiency than $\dfrac{W}{Q_1} = \dfrac{500 - 300}{500} = 0.4$, or 40 per cent.

In order to develop 10 I.H.P., we have $0.4\, Q_1 = \dfrac{10 \times 33,000}{1400}$ 235·7 lb. calories (C.H.U.) per minute, then the quantity of heat supplied to the engine must be $Q_1 = \dfrac{235.7}{0.4} = 589.25$ C.H.U. per minute. The heat rejected at 27° C. is 589·25 − 235·7 = 353·55 C.H.U. per minute, that is, the difference between the quantity

of heat received and the heat equivalent of the work done per minute.

Example 9. A steam engine requires 310 B.Th.U. per minute per horse-power, when working between the temperature limits of 380° F. and 125° F. What is the ratio of its thermal efficiency to that of an ideal engine working between the same temperature limits on the Carnot cycle ?

(*U.L., B.Sc. (Eng.).*)

$$\text{Thermal efficiency of steam engine} = \frac{\text{Heat converted into work}}{\text{Total heat supplied}}$$

$$= \frac{\text{Equivalent of 1 H.P. minute}}{\text{Thermal units taken in per minute}} = \frac{42 \cdot 416 \text{ B.Th.U.}}{310 \text{ B.Th.U.}}$$

$$= 0 \cdot 1368, \text{ or } 13 \cdot 68 \text{ per cent.}$$

Temperature limits are $T_1 = 380° + 460°$ F. $= 840°$ F. absolute, and lower limit $T_2 = 125° + 460°$ F. $= 585°$ F. absolute.

Thermal efficiency of an ideal engine working on the Carnot cycle is $\dfrac{T_1 - T_2}{T_1} = \dfrac{840° - 585}{840} = \dfrac{255}{840} = \cdot 304$, or 30·4 per cent.

Hence the relative efficiency $= \dfrac{0 \cdot 1368}{0 \cdot 304} = 0 \cdot 45$, or 45 per cent.

Answer.

The actual performance of any steam engine is usually compared* with that of an ideal engine working on the Rankine cycle, in which the steam is supplied to the cylinder and rejected to the condenser under corresponding pressure and temperature limits, and measured by the adiabatic heat-drop.

Example 10. A trial of a heavy oil engine gave the following data : Duration of test, 6 hours ; oil used, 192 lb. of calorific value 18,000 B.Th.U./lb ; brake load, 535 lb., and average pull on spring balance at other end of ropes 45 lb., at speed 210 r.p.m. ; effective diameter of brake wheel, 98 in. ; 90 explosions or diagrams per minute of mean effective pressure 96·6 lb./sq. in. ; diameter of engine cylinder, 14·75 in., and the length of stroke, 24 in. ; cooling water circulating through cylinder jacket, 45 lb. per minute, raised from 60° to 110° F. ; and water through exhaust calorimeter, 32 lb. per minute, raised 90° F. Calculate: (*a*) the indicated and brake horse-power ; (*b*) the mechanical efficiency ; (*c*) mean frictional torque on the crankshaft ; (*d*) oil consumption per brake horse-power hour ; (*e*) thermal efficiency, and brake thermal efficiency ; (*f*) draw up a balance sheet showing the heat distribution in the engine.

(*a*) The effective area of piston is $\dfrac{\pi}{4} \times \left(\dfrac{59}{4}\right)^2 = 170 \cdot 9$ sq. in., and length of stroke 2 ft. ; here the indicated horse-power

$$= \frac{plan}{33,000} = \frac{96 \cdot 6 \times 170 \cdot 9 \times 2 \times 90}{33,000} = 90 ;$$

---

* See " Report of the Committee on the Thermal Efficiency of Steam Engines," *Proc. Inst. C.E.*, Vol. 134, 278.

also net brake load = 535 – 45 lb. = 490 lb., and circumference of brake wheel is $\pi \times \dfrac{98}{12}$ ft.

$\therefore$ Brake horse-power $= \dfrac{490 \times 22 \times 98 \times 210}{7 \times 12 \times 33,000} = \underline{80}.$

Answer.

(b) Mechanical efficiency $= \dfrac{80 \ (\text{B.H.P.})}{90 \ (\text{I.H.P.})} = \cdot 889$ or 89 per cent

(c) And $90 - 80$, or 10 H.P. are required to overcome all the resistances in the engine at 210 r.p.m.

The corresponding frictional torque on the crankshaft is

$$T = \dfrac{550 \times \text{H.P.}}{\omega}$$

and

$$\omega = \dfrac{2\pi \times 210}{60} = 7\pi = 22 \text{ radians per sec.}$$

$$T = \dfrac{550 \times 10}{22} = \underline{250} \text{ lb.-ft.}$$

Answer.

(d) The oil used per hour is $\dfrac{192}{6} = 32$ lb., and per brake horse-power hour $= \dfrac{32}{80} = 0\cdot4$ lb

Answer.

(e) The oil used per indicated horse-power hour $= \dfrac{32}{90} = 0\cdot356$ lb.

$\therefore$ Thermal efficiency $= \dfrac{\text{Heat equivalent of 1 H.P. hour}}{\text{Heat in oil supplied per I.H.P. hour}}$

$$= \dfrac{2545 \text{ B.Th.U.}}{\cdot3556 \times 18,000 \text{ B.Th.U}} = \cdot3976, \text{ or } 39\cdot76 \text{ per cent.}$$

Similarly the brake thermal efficiency

$$\dfrac{2545}{\cdot4 \times 18,000} = \cdot353, \text{ or } \underline{35\cdot3} \text{ per cent.}$$

Answer.

(f) The *balance sheet* of the total heat supplied to the engine in the oil *per minute* is

$$\dfrac{32}{60} \times 18,000 = 9600 \text{ B.Th.U.} = 100 \text{ per cent.}$$

		Per cent
Heat converted into I.H.P. = 90 × 42·42 B.Th.U./min.		
= 3818 B.Th.U.		= 39·8
Heat converted into useful work on brake = 80 × 42·42 B.Th.U./min.		
= 3394 B.Th.U. = 35·3 per cent.		
Heat taken away in jacket water = 45(110° – 60°) B.Th.U./min.		
= 2250 B.Th.U.		= 23·4
Heat taken away in exhaust gases = 32 × 90° B.Th.U./min.		
= 2880 B.Th.U.		= 30·0
Heat lost by radiation and unaccounted for (by difference)		= 6·8
		100·0

Mechanical Losses in a Gas Engine. Professor B. Hopkinson* measured the thermal and mechanical efficiency of a Crossley gas engine of cylinder diameter 11·5 in. by 21 in. stroke, intended to give a maximum of 40 b.h.p. when firing a charge every cycle, and running at 180 revolutions, that is, giving an explosion stroke every two revolutions of the crankshaft, or 90 explosions per minute.

The power lost in friction of the mechanism was also estimated.

The engine was run with an average load, which was applied by rope brakes, one all round each flywheel, and as each test only lasted a few minutes, it was unnecessary to have the wheels water-cooled.

While careful observations were taken of the brake load, the dead weight and the spring balance—three photographs of indicator diagrams were taken with the optical indicator, each covering about a dozen explosions. At the same time the number of explosions per minute were noted, and the speed kept constant by the governor at 180 r.p.m.

In the measurement of gas consumption the supply to a small standard gas-holder was cut off, so that the engine took gas only from the holder, and the quantity taken in a definite number of suctions was observed by the number of divisions on the calibration scale in the change of level of the gas-holder. The heating value of the gas was tested during the experiments.

The diagrams taken with the Hopkinson optical indicator were remarkably uniform and made one fine line curve (the diagram is shown in Fig. 86, p. 262) for many consecutive explosions at this load and speed. The *mean effective pressure* (M.E.P.), p_m of the diagram, was measured from the negative on the plate by means of a planimeter.

The effective area of the piston $\frac{\pi}{4} . d^2 = 0.7854(11.5)^2 = 103.9$ sq. in., and the length of stroke l ft. $= \frac{21}{12} = \frac{7}{4}$ ft., so that the work done by the gases on the piston during one explosion cycle is

$$p_m \times 103.9 \times \frac{7}{4} = 181.825\, p_m \text{ ft.-lb.},$$

and at one explosion per minute the horse-power is

$$\frac{181.825}{33,000} \cdot p_m = 0.00551\, p_m.$$

When the engine is running at 180 r.p.m., and firing 90 times per minute, the indicated horse-power is

$$0.00551 \times 90 p_m = 0.4959\, p_m.$$

* *Proc. Inst. Mech. Eng.*, October, 1907.

In one test the average number of explosions per minute was 72·3, and the mean effective pressure by the indicator diagrams was $p_m = 100\cdot3$ lb. per sq. in., giving the indicated power

$$0\cdot00551 \times 72\cdot3 \times 100\cdot3 = 39\cdot95 \text{ h.p.}$$

The useful power on the brake wheels is $\dfrac{2\pi Rn(W-S)}{33,000}$, that is, the effective circumference of each brake wheel in feet multiplied by the number of revolutions per minute of the crankshaft, and by the net load, or difference between the dead weight and pull on the spring balance.

Tabulate the results obtained from three experiments with the optical indicator and brake. when the speed was 180 r.p.m.—

Water-jacket Exit Temperature.	Explosions per Minute.	M.E.P. from Diagram lb. per sq. in.	Indicated H.P.	Brake H.P.	Mechanical Loss H.P.	Gas per suction cub. feet.
149° F.(65°C.)	72·3	100·3	39·9	34·0	5·9	0·1196
— ·	73·8	99·4	40·4	34·6	5·8	0·1182
160°F. (71°C.)	74·1	99·0	40·4	34·9	5·5	0·1164

The mean of the three experiments is 34·5 B.H.P., and 40·2 I.H.P. which gives the mechanical efficiency, $\dfrac{34\cdot5}{40\cdot2} \times 100 = 85\cdot8$ per cent.

Thermal Efficiency. The average number of explosions was 73·4 per minute, and for each charge 0·118 cub. ft. of coal gas was taken in having a lower calorific value 570 B.Th.U. per cub. ft.

The available heat supplied to the engine in the coal gas *per minute* was—

	B.Th.U.	Per cent
$73\cdot4 \times 0\cdot118 \times 570$	= 4937	· · 100
Indicated work = $40\cdot2 \times \dfrac{33,000}{778}$	= 1705·1	= 34·5
Work on the brake = $\dfrac{34\cdot5 \times 33,000}{778}$	= 1463·3	= 29·6
Mechanical loss = $5\cdot7 \times \dfrac{33,000}{778}$	= 241·8	= 4·9

The mechanical loss $= 40\cdot2 - 34\cdot5 = 5\cdot7$ H.P., which becomes less as the temperature of the water jacket rises. This led to further investigation.

After the tests, the brakes were taken off and the engine run without load. the flow of the jacket water being reduced to prevent the engine from cooling rapidly. Indicator diagrams were at once taken, the explosions noted on the average, 12·65 per min., and the gas consumption 0·1252 cub. ft. per suction stroke. The mean

prossure by the indicator diagrams was 105·5 lb. per sq. in., giving the indicated power

$$0·00551 \times 12·65 \times 105·5 = 7·35 \text{ H.P.},$$

or $7·35 - 5·7 = 1·65$ H.P. more than the difference between the indicated and brake power in the full load tests. Had the engine been firing every cycle at full load, instead of about 4 out of every 5 cycles, this difference would have been increased in the ratio $\frac{5}{4}$,

$$\text{or } \frac{90}{73·4} \times 1·65 = 2·02 \text{ H P.}$$

In the idle cycles, without load, only air is admitted to the engine cylinder, and the back pressure on the piston driving the air out during exhaust is much greater than after an explosion, because in the latter, at release, when the exhaust valve opens, gases under high pressure in the cylinder rush down the exhaust pipe, so that there is little or no back pressure on the piston during exhaust. Besides there is loss of heat during the compression of air which is not all recovered during expansion, with the result that there is a negative loop on the compression and expansion shown by the light spring diagrams, and the extra power taken to compress the air at light load, when no gas was taken in, increases the negative work to about 3·5 H P. at 180 r.p.m. That is, the power taken to run the engine *without load* is over 2 H.P. more than the mechanical losses at normal load. This increase of fluid pumping losses nearly balances the decrease in piston friction when running light.

In order to investigate or estimate this power loss in the pumping strokes, i e during exhaust and suction of the air, independently of the optical indicator and brake, the gas engine was driven at 180 r.p m., without load and without allowing it to take gas, by an electro-motor, and the power required was measured electrically.

During one test the engine was closed as usual, and in the other the exhaust valve cover was removed, the cylinder being open to the air, so that there was no loss by compression.

Assuming the loss in the driving belt from the motor to the engine, about 0·5 H.P., the same in the two cases, the difference in the power absorbed should be equal to the power absorbed in the pumping strokes, the water jacket being about 180° F. (82° C.) in both cases.

On the 24th and 25th August, the tests gave the following results—

	H.P.	H.P.
Engine closed	7·72	7·1
Engine opened	4·14	3·77
Difference	3·58	3·33

The average difference, 3·45 H.P., as compared with 3·5 H P., esti-mated from the indicator diagrams, gives tolerably close agreement.

Next, the temperature of the water jacket was varied, and the engine again motored round with the exhaust valve cover taken off, and the pumping losses measured by the electrical power taken to drive the engine running light at 180 r.p.m.—

	H.P. Absorbed.
Engine hot (about 180° F., 82° C.) normal lubrication	4·0
Engine cold (70° F., 21° C.) normal lubrication	6·5
Engine cold (70° F.) excess of oil	4·7
Engine cold (70° F.) water injected into cylinder	2·7

Here the frictional losses of the engine vary considerably with the temperature, and are reduced by the injection of water.

A separate determination of frictional loss was also made with the piston and connecting rod removed ; the loss would then include the friction of the main bearings, side-shaft, valve gear, and driving belt at 180 r.p.m., and was found to be 1·4 H.P.

Deducting this 1·4 H.P. from the total power absorbed gives the power lost in friction of the piston and crank-pin ; the piston friction alone being affected by the changes in the cylinder tem-perature and lubrication. The normal value of the H.P. absorbed due to piston friction with the jacket water at 82° C. was

$$4 - 1·4 = 2·6 \text{ H.P.}$$

Other tests, under varying conditions, with a jacket temperature of 82° to 85° C., gave the mechanical loss about 5 H.P., determined by the optical indicator and brake, and fully half of this total mechanical loss is spent in piston friction.

Professor Hopkinson found that, under normal working con-ditions, at the best jacket temperature 82° C., with the brake 36 H.P., this engine indicated 41 H.P. when running at 180 r.p.m., or piston speed 630 ft. per min., and the mechanical losses were—

	H.P.			
Pumping loss	1·4	3·4 per cent I.H.P.		
Piston friction	2·5	6·1	,,	,,
Other friction, valve lifting, etc.	1·1	2·7	,,	,,
Total mechanical loss	5·0	12·2		

from which the mechanical efficiency was 87·8 per cent.

The motoring test, under nearly actual running conditions, appears to afford one of the best methods of checking the mechanical efficiency and of determining the fluid pumping losses.

With *rise of cylinder temperature* the viscosity of the lubricating oil is reduced, and so also is the piston friction, but the volumetric efficiency is reduced, so that the weight of charge taken into the cylinder is reduced, and with it the indicated horse-power ; yet the

reduction in piston friction may more than counterbalance the decrease of indicated horse-power, so that the brake horse-power rises slightly.

EXAMPLES II

1. A petrol engine, of cylinder diameter 3·6 in. and 4 in. stroke, gives 600 explosions or cycles per minute. The mean indicator diagram (Fig. 2, p. 7) has an area 1·5 sq. in. and length 3 in. ; the scale of the indicator spring is 1/200. Estimate the average indicated horse-power of the engine.

2. In a brake test the diameter of the brake wheel fitted on the crankshaft is 24 in., and that of the rope, around the wheel, 1 in. ; the vertical pulls on spring balances at the ends of the brake rope are 24 lb., and 3 lb. when the speed is 1,200 r.p.m. The mean effective pressure on the piston, 80 lb. per sq. in. during each cycle at 600 explosions per minute. The cylinder is 4 in. in diameter and 4 in. stroke. Calculate: (a) the indicated and brake horse power ; (b) the power spent in engine friction ; and (c) the mechanical efficiency.

3. A four-cylinder petrol engine at a certain speed develops B_1 B.H.P. When firing is stopped in one cylinder, and the engine made to run under the same conditions as to speed, etc., the brake horse-power is B_3. Neglecting pumping losses and assuming that the indicated horse-power developed in each cylinder is the same, find the approximate indicated horse-power of the engine. Tests on a four-cylinder petrol engine under the conditions indicated above gave the following results : B.H.P. with four cylinders firing, 14·7 ; B.H.P. with three cylinders firing, 10·0. Find the approximate indicated horse-power of the engine and its mechanical efficiency.

(*A.M. Inst. M.E.*, 1923.)

4. A test of an oil engine, during *one hour*, gave the following data: Effective circumference of brake wheel and rope, 11·25 ft. ; load on brake rope around the wheel, 48 lb., and the pull on the spring balance at the other end of rope, 4 lb. ; mean speed, 350 r.p.m. ; oil fuel used, 2·84 lb. of calorific value 18,400 B.Th.U. per lb. ; mean indicated horse-power, 6·5 ; jacket cooling water, 252 lb. raised 62·4° F. Calculate: (a) the brake horse-power ; (b) mechanical efficiency ; (c) the thermal efficiency ; and (d) give the percentage of heat energy of the fuel—(1) converted into useful mechanical energy at the crank-shaft, (2) spent in engine friction, and (3) given to the jacket cooling water.

5 A test of an oil engine gave the following figures: Determine from them the five items of the heat balance, and draw up the balance in tabular form. Duration of trial, 30 min. ; oil used, 9·7 lb. ; calorific value of oil, 18,000 B.Th.U./lb. ; jacket water, 725 lb. ; rise of temperature of water, 70° F. ; total revolutions, 6,100 ; brake load, 314 lb. at 2·5 ft. from shaft centre ; mean effective pressure, 82 lb./in.² ; diameter of cylinder, 12 in., and stroke 18 in. (*A.M. Inst. C.E.*, 1923.)

6. A gas engine, working on the constant volume cycle (Fig. 2, p. 7) has clearance 0·25 cub. ft., and the volume swept by piston, 1·25 cub. ft. ; compression curve is $p \cdot v^{1\cdot3}$ constant, and expansion curve $p.v^{1\cdot3}$ constant. At the beginning of the compression stroke the charge is at 14 lb. per sq. in., absolute pressure. While making 110 explosions per minute of maximum pressure 400 lb. per sq. in., the engine uses 12·5 cub. ft. of gas having calorific value 400 B.Th.U. per cub. ft. Calculate: (a) the pressure at end of the compression and expansion strokes ; (b) the net work done per cycle ; (c) the mean effective pressure and indicated horse-power ; and (d) the thermal efficiency of the engine. (*National Diploma of Inst. Mech. Eng.*, 1922.)

7. The following results were obtained from a test on a tank engine : I.H.P., 187·0 ; B.H.P., 162·9 ; fuel used per brake horse-power hour, 0·554 lb. ; calorific value of fuel used, 10,350 C.Th.U. per lb. ; heat carried away by circulating water, 232,500 C.Th.U. per hour. Determine: (a) the thermal

efficiency (measured on the I.H.P.) ; (*b*) the mechanical efficiency ; and (*c*) the heat lost in exhaust and radiation per minute.

<div align="right">(*A.M. Inst.Mech.E.*, 1924.)</div>

8. The following particulars were obtained from a trial of a four-stroke cycle oil engine : Duration of trial, 40 min. ; oil used, 9·77 lb. ; total revolutions, 8142 ; total combustion cycles, 4071 ; jacket water, 738 lb. ; rise of temperature of jacket water, 74° F. ; mean effective pressure in cylinder, 96 lb. per sq. in. ; torque due to brake load, 786 lb.-ft. ; calorific value of 1 lb. oil fuel, 17,000 B.Th.U.'s ; area of piston, 113 sq. in. ; stroke, 18¼ in. Find (*a*) the indicated and brake horse-powers ; (*b*) the oil used per hour per I.H.P. and per B.H.P. ; (*c*) the heat converted into indicated work per minute ; (*d*) the heat rejected by the jacket water per minute ; (*e*) the heat lost by friction, exhaust gases, etc., per minute.

<div align="right">(*U.L., B.Sc.* (*Eng.*).)</div>

9. The following results were obtained from a test on an oil engine : I.H.P., 167 ; B.H.P., 142 ; fuel per B.H.P. hour, 0·457 lb. of calorific value 10,000 C.H.U. per lb. Calculate (*a*) the indicated thermal efficiency, (*b*) the brake thermal efficiency, and (*c*) the mechanical efficiency.

<div align="right">(*A.M. Inst.Mech.E.*, 1925.)</div>

10. What is meant by the " pumping horse-power " of an internal combustion engine ? A gas engine, 8 in. piston diameter and 13 in. stroke, when running at 250 r.p.m., gave indicator diagrams showing mean effective pressures in lb. per sq. in. ; firing cycle—positive loop 85, negative loop 5 ; dead cycle—negative loop 9·5. Find (*a*) the frictional horse-power of the engine if, when running without brake load, there are 31 explosions per minute; (*b*) the I.H.P. (from positive loop), and (*c*) the mechanical efficiency.

11. Calculate the diameter and stroke of a gas engine which can develop 25 brake horse-power at 300 r.p.m., assuming a mechanical efficiency of 80 per cent, a mean effective pressure of 85 lb. per sq. in., and a ratio of stroke to diameter of 1·5.   (*U.L., B.Sc.* (*Eng.*), 1924.)

CHAPTER III

PROPERTIES OF GAS AND VAPOUR

THE working substance in heat engines is a fluid, usually in the state of vapour, and so-called *gas*, which may be regarded as highly heated vapour. When a vapour is heated above a certain temperature it becomes a gas, which cannot be liquefied by pressure until it is cooled below a certain temperature, which is called the *critical temperature* for that substance. Nitrogen, oxygen, and even hydrogen are really vapours far above their critical temperatures at ordinary atmospheric conditions. For instance, the critical temperature of hydrogen is $-241°$ C. below the freezing point of water, and the melting point of solid hydrogen, under atmospheric pressure, is about $-258°$ C Under the same conditions air boils below $-194°$ C.

By **temperature** is meant the degree of *hotness* or intensity of heat in a body, and is that property in virtue of which heat tends to flow from a warm body to a colder one.

Measurements of temperature are based on some of its effects on substances—

(1) In the ordinary thermometer the property made use of is the expansion of mercury or other liquid enclosed in a fine glass tube of uniform bore, and the temperature is indicated by the rise of the column of mercury. The indications depend on the difference of the expansion of the liquid and that of the glass.

(2) In the gas thermometer the increase of volume of air or gas when heated and kept at constant atmospheric pressure is about 20 times that of mercury for the same change of temperature, and furnishes an accurate scale of temperature as a standard of reference over a very wide range.

(3) The *platinum thermometer** has been made a standard of reference for work of precision by Callendar and Griffiths, and measures temperature by the variation of the electrical resistance of *pure* annealed platinum wire, and is suitable for direct use at very high temperatures,† as well as at extremely low ones. A special advantage secured by this thermometer is the close agreement of the thermodynamical scale with the platinum scale of temperature, as given by the expression $p_t = 100(r - r_0) \div (r_1 - r_0)$, in which p_t denotes platinum temperature Centigrade, and r, r_1, and r_0 are the observed electrical resistances of the thermometer wire at the

* *Phil. Trans.*, R.S. 1887, p. 161 ; 1891 and 1893.
† Heycock and Neville, *Journal of Chem. Soc.*, Feb., 1895. Harker and Chappins, *Phil. Trans.*, 1900. Chree, *Proc. R.S.*, 1900.

temperatures p_t, 100°, and 0° C. respectively   The change of resistance of the platinum thermometer wire between 100° C and 0° C., $r_1 - r_0$, is called the *fundamental interval* of the thermometer, and is taken as 100° C., so that on the platinum temperature scale a rise of 1° C. at any temperature would increase the electrical resistance of the platinum wire by one-hundredth of the difference between its resistance at 100° and 0° C. The reduction to the scale of the gas thermometer is effected by the parabolic *difference formula*

$$t - p_t \quad \cdot 5t\,(t - 100) \quad 10^{-1}$$

(4) A *thermo-electric couple* is composed of the junction of two different metals, as a piece of cast iron with wrought iron wire inserted, and leading to a cold junction to complete the electric circuit at the other end   A junction may also be formed by simply twisting together at one end a fine wire of pure platinum and another wire of platinum containing 10 per cent of rhodium or iridium connecting them with a dead-beat D'Arsonval galvanometer or suitable voltmeter, and inserting the other pair of ends in melting ice. On heating this thermo-couple by dipping the junction in melting metals, a small electromotive force is generated, which is plotted against the known fixed temperatures, and a calibration curve obtained giving the relation of electromotive force to temperature. Then the junction may be exposed to hot gases and their temperature measured   The platinum-iridium couple gives about 1½ times the electromotive force of the rhodium alloy, but the iridium volatilizes above 1000 C., when the platinum-rhodium alloy should be used , while for the highest temperatures a couple of tungsten and tantalum may be used.

The chief use of the thermo-electric couple is for measuring very high and low temperatures or difference of temperatures ; and it has proved specially suitable in the determination of the cyclical variation of temperature at any particular *point* or depth in the cylinder walls during the working of the steam engine,* and in the cylinder of the gas or petrol engine.†   (See pp. 329 and 337)

(5) Lord Kelvin devised a *thermodynamic scale* of temperature which is independent of the physical properties of any substance, and is deduced from the fact that two temperatures on this absolute scale are to one another in the same ratio as the quantities of heat taken in and rejected by a perfect reversible heat engine which works between the higher and lower of these temperatures (p. 103).

In the mercury thermometer, the glass, after being heated, does not quite return to its original volume at once when cooled, so that the zero point is altered during the slow and gradual shrinkage of the glass.

In order to form a scale, the two definite temperatures selected as

* Callendar and Nicolson, *Proc. Inst. C.E.*, Vol. CXXXI, p. 1.
† Coker and Scoble, *Proc. Inst. C.E.*, Vol. CXCVI, p 1.

fixed points are the temperature of melting ice and the temperature of steam as it escapes from pure water boiling under standard atmospheric pressure. On the Centigrade scale the boiling point is marked 100° C. and the freezing point 0° C., while these temperatures on the Fahrenheit scale are 212° and 32° F.

The range of temperature between these fixed points is divided into equal parts called degrees, by comparison with standard instruments ; 100° on the Centigrade scale and 212° − 32° or 180° F. Hence to reduce from one scale to the other, if readings of the same temperature are denoted by C and F, we have $\dfrac{C}{100} \quad \dfrac{F-32}{180}$ Th $C = \dfrac{5}{9}(F-32)$, and $F = 1\cdot 8\,C + 32$.

The mercury in glass thermometer is portable and convenient for ordinary purposes, where strict accuracy is not required. Although a temperature cannot be determined in this way with very great accuracy, without several corrections, *differences of temperatures* may be measured with more precision

It is essential for accuracy in indications that mercury or other liquid thermometers should be standardized, that is, calibrated for corrections at all points on their scale by comparison with a standard instrument under the same conditions as when in use.

The *gas thermometer* is a delicate piece of apparatus requiring careful manipulation, and is not suitable for ordinary work. Perfectly dry air, hydrogen and nitrogen expand nearly the same fraction of their volume at 0° C. for a given rise of temperature, if the pressure is kept constant, and are used to measure temperature in two ways : (a) by change of volume while the pressure of the gas is kept constant, and (b) by change of pressure while the volume is kept constant

The scale of temperature of the hydrogen thermometer is the standard adopted below 100° C., and that of nitrogen at higher temperatures.

Professor H. L. Callendar has, by various investigations, deduced the correction or deviation of the scale reading on the gas thermometer[*] from that of the absolute thermodynamic scale, based on the " cooling effect " observed by Joule and Thomson in their porous plug experiment. A few values of the correction found for hydrogen are given, to show the degree of accuracy—

Temperature, C.	− 200°	− 10°	40° to 50°	200°	1000°
Constant Volume	0·0311	0·00021	0·0004	0·00236	0·0438
Constant Pressure	0·201	0·00075	0·00132	0·0068	0·1040

At the fixed points, 0° and 100° C., the readings agree, and the greatest difference between the fixed points is at 40° to 50° C.

* " Thermodynamical Correction of the Gas Thermometer," by Professor H. L. Callendar, *Proc. Phys. Soc.* (*London*) Vol. XVIII, May, 1903, pp. 282–334.

Even at the extremely low temperature $-250°$ C., the correction of the constant volume hydrogen scale is only $0.1005°$ C.

Callendar takes the *absolute zero of temperature* $273.1°$ C. below the freezing point, and for his steam tables the exact values of absolute temperature,

$$T = t° \text{ C. } + 273.1 \text{ on the Centigrade scale,}$$

and $$T = t° \text{ F. } + 459.58 \text{ on the Fahrenheit scale.}$$

In ordinary calculations, except for tables, the numbers 273 and 460 may be used, as within the limits of accuracy of all practical observations.

Boyle's Law. Boyle discovered by experiment that if the *temperature* of a given quantity of any gas is kept *constant*, during a change of pressure and volume, the volume, v, varies inversely as pressure, p, of the gas, or $pv =$ *a constant*, which is represented by a rectangular hyperbola and called an *isothermal curve*.

Law of Charles. If the *pressure* be kept *constant*, the permanent gases expand nearly the same fraction of their volume at $0°$ C. for a given increase of temperature. The results of experiments show that hydrogen and other permanent gases, when kept at constant pressure, expand nearly $\dfrac{1}{273}$ of their volume at $0°$ C for every $1°$ C. rise of temperature, or $\dfrac{1}{492}$ of their volume at $32°$ F. for every $1°$ F. rise of temperature. For example, at constant pressure, 273 cub. in. of gas at $0°$ C. became $273 + 1$ cub. in. at $1°$ C., $273 + 2$ cub. in. at $2°$ C., and $273 + t$ cub in. at t C. Thus at $t°$ C. the volume at constant pressure becomes

$$v_t = v_0 \left(1 + \frac{1}{273} \times t \right) = v_0 \frac{273 + t}{273},$$

and $$\frac{v_t}{v_0} = \frac{273 + t}{273} = \frac{T}{T_0}.$$

The equation becomes $\dfrac{v_0}{T_0} = \dfrac{v_t}{T} =$ a constant.

Also, at *constant pressure*, when the same quantity of gas is cooled $1°$ C. below the freezing point to $-1°$ C., the volume becomes $273 - 1$ cub. in., and at $-2°$ C. the volume is reduced to $273 - 2$ cub. in. ; and the volume would become nothing at $-273°$ C. if the law could be held to apply down to the lowest possible temperature.

Before this temperature would be reached any actual gas would become a vapour, change its physical state to a liquid, and then become frozen to a solid. By the equation, if the temperature be reckoned from this absolute zero, the volume of a gas, heated at constant pressure, is proportional to the temperature reckoned

from absolute zero Now if the pressure be changed from p_0 to p, and the gas obeys Boyle's law, it follows that

$$\frac{pv}{p_0v_0} = \frac{T}{T_0} \text{ or } \frac{pv}{T} = \frac{p_0v_0}{T_0} = \text{ a constant } (R)$$

and we have

$$pv = w \cdot RT, \text{ or } \frac{pv}{T} = w \cdot R \quad . \quad . \quad . \quad . \quad (1)$$

the **characteristic equation** of the relations between pressure, volume and absolute temperature for w lb. weight of a gas, where

$p =$ absolute pressure in pounds per square foot,

$v =$ volume in cubic feet,

$R =$ a constant

depending on the particular gas and whether the absolute temperature, T, is expressed on the Centigrade or Fahrenheit scale

At $0°$ C and standard atmospheric pressure of 14.7 lb per sq. in., or $2,116$ lb. per sq. ft , the weight of *dry air* is 0.0807 lb. per cub. ft , hence the volume of 1 lb. weight is $\frac{1}{.0807}$, or 12.391 cub. ft.

During expansion under absolute pressure of $2,116$ lb per sq. ft. the increase in volume when the temperature is raised $1°$ C. is $\frac{12.391}{273}$ cub. ft., and the external work done in overcoming the constant pressure is

$$p(v_2 - v_1) = \frac{2116 \times 12.391}{273} = 96 \text{ ft.-lb.} = R,$$

in the equation $\frac{pv}{T} = R$.

Also in the Pound Fahrenheit system,

$$R = \frac{pv}{T} = \frac{2116 \times 12.391}{492} = 53.2 \text{ ft.-lb}$$

per lb. weight of air (nearly).

In the metric system, Regnault found that 1 cub. metre of dry air at $0°$ C and atmospheric pressure of 1.03276 kg. per sq. cm., or 10327.4 kg. per sq. metre, weighed 1.2928 kg.

Hence the volume of 1 kg of dry air $= \frac{1}{1.2928}$, or $.7735$ cub metre, and the increase in volume for the rise of temperature $0°$ C. to $1°$ C., if the air were a perfect gas, would be $\frac{.7735}{273.1}$ cub. metre. The work done by the air during expansion against constant atmospheric pressure while being warmed is $R = \frac{10327.4 \times .7735}{273.1} = 29.25$ kg.-metres per kilogramme.

To check this result, multiply 29·25 by 3·28085, the feet in a metre, gives the value of R in ft.-lb. per lb. $= 95·965$, or 96, nearly.

In the case of hydrogen, the specific volume, at 0° and atmospheric pressure of 2,116 lb. per sq. ft., is 178·8 cub. ft. per lb.

$$R = \frac{2116 \times 178·8}{273·1} = 1385 \text{ ft -lb. per lb. (nearly).}$$

The ideal *perfect gas* follows Boyle's law and satisfies the characteristic equation exactly. No actual gas does, but hydrogen probably comes nearest to it, since the deviation is very small over a wide range of temperature (p. 56).

Joule's Law. The **Internal Energy** *of a given quantity of gas* **depends only on its temperature,** *and is practically independent of its pressure and volume.*

Joule first investigated the assumption that practically no *internal work* is done, and no appreciable quantity of heat energy is absorbed
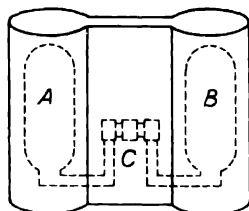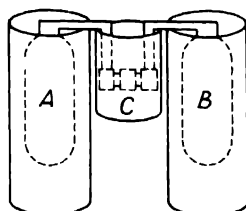
Fig 22   Fig 23

FREE EXPANSION OF GAS

by a gas during free expansion into vacuum, without doing external work. To do this, two copper vessels of equal capacity were connected by a short tube with stop-valve. One of them, *A*, Fig. 22, was filled with dry air compressed to about 21 atmospheres, and the other, *B*, was exhausted. Both vessels were immersed in a bath containing 16½ lb of water, which was stirred and its steady temperature taken by a sensitive thermometer reading to $\frac{1}{200}$ of a degree Fahr. The stop-valve at *C* was opened by means of a key and the compressed air allowed to pass from *A* into the empty vessel *B* until equilibrium was established. The water was again stirred, and no change of temperature could be detected by the thermometer.

In order to analyse this result, Joule inverted the vessels and immersed them, as well as the connecting pipe and stop-valve, *C*, each in a separate bath, Fig. 23.

After opening the stop-valve to allow the compressed air in *A* to expand into the exhausted vessel *B*, as before, the water was again stirred and a drop of temperature was observed in *A*, while that around *B* and *C* was raised to the same amount, showing that the

heat absorbed by part of the air as it rushed out of A was given up to B and C; but, on the whole, there was neither gain nor loss of heat by the air, the heat lost by A being compensated by that gained by B and C. It follows that no internal work is done by a gas during free expansion; or in a perfect gas the molecules are so far apart, compared with their size, that there is no appreciable attraction between them, consequently no internal work has to be done to separate the molecules.

Joule and Thomson (Lord Kelvin) found in later experiments[*] that the 0·13 lb. weight of air in Joule's experiment, under 21 atmospheres expanding to 10·5 atmospheres, without doing work during free or unresisted expansion required only 0·063 heat units to make up for the total internal cooling effect. This quantity of heat taken from 16·5 lb. of water, 28 lb. of copper, and 7 lb. of tinned iron, would produce a drop of temperature of only 0·003° C.

Little wonder they remarked that no appreciable external thermal effect was observed, and that similar experiments afterwards made by Regnault should have led only to the same negative conclusion.

By *taking the heat from the air itself* the drop of temperature was 2·8° C., that is, 900 times the thermometric effect in the earlier experiments.

In the **porous plug experiments** of Joule and Thomson this extremely sensitive method was devised of detecting the deviations of actual gases from the ideal perfect state, which has led to far-reaching results

It was proved that when there is molecular attraction or any trace of friction amongst the particles of actual gases, mere expansion, without doing work, produces cooling or reduction of the stock of internal energy equivalent to the internal work done in overcoming the molecular forces or friction.

A steady continuous stream of gas supplied at constant pressure and temperature is forced through a porous plug, or small orifice, from which it issues at a lower pressure, where its temperature is observed. Care is taken to prevent external loss or gain of heat by the plug or orifice, and the walls of the pipe are non-conductors. The issuing jet has kinetic energy of eddies which quickly subside, being converted into heat by fluid friction.

Suppose in the pipe there is a diaphragm with a small orifice. Consider a short length of the stream at cross-sections A and B on each side of the orifice but not close to it. At A, approaching the orifice the internal energy per lb. of fluid is E_1, and p_1, v_1 the pressure and volume, and at B, E_2, p_2 and v_2 respectively. At A the gas forced forward by the pressure p_1, with velocity v_1, escapes through the orifice, and pushes the gas in front with pressure p_2 at

* Joule and Thomson ' Thermal Effects of Fluids in Motion," *Joule's Scientific Papers*, Vol. II, pp. 216 362; or Thomson (Lord Kelvin) Papers Vol. I, p. 333.

velocity v_2, beyond the region of the "rapids" or eddies which subside at some distance from the orifice. These are greatly reduced by the porous plug device.

In the case of STEADY FLOW the temperature may be higher or lower at B than that upstream at A, depending on the nature of the fluid. The decrease of energy $E_1 - E_2$ must be equal to the work done by the fluid. Now the work done *by* the gas per pound on the portion in front at B is $p_2 v_2$, while at A, $p_1 v_1$ is the work done *on* the gas by the stream behind it. Therefore,

$$E_1 - E_2 = p_2 v_2 - p_1 v_1,$$

that is, $E_1 + p_1 v_1 = E_2 + p_2 v_2,$

a constant at any two sections of the stream, which is defined as the Total Heat (H) of the gas (see p. 42).

Suppose the product $p_1 v_1$ has not changed and is equal to $p_2 v_2$, then $E_1 = E_2$, and *if the internal energy* depends only on the temperature, then the temperature will be the same at A and B. Hence, if the temperature be found constant, the gas obeys Boyle's law and Joule's law, $E_1 = E_2$, even though the pressure and volume vary; consequently the internal energy must be a function of the temperature only. This is only strictly true of the ideal perfect gas, but if the temperature changes, then E must vary with p and v, as well as with temperature.
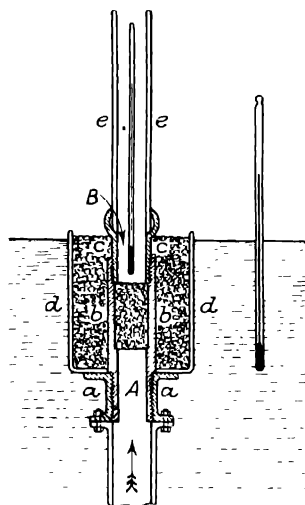
FIG. 24. POROUS PLUG

Joule and Thomson had a pump driven by an engine to force dried air or other gas in a tranquil stream through a long spiral of copper pipe immersed in a bath of water, which was constantly stirred and kept at a uniform temperature. A brass cylinder, *ce*, Fig. 24, was secured to the flange on the upper end of the copper pipe, to which a boxwood nozzle, *bb*, 1·5 in. diameter, was screwed, and in this a plug of cotton wool was slightly compressed between two perforated brass plates resting on ledges 2·72 in. apart. A short piece of boxwood, *cc*, was screwed above the top of the upper plate to hold it securely. A tin can, *dd*, screwed on the brass cylinder, *aa*, also filled with cotton wool, prevented heat from the bath passing to the boxwood nozzle.

When the stop-valve was opened to allow the stream of compressed air or gas up through the plug, fluctuations of temperature were observed, due to the sudden cooling effect and changes of temperature at the nozzle, as well as to expansion and compression in the long coil of copper pipe in the bath. After about an hour or

two, a steady condition of heat-flow allowed temperatures to be noted on sensitive thermometers, one in the bath and the other the same distance above the plug, observed through the glass tube *ee* ; also the pressure gauge was observed, every two minutes, and the mean values recorded.

The experiments showed that for pressures from 1 to 6 atmospheres, hydrogen, nitrogen, oxygen, common dry air, and carbonic acid passed towards, through, and from the porous plug in a tranquil stream, almost without change of temperature, carbonic acid much less so than any of the others, hence the inference that absolute temperature is approximately proportional to the volume of these fluids   The Joule-Thomson *cooling effect* is defined as the ratio of drop of temperature to drop of pressure between the two sides of the plug   For dry air it was at 17° C, nearly 0·0176° C. for each pound per square inch difference of pressure, and decreased at 91·6° C   to 0·014° C., or about a quarter of a degree for each atmosphere drop in pressure.

In the case of CO_2, the cooling effect was 0·0833° C per lb. on the square inch difference of pressure at 12·8° C while at 91·5° C it was reduced to 0·0474° C   Callendar finds that a curve for the mean corrected results for carbonic acid gas gives 0·0944° C. per lb pressure at 0° C, and 0·0421° C. per lb at 100° C. At higher temperatures these gases approximate more nearly to Boyle's law ; and the results showed that *the cooling effect was proportional to the difference of pressure on the two sides of the porous plug, and varied inversely as the square of the absolute temperature for dry air and carbonic acid.*

For air, the drop of temperature (Centigrade) is

$$0\,275(P_1 - P_2)\left(\frac{273}{T_1}\right)^2.$$

where the pressure is in atmospheres   For carbonic acid, the constant in this expression is 1·388.

In the case of hydrogen there was a *heating effect* which agrees with the results of Amagat's experiments that the product *pv increases* with the pressure, whereas for carbonic acid and air, *pv decreases* nearly uniformly and the curve has a steep slope downwards to a minimum near the critical temperature, where it becomes vertical.

Kelvin also deduced from the cooling effect a method of comparing the absolute thermodynamic scale of temperature with that of the gas thermometer   Taking the work equivalent of the heat required to compensate or neutralize the cooling effect, and Regnault's measurements of the specific heat at constant pressure and the coefficients of expansion between 0° and 100° C., Kelvin concludes that the absolute temperature of melting ice is 273·1° C.,* or " for

* Article on " Heat," *Ency. Bru.* ; also Kelvin's *Mathematical and Physical Papers*, Vol 1, p. 333.

every 100 units of heat converted into work by a perfect thermo-dynamic engine, 373·1 are taken from the source, and 273·1 rejected to the refrigerator, if the temperature of the source be that at which steam of water has a pressure of 1 atmosphere and the temperature of the refrigerator that at which ice melts."

The cooling effect has been thoroughly investigated and applied by Callendar in the determination of the properties of steam and other vapours at low and moderate pressures common in practice. He pointed out that the cooling effect indicates a corresponding variation of specific heat with pressure by the relations between specific heat, cooling effect, and variation of the total heat of gases.

This Joule-Thomson cooling effect, or small drop in the tem-perature of compressed air and vapours escaping through a fine orifice, as in a throttling operation, through a valve without gain or loss of heat, is most important in applied thermodynamics.

Professor Linde took advantage of it in a *regenerative process* which accumulates the cooling effect, with an *interchanger*, and air pump for the production of extreme cold below the critical tem-perature of air – 140° C., and nearly to absolute zero (see p. 186).

Obviously, Joule's law only holds strictly for the ideal perfect gas, the deviation for the permanent gases is so small as to be negligible, except in the case of vapours near their critical tem-peratures.

Properties of a Vapour. As defined above, any fluid in a state above its **critical temperature** is known as a **gas,** which cannot be changed into the liquid state by any pressure, however great Below its critical temperature the same fluid is a **vapour,** which may be liquefied by pressure at constant temperature

Dalton observed that, for a given volume and temperature, the pressure of a mixture of gas and vapour is very nearly equal to the sum of the pressures which each constituent would exert if it alone occupied the space. This law is only true within certain limits for the vapours of liquids which have no chemical action on each other, but when the vapours mutually dissolve each other, the total pressure is always less than the sum of the vapour pressures acting separately. Professor T. Andrews showed by experiment that in a mixture of two ordinary gases, carbonic acid and nitrogen, Dalton's law is deviated from at high pressures, and only holds true when the gases are in the state of a perfect gas, far above their temperatures of liquefaction.

Suppose a closed vessel, of volume v cub. ft., contains only *dry air, w* lb. weight at temperature $T°$ C. absolute, by the charac-teristic equation, the pressure in pounds per square foot is $p = \dfrac{96wT}{v}$

If a small quantity of water be added, it will evaporate until the air is saturated with dry water vapour. The quantity of vapour required to saturate the given volume depends only on the

temperature T° C. (kept constant), and is very nearly the same as would be taken if the air were not present. At that temperature the corresponding pressure, p_1, of the water vapour is found from steam tables. Then, by Dalton's law, the actual total pressure, as measured in the vessel, is the sum of the *partial pressures*, $p + p_1$, which the air and water vapour would exert separately.

The air present does not appreciably alter the quantity of water vapour which the vessel will hold at T° C., so long as the pressure is not too high.

In case all the water is in the state of vapour, and the air saturated with the vapour, then by increase of temperature the steam is said to be *superheated*. On the other hand, by cooling the mixture below the saturation or *dew point* at constant pressure, part of the vapour condenses as mist or exceedingly small drops on the cold walls of the containing vessel. If the surface is below the freezing point, as that of pipes through which the cold brine circulates in refrigeration, the water vapour in the air forms a coating of fine particles of ice or frost on the cold pipes.

When coal gas at a given temperature and pressure is saturated with the vapour of volatile liquid hydrocarbons, the gas cannot retain any more of that vapour, and fall in temperature or increase of pressure causes some of the vapour to condense. Hence the trouble in cold weather when part of the heavy hydrocarbon vapour, carried by enriched coal gas, condenses and is deposited in the distributing and house supply pipes exposed to cold air, thereby reducing the heating value of the gas.

A dry saturated vapour begins to condense when it is cooled, or when its volume is reduced at constant temperature and pressure ; it is then said to be *wet*. The wet portion of the vapour may be dried by evaporation during isothermal expansion at constant pressure, taking in its latent heat of evaporation ; or, again, the saturated vapour may be gradually reduced to the liquid state by isothermal compression at constant pressure, giving out its latent heat. The *dryness fraction* of the mixture is the fraction of the whole mixture of vapour and liquid that is in the state of vapour. A dry saturated vapour, when heated apart from its liquid, is said to be *superheated*.

The isothermal lines in Fig. 25 were plotted by Professor Thomas Andrews from the results of his experiments* on the behaviour of carbon dioxide (CO_2), known as carbonic acid. The CO_2 "gas" was compressed over mercury in a capillary glass tube, carefully calibrated. Another similar tube, placed beside it and connected with it, contained dry air, which acted as a pressure gauge. Both tubes were kept at any desired temperature in a bath, and the pressure altered by a screw plunger.

Starting with dry CO_2 at D, keeping the temperature constant

*. *Phil. Trans.* 1869, *part II*, p. 575 ; *ibid.* 1876, *part II.*

at 13·1° C., the volume is gradually reduced by compression to 49 atmospheres at the point *A*, when the vapour is saturated, and with a slight increase of pressure condensation begins. The pressure remains practically constant as the volume is reduced and the vapour condensed from *A* to *B*. Only a slight increase of pressure is required to convert all the vapour into liquid at *B*. By increasing
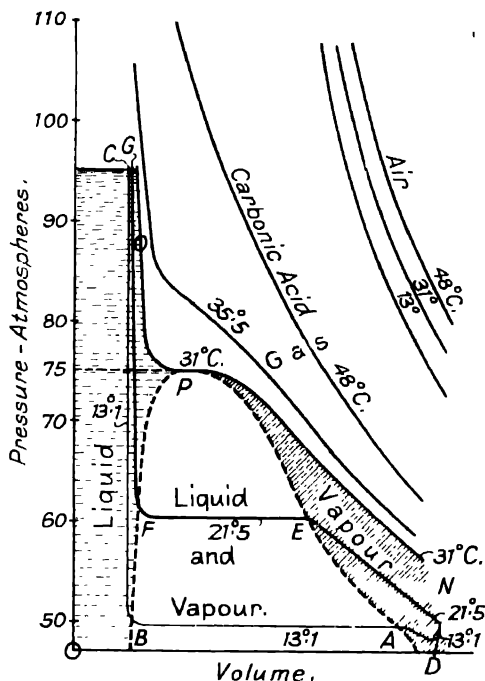
FIG. 25. ISOTHERMALS OF CO_2

the pressure up to 95 atmospheres the volume of the liquid is only slightly reduced from *B* to *C*.

At the constant temperature 21·5° C. a similar isothermal, *DEFG*, is obtained. The pressure is gradually increased from *D* to 61 atmospheres at *E*, the saturation pressure of the vapour, when a sharp, clearly marked change is observed to the liquid state, and condensation continues at constant pressure as the volume is reduced from *E* to *F* until all the mixture becomes liquid.

Starting from *N*, at 31·1° C., the compression curve shows that the volume diminishes regularly until a pressure of about 73 atmospheres is reached. The volume is then *reduced very rapidly* to about one-half, as seen by the points of inflection in the isothermal, while the pressure is increased steadily to 75 atmospheres, and there is no evidence of condensation or separation of liquid in the tube,

the space being then occupied by a homogeneous fluid. " No apparent liquefaction of CO_2, or separation, into two distinct forms of matter could be effected, even when a pressure of 300 or 400 atmospheres was applied." The isothermal for 35·5° C. is similar, but the inflection by change in volume is less abrupt

Further, if the CO_2 gas is heated above 31° C. and kept under a pressure, say, of 80 atmospheres, and subsequently cooled below 31° C. at this constant pressure, the substance remains homogeneous, but is gradually converted into the liquid state without any abrupt change or breach of **continuity of state.**

When the pressure of the liquid is reduced while cooling at constant volume, say to the state point, *F*, the vapour begins to form, and evaporation continues during isothermal expansion at constant pressure, *FE*, until all the liquid becomes saturated vapour at *E*. Then, if the pressure is gradually reduced, isothermal expansion of the vapour takes place as shown by the curve *ED*.

Similar isothermals may be traced below the **Critical Point,** *P*. As the temperature is raised, the pairs of state points *A*, *B*, and *E*, *F*, etc., come closer together, forming two continuous curves, showing the saturation points of vapour and the liquid line, which merge into one **boundary curve** with rounded top at the **Critical Temperature** and **Critical Pressure.** The state to the left of the *boundary line PFB* is wholly liquid, that to the right of the *dry saturation line PEA* is superheated vapour, and the region between the curves a mixture of liquid and vapour.

In the same way, ammonia (NH_3), sulphur dioxide (SO_2), and other vapours below their critical temperatures can be easily changed to the liquid state by compression and cooling. Again, when the pressure is reduced, these liquids readily evaporate and take in their latent heat of evaporation from their surroundings This property is commonly used for refrigeration purposes.

Professor Dewar found by experiment that in a mixture of carbonic acid with benzol, the CO_2 liquefied at 18° C. under a pressure of 25 atmospheres ; and on reducing the pressure, the carbonic acid boiled away rapidly from the benzol.

The saturation pressure and volume, latent and specific heats, and other physical properties of various vapours below their critical temperatures have been determined and given in Tables (p. 192), or plotted on charts for practical purposes of calculation.

Van der Waals devised a general equation giving the relation of *p*, *v* and *T* for any fluid through any range of temperature and density from the state of liquid to that of a vapour and perfect gas :

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT, \text{ or } p = \frac{RT}{v - b} - \frac{a}{v^2},$$

where *a* and *b* are constants, determined by experiment for any fluid.

Here, a makes allowance for the pressure *in* the gas due to the mutual attraction between the molecules, assumed proportional to the square of the density, but independent of the temperature. This internal molecular attraction assists the external pressure of the enclosure in preventing the fluid from expanding. The effect is to add a term to p of the form a/v^2 where a is assumed constant, although it is probably less the higher the temperature.

The other correction is for the size of the molecules, which reduces the free space and volume of the gas by an amount b per unit volume. This co-volume b represents the least volume the gas can occupy when the pressure is exceedingly high ; and by the equation, at absolute zero, when $T = 0$, we have $v = b$.

The equation of Van der Waals, plotted on squared paper, agrees approximately with the general form of the isothermals (Fig. 25) for carbonic acid in the liquid and vapour states obtained by Dr. Andrews, and shows the deviation of vapour from the ideal gas when suitable constants are chosen for each substance, but fails to represent with sufficient accuracy the exact quantitative results of these experiments, or the behaviour of a fluid in all states. There is discrepancy between the actually observed values of the saturation pressures and volumes and those calculated from this equation.

Callendar's Characteristic Equation gives the behaviour of any vapour, saturated or superheated, at low and moderate pressures and temperatures, and may be written in the form

$$v = \frac{RT}{P} - c + b, \text{ or } v = \frac{RT}{P} - (c - b) \qquad . \qquad . \quad (1)$$

where $v =$ volume in cubic feet per pound ; $P =$ pressure in pounds per square foot ; $T =$ absolute temperature in degrees Centigrade ; c is a function of the temperature only ; R and b are constants. This equation does not hold even approximately over an unlimited range, but it is of such a form that *simple thermodynamic* relations are readily deduced from it, between the various properties of steam, and give results which agree very closely with the values observed by experiment. Callendar has applied it to calculate his tables of the properties of dry steam for pressures up to 500 lb. per sq. in., such as are commonly required in modern steam engine practice. This is well below the *critical point* for water, which is about 374° C., and critical pressure 200 atmospheres.

For 1 lb. of the *ideal* perfect gas $PV = RT$, and the volume $V = \dfrac{RT}{P}$. The volume of any actual gas, having the same values of P, T, and R, is less than this ideal volume. The diminution of volume from that of the ideal gas is regarded as due to co-aggregation or pairing of the molecules, which reduces the volume. Callendar deduced the " co-aggregation volume," c, from the cooling effect in throttling, and showed that, for a vapour at low and moderate

pressure, it may be regarded as a function of the temperature of the form $c = c_1 \left(\dfrac{T_1}{T}\right)^n$, where c_1 is the value of c at $100°$ C., or $373 \cdot 1°$ C. absolute.

All the experiments on a fluid, in the state of gas or vapour, show that the defect $(c - b)$ of volume from the ideal gas volume $\dfrac{RT}{P}$ is a function of the temperature only at low pressures, and the increment of $\log_e p = (c - b)/\dfrac{RT}{P}$.

The numerical value of n depends on the nature of the fluid, and for steam $n = \dfrac{10}{3}$. The value of c agrees very closely with the results of experiments on the cooling effect in the porous plug by Joule and Thomson, and is mainly based on the more recent throttling experiments of Jenkin and Pye, Grindley, Peake, as well as those by Nicolson and Callendar. The cooling effect is the ratio of the drop of temperature for every pressure drop of 1 lb. per sq. in., $-\left(\dfrac{dt}{dp}\right)$ in a throttling process at constant total heat H.

The co-volume, b, is taken as the minimum volume of the fluid, that is, when reduced to the liquid state. For water, the co-volume of 1 lb. at $0°$ C. $= 0 \cdot 01602$ cub. ft.

Taking $n = \dfrac{10}{3}$ for steam, the constant $R = \dfrac{PV}{1400T} = 0 \cdot 11012$ mean calorie per lb., or $R = \dfrac{PV}{T}$ ft.-lb. per lb. of vapour,

$$RJ = 0 \cdot 11012 \times 1400 = 154 \cdot 168 \text{ ft.-lb.,}$$

and $c_1 = 0 \cdot 4213$ cub. ft. per lb at $100°$ C.; then the co-aggregation volume

$$c = 0 \cdot 4213 \left(\frac{373 \cdot 1}{T}\right)^{10} , \qquad \frac{157 \cdot 52 \times 10^6}{T^{\frac{10}{3}}}$$

Substituting these values, the Callendar equation for dry saturated and superheated steam is

$$v = \frac{154 \cdot 168 T}{P} - \frac{157 \cdot 52 \times 10^6}{T^{\frac{10}{3}}} + 0 \cdot 01602 \qquad . \qquad (2)$$

Example 1. Calculate the volume of 1 lb. of dry saturated steam at $170°$C. The saturation pressure is 115 lb. per sq. in.

We have $T = 170° + 273 \cdot 1° = 443 \cdot 1°$ C. absolute;
and $P = 115 \times 144$ lb. per sq. ft.

$$v = \frac{154 \cdot 168 \times 443 \cdot 1}{115 \times 144} - \frac{157 \cdot 52 \times 10^6}{443 \cdot 1^{\frac{10}{3}}} + 0 \cdot 016$$

$$= 4 \cdot 1251 - 0 \cdot 2375 + \cdot 016 = 3 \cdot 9036 \text{ cub. ft.}$$

which agrees closely with the value given in the tables.

Suppose this dry saturated steam is superheated 100° C. at constant pressure 115 lb. per sq. in. The temperature is now

$$T = 270° + 273·1° = 543·1° \text{ C. absolute},$$

and

$$v = \frac{154·168 \times 543·1}{115 \times 144} - \frac{157·52 \times 10^6}{543·1^{\frac{1}{3}}} + 0·016$$

$$= 5·0561 - 0·1205 + 0·016 = 4·95 \text{ cub ft.}$$

Again, in Example 1, starting with 1 lb. of dry saturated steam, suppose the temperature is kept constant 170° C., whilst the pressure is reduced from 115 lb. to 20 lb. per sq in. The term c remains the same, the volume per pound is found by the characteristic equation (2), or from the total heat, H, 673·35 C.H.U. by (3), to be 23·497 cub. ft., and the steam is superheated.

Example 2. Find the volume of 1 lb. of steam at 230° C. and 120 lb. per sq. in.

$T = 503·1°$ C. absolute; and the total heat $H = 695·95$ C.H.U. per lb.

Substituting values in equation (2), gives

$$v = \frac{154·168 \times 503·1}{120 \times 144} - \frac{157·52 \times 10^6}{503·1^{\frac{10}{3}}} + 0·016$$

$$= 4·4885 - 0·1555 + 0·016 = 4·349 \text{ cub ft.}$$

In this case the steam is superheated, since the saturation temperature for pressure 120 lb. per sq. in. is 171·75° C.

Professor Dalby derived from the characteristic equation and that of total heat (p. 414) a simple expression for the volume in cubic feet per pound of dry saturated or superheated steam —

$$v = \frac{2·2436(H - 464)}{p} + 0·0123 \qquad (3)$$

where $p =$ lb. per sq. in , and the corresponding value of H is given in the steam tables.

Thus, for the case of saturated steam at 115 lb. per sq. in. (as in Example 1), from tables the total heat $H = 663·44$, and

$$v = \frac{2·2436(663·44 - 464)}{115} + 0·0123$$

$$= 3·8910 + 0·0123 = 3·9033 \text{ cub. ft. per lb.}$$

The constant 0·0123 is negligible at low pressures, and up to 400 lb. per sq. in. is less than 1 per cent.

The volume in Example 2, works out by formula (3), to 4·349 cub. ft.

Supersaturation of Steam. C. T. R. Wilson showed by experiment* that water vapour, mixed with air free from dust and saturated at

* *Phil. Trans. R.S.*, 1897, p. 301

20° C., can be suddenly expanded until its pressure is *eight times the normal saturation pressure* corresponding to the temperature after expansion, and its temperature is much lower than that of saturation at the actual pressure reached, without condensation taking place. With expansion beyond this *limit*, or Wilson curve (p. 492), extremely rapid condensation begins, a dense cloud of thick fog being formed of very fine particles as centres or nuclei of condensation.

Callendar estimates the radius of each nucleus about 5×10^{-8} cm., of the same order of magnitude as the co-aggregated or paired molecules in water-vapour at 20° C.

Professors Callendar and Nicolson first pointed out the loss of heat-drop due to supersaturation in the rapid expansion of steam observed in their experiments * The adiabatic law of expansion : $P (v - b)^{1.3} =$ constant, still holds while the steam remains dry and supersaturated. The time taken to reach the throat of the nozzle is only about 0·0001 second, and it is probable that the steam flows through the nozzle in a supersaturated condition, while the condensation at the throat must be very small, seeing it is retarded by the surface tension of the small drops, and it takes place very rapidly after passing the throat.

From data obtained in trials of the performance of marine steam turbines, Callendar defined the *supersaturation limit at 3 per cent equivalent wetness of saturated steam*, from which the total heat, H, is more easily deduced than from the Wilson limit, and gives results in agreement with observations on both high and low pressure steam turbines

Specific Heats of Gas. The specific heat of any substance is the quantity of heat required to raise the temperature of unit weight (1 lb.) of it 1°. A gas has two specific heats, depending on whether it is kept at constant volume or at constant pressure while being heated.

By the specific heat of a gas at *constant volume*, C_v, is meant the quantity of heat (thermal units) taken in by unit weight (1 lb.) of the gas to raise the temperature 1°, while the volume of the gas is kept constant, and the pressure increases. All the heat goes to increase the stock of internal energy in the gas, since no external work is done by the gas

The specific heat of a gas at *constant pressure*, C_p, is the quantity of heat required to raise the temperature of 1 lb. weight of the gas 1°, while the pressure is kept constant and the gas expands under constant pressure, doing an amount of external work equal to the product of the pressure and increase of volume.

Difference of the Specific Heats. When the volume of the gas changes from v_1 to v_2 during expansion at constant pressure p, the

* " On the Law of Condensation of Steam," *Proc. Inst. C.E.*, 1898, Vol. CXXXI, p. 147 ; also *Proc. Roy. Soc.*, 1900, Vol. LXVII, p 266 ; and " Flow of Steam Through a Nozzle," *Inst. Mech. E.*, 1915, p. 53.

external work done by the gas is $p(v_2 - v_1)$. It follows that, for a perfect gas, the heat equivalent of the external work done by 1 lb. weight of gas during expansion while the temperature rises 1°, is

$$\frac{R}{J} = C_p - C_v$$

Take $K_v = J \cdot C_v$ ft -lb, and $K_p = J \cdot C_p$ ft.-lb., then $K_p - K_v = R$.

Since no internal work has to be done, nor heat required to overcome the molecular attraction of a perfect gas during expansion, any heat given to the gas must go either to raise the temperature of the gas and so increase its stock of internal energy, or to do external work, according to the fundamental energy equation—

Heat received = Increase of internal energy + External work done

$$Q = E + \frac{W}{J}$$

Let w lb weight of gas be heated from T_1 to T_2 absolute, at constant volume, the heat taken in is $wC_v(T_2 - T_1) =$ increase of internal energy.

Again, if the volume increases from v_1 to v_2, under constant pressure p, while the temperature is raised from T_1 to T_2, the heat taken in by the gas to raise its temperature is $w \cdot C_p(T_2 - T_1)$, and the external work done by the gas while it is being warmed is

$$W = wp(v_2 - v_1) = wR(T_2 - T_1) \text{ ft.-lb.}$$

since the perfect gas follows the law $pv = wRT$. Also, the change of internal energy is $w \cdot C_v(T_2 - T_1)$. Substituting these values in the energy equation, we have

$$w \cdot C_p(T_2 - T_1) = w \cdot C_v(T_2 - T_1) + \frac{wR}{J}(T_2 - T_1)$$

from which $C_p = C_v + \dfrac{R}{J}$, or $C_p - C_v = \dfrac{R}{J}$ heat units . . (2)

that is, $\qquad\qquad K_p - K_v = R \text{ ft.-lb.}$

Therefore the difference in heat units between the specific heat of the perfect gas at constant pressure and the specific heat at constant volume is equal to the constant R divided by J, Joule's equivalent.

Combining this result with the characteristic equation (1), (page 58) we have

$$\frac{pv}{T} = wR = w(K_p - K_v) = wJ(C_p - C_v) \text{ ft.-lb.} \tag{3}$$

This relation is only strictly true for a perfect gas, and very nearly holds for hydrogen, nitrogen, dry air, and other gases when above their critical temperatures of liquefaction and at moderate pressure .

The ratio of the specific heats $\dfrac{K_p}{K_v}$, denoted by γ, is a very important factor, and substituting this value in equation (2), gives

$$\gamma - 1 = \frac{R}{K_v}, \text{ or } K_t = \frac{R}{\gamma - 1}.$$

Example 3. Find the relation between the specific heats of a perfect gas. Taking the specific heat of air at K_p 183·4 ft.-lb and K_v 130·2 ft.-lb.; find the volume of 3 lb. air at a pressure of 40 lb. per sq in. absolute, and temperature 75° F.   (*U.L. . B.Sc. (Eng.*).)

We have $R = K_p - K_v = 183\cdot4 - 130\cdot2 = 53\cdot2$ ft.-lb per lb. ° F., and absolute temperature of the air $= 460 + 75° = 535°$ F. (abs.).

Now $pv = wRT$,

and $r = \dfrac{wRT}{p} = \dfrac{3 \times 53\cdot2 \times 535}{40 \times 144} = \underline{14\cdot824 \text{ cub. ft.}}$

Answer.

For dry air, Regnault found $C_p = 0\cdot2375$ calorie at 0° C., and 1 atmosphere. Then, deducting the heat equivalent of the external work done by unit weight of the air during expansion under constant atmospheric pressure while being warmed 1° C, namely ·0684 calorie, gives

$$C_t = \cdot2375 - \cdot0684 = 0\cdot1691 \text{ calorie}$$

Hen $\dfrac{C_p}{C_v} = \dfrac{\cdot2375}{\cdot1691} = 1\cdot405.$

To find the values of the specific heats K_p and K_t, in work units, foot-pounds, multiply C_p and C_t in heat units by the appropriate value of J.

In more recent determinations of C_p for air and other gases under constant pressure, Swann, by the continuous electric heating* method of Callendar, found the value for air at 0° C., 0·2413 ; and ·243 at 100° C. and atmospheric pressure   Taking R at 0·0686, the result of C_t for air at 0° C. is 0·1727 calorie.

Joly finds C_v for air at 0° C. and atmospheric pressure, by direct measurement,† 0·1729 calorie.

Joly, by means of his **differential steam calorimeter,** determined by direct measurement the specific heats of gases at *constant volume,* by the condensation of steam that heats the gases. Two exactly equal and similar globes or spherical copper vessels hang, in a steam chamber, from the arms of a balance in a case above. One vessel is empty and the other contains gas at a given pressure or density. The difference of weight, W, is that of the gas. The spherical shells, when empty, have equal thermal capacity and are both supported

* See *Brit. Assoc. Report*, 1908 ; *Phil. Trans.*, 1910, p. 199.
† *Phil. Trans. A.*, Vol 182 (1891), p. 73 , and A (1894), p. 943.

on small scale pans, or " catch waters," to collect the water from condensation on the surface of the vessels. After the temperature t° becomes stationary, steam is admitted to the chamber, and more condenses on the shell which contains the gas, and this excess of condensation, w lb. of condensate water, gives the heat wL required to raise the given weight W lb. of gas through the observed temperature, t° to t_1°, at constant volume. Hence $W(t_1 - t)C_v = wL$, or

$$C_v = \frac{wL}{W(t_1 - t)}, \text{ where } L \text{ is the latent heat of steam.}$$

Small corrections are needed for the expansion of the copper globe, due to rise of temperature and extension with increase of
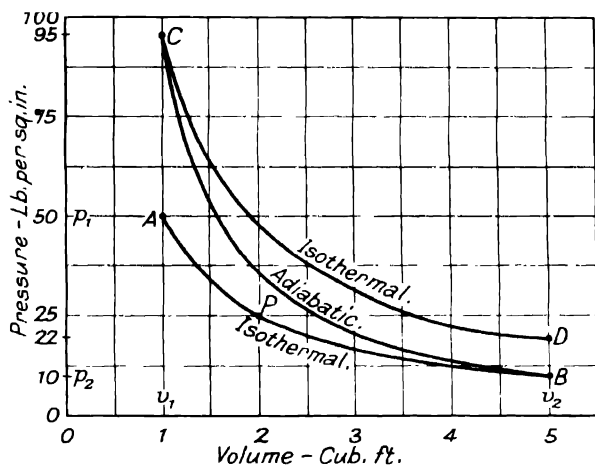
Fig 26. Isothermals and Adiabatics

internal pressure, also for the external work done by the gas in expanding this amount. The results of experiments by Dr. J. Joly show that the *specific heat of gases at constant volume is not constant, but increases with pressure and temperature.* For carbonic acid gas there was rapid increase of specific heat at higher pressure, as the temperature was reduced below 30° C.

Swann also found C_p for carbonic acid 0·202 calorie at 20° C., and 0·221 at 100° C., the mean value from 0° to 100° C. being about 0·21, and $\gamma = 1·3$. The corresponding values of K_p, expressed in *o ot-pounds per cubic foot of gas* are 27·4 at 20° C. and 30·7 at 100° C. (See page 367.)

Isothermal Expansion of a Perfect Gas. When heat is transformed into mechanical energy by the expansion of a perfect gas, all the work done is due to the pressure of the gas overcoming external resistance, and there is supposed to be no loss of mechanical energy from internal friction of the gas itself ; and in every operation the

expansion and compression are reversible, as explained at the outset, whether the substance receives or gives out heat.

When the *absolute temperature* is kept *constant*, the *isothermal curve*, APB, Fig 26, gives the relation of pressure and volume of gas during expansion from the initial state p_1, v_1, T_1, to the state p_2, v_2, T_1; the characteristic equation becomes $p_1v_1 = p_2v_2 = wRT_1 = k$ a constant.   (Boyle's law.)

The product of any pair of values of pressure and volume at state points A, P, and B, Fig. 26, on the curve is equal to 50.   Hence the curve is a rectangular hyperbola.

The same quantity of gas at a different constant temperature will have another similar isothermal curve, CD, and in this way we can draw any number of isothermals for the same mass of gas, by increasing its stock of internal energy.

The work done, W, *by* the gas during expansion, and *on* the gas during compression, is calculated by the equation (6) (p. 10), substituting for k the value wRT; or graphically by measuring the area under the curve to the volume axis, and bounded by the pressure ordinates at the initial and final states, A and B.

$$W = p_1v_1 \log_e \left(\frac{v_2}{v_1} \right), \text{ or } wRT_1 \log_e r \quad . \qquad . \qquad . \quad (4)$$

Since there is no change of temperature, the internal energy does not change, and the energy equation

$$Q = E + \frac{W}{J}, \text{ becomes } JQ = O + p_1v_1 \log_e r,$$

so that during the expansion of a perfect gas the heat received is converted into the equivalent work during the change of state.

The isothermal operation must be *performed very slowly* to allow time for the transfer of heat to or from the working substance, and for this heat flow some difference of temperature is actually required. Obviously, during isothermal compression the heat generated must be taken away continuously from the working substance, hence the necessity for the circulation of cooling water around the compression cylinder; but the temperature cannot be kept rigorously constant in practice, except during the condensation of a vapour in compression, or by the evaporation of the liquid during expansion.

Adiabatic Operations of a Perfect Gas.   Expansion or compression is adiabatic when the operation is reversible and the working substance neither gains nor loses heat by conduction and radiation, or by internal chemical action.   We must imagine a given quantity of perfect gas to be enclosed in a cylinder, fitted with a piston, both perfect non-conductors of heat, which neither take up any heat themselves nor allow heat to pass through them to or from the gas. An adiabatic operation cannot be quite realized because every

substance absorbs heat, more or less, and a cylinder cannot be perfectly heat-tight, or impervious to heat. An operation is nearly adiabatic when *performed quickly* to allow very little time for the transfer of heat between the gas and the surface of the cylinder walls.

In adiabatic expansion *CB*, Fig. 26, work is done by the given weight of gas solely at the expense of its own store of internal energy, hence the temperature and pressure fall below that of the isothermal *CD* for the same change in volume, since during isothermal expansion heat is supplied to keep the temperature constant. During adiabatic *compression*, *BC*, the work done on the gas generates heat, which increases its stock of internal energy since no heat is given out, and the temperature rises accordingly and gives rapid increase of pressure, whereas, during isothermal compression between the same volumes, *B* to *A*, the heat produced must be taken away in order to keep the temperature constant, therefore the isothermal compression curve must be below the adiabatic.

Hence, *the adiabatic curve has a steeper slope than the isothermal, during both expansion and compression.*

Let dQ denote a small quantity of heat taken in by 1 lb. weight of a perfect gas, and dT and dv the resulting small increments of temperature and volume, then the fundamental energy equation.
Heat received = Gain of internal energy + External work done, becomes $dQ = C_v \cdot dT + p \cdot dv$.

Since in adiabatic expansion or compression no heat is taken in or given out by the gas, $dQ = 0$, the *work done is equivalent to the change of internal energy*, and we have $C_v \, dT + p \cdot dv = 0$

Differentiate the characteristic equation of a perfect gas $pv = RT$, and we obtain $p \, dv + v \, dp = R \, dT$, or $dT = (p \cdot dv + v \cdot dp)/R$.
Substitute this value of dT in the above equation, and taking $R = C_p - C_v$ we obtain

$$C_p \cdot p \cdot dv + C_v \cdot v \, dp = 0, \text{ and } \frac{C_p}{C_v} = \gamma$$

from which $$\gamma \cdot \frac{dv}{v} + \frac{dp}{p} = 0$$

and, by integration, $\gamma \cdot \log_e v + \log_e p = $ a constant.

Therefore, $p \cdot v^\gamma = $ a constant . . . . . (5)

the **equation to the adiabatic curve** of a perfect gas, giving the relation between the absolute pressure and volume during expansion or compression.

The value of the constant can be found for a given state of the working substance for any pair of values of p and v on the adiabatic curve.

The work area during an adiabatic change of state can now be found by substituting γ for n in the general equations (1) to (5) (p. 10).

We have $pv = wRT$, and $p_1v_1 - p_2v_2 = wR(T_1 - T_2)$
and, for adiabatic expansion,

$$W = \frac{p_1v_1 - p_2v_2}{\gamma - 1}, \text{ becomes } W = \frac{wR(T_1 - T_2)}{\gamma - 1} \qquad . \qquad (6)$$

or $$W = \frac{p_1v_1}{\gamma - 1}\left(1 - \frac{T_2}{T_1}\right) - \frac{p_1v_1}{\gamma - 1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right\} \qquad . \qquad (7)$$

Then $\frac{W}{J}$, represents the reduction of the internal energy of the gas or the amount converted into mechanical energy during expansion from the state p_1, v_1, T_1, to the state p_2, v_2, T_2.

The internal energy of 1 lb. weight of the gas is gradually spent in doing work on the piston as the adiabatic curve is indefinitely prolonged to cut the volume axis at infinity and the pressure falls to zero, when the gas can do no more external work and its stock of energy is used up. If $p_2 = 0$, the equations representing the work area under the adiabatic curve become

$$\text{Work done} = \frac{p_1v_1}{\gamma - 1} = \frac{RT_1}{\gamma - 1} = K_vT_1 \text{ ft.-lb.} \qquad . \qquad . \qquad (8)$$

which may be taken as the mechanical equivalent of the *internal energy* of 1 lb weight of gas in the initial state at absolute temperature T_1.

Change of Temperature of a Perfect Gas during Adiabatic Expansion and Compression. During an adiabatic operation from the state p_1, v_1, T_1, to that of p_2, v_2, T_2, the relation between pressure and volume is

$$p_1 \cdot v_1{}^\gamma = p_2 \cdot v_2{}^\gamma, \text{ where } \gamma \text{ denotes the ratio } \frac{C_p}{C_v} \qquad . \qquad (a)$$

and, for a perfect gas, the characteristic equation is

$$\frac{p_1 \cdot v_1}{T_1} = \frac{p_2v_2}{T_2} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (b)$$

To eliminate p, divide (a) by (b) and we obtain

$$T_1 \cdot v_1{}^{\gamma-1} = T_2 \cdot v_2{}^{\gamma-1}, \text{ or } \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \qquad . \qquad . \qquad (9)$$

where $\frac{v_2}{v_1}$ is the ratio of expansion.

Again, to eliminate v, raise (b) to the γ power, and divide by (a),

gives $$\frac{p_1{}^{\gamma-1}}{T_1{}^\gamma} = \frac{p_2{}^{\gamma-1}}{T_2{}^\gamma}, \text{ or } \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

and the temperature may be calculated from a given pressure

change. Hence, the adiabatic law for a perfect gas may also be written—

$$T \cdot v^{\gamma-1} = \text{constant, and } \frac{T}{p^{\frac{\gamma-1}{\gamma}}} = \text{constant.} \qquad . \qquad . \quad (10)$$

In the application of the energy law to a perfect gas—

(a) In isothermal expansion $pv = $ constant, when T is constant $dQ = dE + p \cdot dv$; all the heat given to the gas goes to do external work in expansion, and the internal energy remains constant; $dE = 0$, and $dQ = p \cdot dv$.

(b) Adiabatic expansion, $dQ = 0$, and $\therefore p \cdot dv = -dE$. In this case, external work is done at the expense of the internal energy of the gas.

(c) Heating gas at constant pressure : Internal energy increases and external work is done by the heat $\int_1^2 dQ = \int_1^2 dE + \int_1^2 p \cdot dv$, $\therefore$ heat given to the gas, $Q = E_2 - E_1 + p(v_2 - v_1)$.

(d) Heating at constant volume, $\therefore p \cdot dv = 0$, and $dQ = dE$. All the heat goes to increase the internal energy of the gas.

Example 4.   The temperature of the mixture of gas and air in a gas engine at the end of the admission stroke is 90° F., and the pressure 15 lb. per sq. in. absolute.   The clearance volume is 4·6 cub. ft., and the total volume of clearance plus piston displacement is 12 cub. ft.   Assuming adiabatic compression $p v^{1·4}$ constant. determine the temperature at the end of the compression stroke.

If the pressure after ignition is 240 lb. per sq. in., find the temperature in the cylinder.   (U.L . B.Sc. (Eng.).)

The student should draw a compression curve above the atmospheric line and a vertical to represent increase of pressure at constant volume during explosion after compression and ignition, like the lower curve and constant volume line in Fig. 2 (p. 7).

Here the temperature before compression is

$$T_1 = 90° + 460° = 550° \text{ F. (absolute).}$$

Let $T_2 = $ absolute temperature at end of adiabatic compression

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{1·4-1} \text{ and } T_2 = 550\left(\frac{12}{4·6}\right)^{0·4}$$

taking logarithms

$$\log T_2 = \log 550 + 0·4 (\log 12 - \log 4·6)$$

$$= 2·7404 + 0·4(1·0792 - 0·6628) = 2·907$$

hence, $T_2 = 807° \text{ F. (abs.), or } 807° - 460° = 347° \text{ F.}$

Again, since the *volume* is constant during explosion, by the characteristic equation, the temperature is proportional to pressure.

Now, $p_2 \times 4 \cdot 6^{1 \cdot 4} = 15 \times 12^{1 \cdot 4}$

that is, $p_2 = 15 \left(\dfrac{12}{4 \cdot 6}\right)^{1 \cdot 4} = 57 \cdot 42$ lb. per sq. in.

and, at constant volume,

$$\frac{T}{807} = \frac{240}{57 \cdot 42}, \text{ or } T = \frac{807 \times 240}{57 \cdot 42} = 3373° \text{ F. (abs.)}$$

∴ temperature of the charge at 240 lb. per sq. in. is

$$3373° - 460° = \underline{2913°} \text{ F.} \qquad\qquad Answer.$$

Example 5. If 0·1 lb. of gas occupying 0·5 cub. ft. is expanded in a cylinder at constant pressure of 150 lb. per sq. in. absolute until its volume is 1 cub. ft., and is then expanded adiabatically to 5 cub. ft., find the temperature of the gas (*a*) at the end of the constant pressure stage, (*b*) at the end of the adiabatic expansion ; and calculate the heat expended and the work done during each portion of the process.   Take $K_p = 198$ ft.-lb. and $K_v = 144$ ft.-lb. (*U.L., B.Sc. (Eng.).*)

Given, $wR = w(K_p - K_v) = \dfrac{1}{10}(198 - 144) = 5 \cdot 4$ ft.-lb.

also $pv = wRT$, and initial temperature $T_1 = \dfrac{pv}{wR}$.

Substituting values,

$$T_1 = \frac{(144 \times 150)0 \cdot 5}{5 \cdot 4} = 2000° \text{ F. (abs.) o } 1540° \text{ F}$$

(*a*) At constant pressure,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}, \text{ that is, } \frac{T_2}{T_1} = \frac{v_2}{v_1} = \frac{1}{0 \cdot 5} = 2,$$

therefore, at the end of the constant pressure stage,

$$T_2 = 2T_1 = 4000° \text{ F. (abs.), or } 3540° \text{ F.}$$

Heat energy received by the gas during expansion at constant pressure is

$$w \cdot K_p(T_2 - T_1) = \frac{1}{10} \times 198(4000° - 2000°) = 39{,}600 \text{ ft.-lb.,}$$

Equivalent to $39{,}600/778 = 50 \cdot 9$ B.Th.U.

The work done at constant pressure

$$= p(v_2 - v_1) = (144 \times 150) \times \frac{1}{2}$$
$$= \underline{10{,}800} \text{ ft.-lb.} \qquad\qquad Answer$$

(*b*) For adiabatic expansion,

$$\gamma = \frac{198}{144} = \frac{11}{8},$$

and $\dfrac{T_3}{T_2} = \left(\dfrac{v_2}{v_3}\right)^{\gamma - 1} = \left(\dfrac{1}{5}\right)^{\frac{3}{8}}$; taking logarithms

$$\log T_3 = \log 4000 - \tfrac{3}{8} \log 5 = 3 \cdot 3400,$$

we obtain, $T_3 = 2188°$ F. (abs.), or $1728°$ F.

The gas neither receives nor gives out heat during an adiabatic change, therefore the work done *by* the gas is equal to its loss of internal energy due to change of temperature, which is $w.K_v(T_3 - T_2)$, namely,

$$\frac{1}{10} \times 144(4000 - 2188) = 26,093 \text{ ft.-lb.},$$

Equivalent to $26,093/778 = 33.54$ B.Th.U.

To check this result, for the adiabatic curve, $p_3 \cdot v_3{}^\gamma = p_2 \cdot v_2{}^\gamma$,

that is, $\dfrac{p_3}{p_2} = \left(\dfrac{v_2}{v_3}\right)^\gamma = \left(\dfrac{1}{5}\right)^{\frac{11}{8}}$, taking logarithms,

and $\log p_3 = \log 150 - \dfrac{11}{8}\log 5 = 1.2150$.

We have the pressure $p_3 = 16.41$ lb. per sq in., and the work done during expansion.

$$W = \frac{p_2 v_2 - p_3 v_3}{\gamma - 1},$$

is obtained, $W = \dfrac{144(150 \times 1 - 16.41 \times 5)}{\frac{3}{8}}$.

$$= 384 \times 67.95, \text{ or } \underline{26,093 \text{ ft -lb}} \qquad \textit{Answer}.$$

A pressure volume diagram, drawn to scale, would show these changes clearly.

Example 6. Four cub. ft. of gas at 20 C. and 14.7 lb. per sq. in. is compressed adiabatically to one-fifth of this volume. Determine (*a*) the weight of the gas; (*b*) the pressure and temperature at the end of the adiabatic compression; (*c*) the work done *on* the gas; (*d*) the heat given out while the gas is cooled to 20 C., at the reduced constant volume; and (*e*) the pressure after cooling. The specific heat at constant pressure is 0.2404, and at constant volume 0.1718.

We have $\gamma = \dfrac{.2404}{.1718} = 1.4$;

and the gas constant,

$$R = (C_p - C_v) J = (.2404 - .1718)\, 1400 = 96 \text{ ft.-lb.}$$

(*a*) Then the weight of the gas is found by the characteristic equation

$$w = \frac{p_1 v_1}{RT_1} = \frac{(144 \times 14.7)4}{96 \times 293} = 0.301 \text{ lb}$$

(*b*) By the adiabatic law, $p_2 \cdot v_2{}^\gamma = p_1 \cdot v_1{}^\gamma$, or $p_2 = p_1\left(\dfrac{v_1}{v_2}\right)^\gamma$, and the compression ratio $\dfrac{v_1}{v_2} = 5$; substituting the given values

$$p_2 = 14.7 \times 5^{1.4}, \text{ and } \log p_2 = \log 14.7 + 1.4 \log 5$$
$$= 2.1459,$$

we obtain the pressure at end of compression,

$$p_2 = 139.93 \text{ lb. per sq. in.}$$

Again, the absolute temperature, $T_2 \cdot v_2^{\gamma-1} = T_1 \cdot v_1^{\gamma-1}$

or $\qquad T_2 = T_1 \cdot \left(\dfrac{v_1}{v_2}\right)^{\gamma-1}$

which becomes,

$$T_2 = 293 \times 5^{0\cdot4},$$

and $\qquad \log T_2 = \log 293 + 0\cdot4 \log 5 = 2\cdot7465,$

and $\qquad T_2 = 557\cdot8° \text{ C. (absolute), or } 284\cdot8° \text{ C.}$

Check this value by the relation

$$\frac{T_2}{T_1} = \left(\frac{139\cdot93}{14\cdot7}\right)^{\,\,\,\,\,\,\,}$$

(c) Work done *on* the gas during adiabatic compression is

$$\frac{p_2 \cdot v_2 - p_1 \cdot v_1}{\gamma - 1} = \frac{144(139\cdot93 \times 0\cdot8 - 14\cdot7 \times 4)}{0\cdot4}$$

$$= 360 \times 53\cdot144 = 19{,}132 \text{ ft.-lb.}$$

The equivalent of this work is the heat energy given to the gas

$$= \frac{19{,}132}{1400} = 13\cdot67 \text{ C.H.U.},$$

since, by the energy equation, $Q = E + \dfrac{W}{J}$.

(d) Now in cooling 0·301 lb. weight of gas at *constant volume*, from 284·8° C. to 20° C., the heat given out by the gas

$$= 0\cdot301 \times 264\cdot8° \times 0\cdot1718, \text{ or } 13\cdot69 \text{ C.H.U.}$$

(e) Also the pressure drops to that on the isothermal curve through the initial state point, and the internal energy of the gas is the same as at the outset, while the pressure is reduced to that on the isothermal, which, by Boyle's law, is 14·7 × 5, or 73·5 lb. per sq. in. *Answer.*

If a pv diagram were drawn similar to BCA, Fig. 26, but to a suitable scale, the point B would represent the initial state of the gas as regards pressure and volume. During compression without gain or loss of heat, the change of pressure and volume would be shown by the *adiabatic curve BC*. Heat is rejected in the constant volume change CA, and no work is done. During the isothermal expansion, as the state point moves from A to B along the isothermal curve, the gas receives heat at the rate required to keep the temperature constant, while the gas does work represented by the area under AB. The difference between the work area under BC done *on* the gas, and that done *by* the gas under AB, is BCA. When the state point moves round the closed area in the direction $BCAB$,

opposite to the hands of a clock, on the whole cycle, work is converted into heat.

Example 7. One pound of air at 354° F. (178·9° C.) expands adiabatically to three times its original volume, and in the process falls in temperature to 60° F. (15·6° C.). The work done during the expansion is 38,410 ft.-lb. Calculate the two specific heats.   (*U.L., B.Sc.* (*Eng.*).)

$$T_1 = 178·9 + 273 = 451·9° \text{ C (abs.),}$$

and $$T_2 = 15·6 + 273 = 288·6° \text{ C. (abs.).}$$

Substitute given values in equation (9) (p 76), $\dfrac{451·9}{288·6} = 3^{\gamma-1}$

Take logarithms $(\gamma - 1) \log 3 = 0·19474$,

and $\gamma \log 3 - \log 3 = 0·19474$,

that is, $\gamma \times 0·47712 = 0·19474 + 0·47712$,

from which,

$$\frac{C_p}{C_v} = \gamma = \frac{·67186}{·47712} = 1·408, \text{ and } C_p = 1·408 \, C_v.$$

Again, by equation (6),

$$W = \frac{wR(T_1 - T_2)}{\gamma - 1}, \text{ where } wR = wJ(C_p - C_v),$$

substitute values,

$$38,410 = \frac{1400(C_p - C_v)(451·9 - 288·6)}{\frac{C_p}{C_v} - 1} \text{ ft.-lb },$$

and, $\gamma - 1 = \dfrac{C_p - C_v}{C_v}$, the equation reduces to

$$38,410 = 1400 C_v \times 163·3, \text{ and } C_v = \frac{38,410}{1400 \times 163·3} = 0·168.$$

$$C_p = 1·408 \times 0·168 = 0·23655. \qquad\qquad Answer.$$

To check calculations, substitute the values obtained, as above, for the work done

$$C_p - C_v = 0·23655 - 0·168 = 0·06855,$$

and $$\gamma = \frac{·23655}{·168} = 1·408$$

also $$W = \frac{wR(T_1 - T_2)}{\gamma - 1} = \frac{1 \times 1400 \times 0·06855 \times 163·3}{1·408 - 1}$$

$$= 38,410 \text{ ft.-lb.}$$

Example 8. The characteristic constant for hydrogen is 1382 ft.-lb. units, and its specific heat at constant pressure is 3·41 ; 3 cub. ft. of hydrogen, measured at 15 lb. per sq. in. and 18° C., are compressed adiabatically to 200 lb. per sq. in. and then expanded isothermally to the original volume of 3 cub. ft. Determine the final pressure of the gas. Calculate the amount of

heat which must be added to the gas during isothermal expansion, and also the heat which must be abstracted from the gas after expansion in order to reduce it to its initial state of pressure. (*U.L., B.Sc. (Eng.), 1923*).

Let R represent the gas constant, then the difference of the specific heats is

$$C_p - C_i = \frac{R}{J}, \text{ that is, } C_i = 3\cdot41 - \frac{1382}{1400} = 2\cdot423$$

Then, $\gamma = \frac{C_p}{C_i} = \frac{3\cdot41}{2\cdot423} = 1\cdot4074$

Fig. 27

The adiabatic law is

$$p_1 \cdot r_1{}^\gamma = p_2 \cdot r_2{}^\gamma, \text{ or, } \frac{r_1}{r_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{200}{15}\right)^{\frac{1}{1\cdot4074}}$$

Taking logarithms.

$$\log\left(\frac{r_1}{r_2}\right) = \frac{1}{1\cdot4074}(\log 200 - \log 15) = \frac{1\cdot125}{1\cdot4074} = 0\cdot7993.$$

Hence the compression ratio $r = \frac{r_1}{r_2} = 6\cdot3$; and the volume at end of the adiabatic compression, at state point B, on the sketch diagram (Fig. 27),

$$r_2 = \frac{3}{6\cdot3} = 0\cdot4762 \text{ cub. ft}$$

During isothermal expansion, the state point moves along the curve BC.

Let p_1, p_2 and p_3 be the pressures at A, B and C (Fig. 27).

At B and C, on the isothermal,

$$p_2 \cdot r_2 = p_3 \cdot r_1, \text{ and } p_3 = p_2\left(\frac{r_2}{r_1}\right)$$

Hence, $p_3 = \frac{200}{6\cdot3} = 31\cdot75$ lb. per sq. in. at C.

By Joule's energy law, the internal energy of a perfect gas depends only on its temperature, and (p. 56) the deviation by volume of hydrogen is practically negligible : it follows that the internal energy of hydrogen is constant at every point on the expansion curve *BC*, Fig. 27. Therefore, the amount of heat received by the gas, to keep its temperature constant, is equivalent to the work done by the gas during expansion, and represented by the work area *BCNF*. The work done during expansion is

$$W = p_2 r_2 \times 2 \cdot 3026 \log_{10} r$$

$$= 144 \times 200 \times 0 \cdot 4762 \times 2 \cdot 3026 \log 6 \cdot 3$$

$$= 25,240 \text{ ft.-lb}$$

which is equivalent to

$$\frac{W}{J} = \frac{25,240}{1400} = \underline{18 \cdot 03} \text{ C H U} \qquad \text{\textit{Answer.}}$$

Hence, also, the heat energy to be extracted from the gas at constant volume, *C.1.* to reduce the gas to its initial state must be the equivalent of the work done on the gas during adiabatic compression, *AB*, and represented by the area *ABFN*, equal to

$$\frac{p_2 r_2 - p_1 r_1}{\gamma - 1} = \frac{144(200 \times 0 \cdot 4762 - 15 \cdot 3)}{1 \cdot 4074 - 1} = 17,760 \text{ ft.-lb.}$$

which is equivalent to

$$\frac{17,760}{1400} = \underline{12 \cdot 68} \text{ C.H.U.} \qquad \text{\textit{Answer.}}$$

The cycle *ABC* is closed, and the gas returned to its initial state.

The *difference* between the isothermal work done *by* the gas, and the work done *on* it, which is $18 \cdot 03 - 12 \cdot 68$ C.H.U $= 5 \cdot 35$ C.H.U., the heat converted into work, is represented by the area of the indicator diagram *ABC*.

It is to be noted, when the state point moves round a closed curve in the direction of the hands of a clock, the gas, on the whole, receives more heat than it rejects, and the difference is converted into work

By the principle of the conservation of energy, and Joule's first law of the equivalence of heat and work —

Heat received = Work done by the gas + Heat rejected,

becomes, in this case, $18 \cdot 03$ C.H.U. $= 5 \cdot 35 + 12 \cdot 68$ C H.U.

Further, the temperature in the initial state, *A*, is

$$T_1 = 18^\circ + 273 = 291 \text{ C (abs.),}$$

and we may calculate the temperature at B, due to the adiabatic compression,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\gamma-1} = \left(\frac{200}{15}\right)^{0.289,}$$

since $\dfrac{\gamma-1}{\gamma} = \dfrac{1.4074-1}{1.4074} = 0.2895,$

from which, $T_2 = 616°$ C. (abs.), that is, $343°$ C.

The same result is obtained by taking the adiabatic change of volume, thus,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = 6.3^{0.4074}, \text{ and } T_2 = 616° \text{ C. (abs.).}$$

As a check on the above calculations, estimate the quantity of heat given out by the hydrogen in cooling at constant volume, from $343°$ C. at the state C, to $18°$ C., the initial state at A. The temperature drop is $325°$ C. From the hydrogen gas constant, R, given equal to 1,382 ft.-lb. per lb., we may calculate the density or weight of hydrogen in pounds per cubic foot at $0°$ C. and 14·7 lb. per sq. in., by the characteristic equation $pv = wRT$, which gives 0·00561 lb. per cub. ft. By the same equation, reduce the 3 cub. ft. measured at 15 lb. per sq. in. and $18°$ C. to its volume at $0°$ C. and 14·7 lb. per sq. in., which comes to 2·873 cub. ft., and so find its weight $2·873 \times 0·00561$, or 0·01611 lb. The weight is also obtained directly from the characteristic—

$$w = \frac{pv}{RT} = \frac{15 \times 144 \times 3}{1382 \times 291} = 0·01611 \text{ lb.}$$

The heat given out by this weight of gas in cooling at constant volume is

$$w \times C_v \times \text{drop of temperature} = 0·01611 \times 2·423 \times 325°$$
$$= 12·68 \text{ C.H.U.}$$

The pressure at C is also determined from the temperature, since the volume is constant during cooling. The relation is

$$\frac{p_1}{T_1} = \frac{p_3}{T'}, \text{ that is, } \frac{p_3}{p_1} = \frac{T_3}{T_1}, \text{ and } p_3 = p_1 \times \frac{T_3}{T_1}$$

from which $p_3 = 15 \times \dfrac{616°}{291°} = 31·75$ lb. per sq. in.

This method is important in the determination of extremely high and quickly varying temperature in the gas engine cylinder, from measurements of pressure and volume on the indicator diagram.

Rate of Heat Reception or Rejection by a gas in the engine cylinder. Assume the specific heats of the gas constant. Given a pv indicator diagram traced to scale, without knowing the temperature.

Let dp be the infinitely small change of pressure while the piston moves through a very short length of stroke and sweeps out the infinitely small change of volume dv. Then $\dfrac{dp}{dv}$ is the rate of change of pressure per unit change of volume, at the point corresponding to the mean value of dv. If dp gets less, this is negative, as in expansion; if dv is negative, the volume is decreasing, as in compression, and a positive value of $\dfrac{dQ}{dv}$ is the *rate of rejection of heat* by the gas to the cylinder walls.

Let dQ be the small quantity of heat given to the gas during the small change of pressure and volume, and dT the corresponding change in temperature, then $\dfrac{dQ}{dv}$ will represent the rate at which the expanding gas receives heat *per unit change of volume.* Also the *rate of heat reception per second,* during the short interval of time dt is

$$\frac{dQ}{dt} = \frac{dQ}{dv} \cdot \frac{dv}{dt} = \frac{dQ}{dv} \times \text{ volume swept by the piston per sec.}$$

Now a pound weight of perfect gas follows the law

$$pv = RT, \text{ or } T = \frac{pv}{R},$$

by differentiating, we have

$$\frac{dT}{dv} = \frac{1}{R}\left(p + v\,\frac{dp}{dv}\right) \quad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (11)$$

Again, when heat dQ is given to a pound of gas for an infinitely small change of state, the energy equation is

$$dQ = p \cdot dv + C_v \cdot dT, \text{ divide by } dv,$$

and

$$\frac{dQ}{dv} = p + C_v \frac{dT}{dv}$$

Substituting the value $C_v = \dfrac{R}{\gamma - 1}$, and the value of $\dfrac{dT}{dv}$ from (11),

gives

$$\frac{dQ}{dv} = p + \frac{R}{\gamma - 1} \times \frac{1}{R}\left(p + v \cdot \frac{dp}{dv}\right)$$

from which,

$$\frac{dQ}{dv} = \frac{1}{\gamma - 1}\left(\gamma p + v \cdot \frac{dp}{dv}\right) \qquad\qquad (12)$$

the rate of reception of heat by the gas per unit change of volume.

If γ is known for the gas, and the values of p and v, given on the indicator diagram, the rate of gain of heat by the gas may be calculated.

Since the expansion or compression curve follows the law,

$$p \cdot v^n = k, \text{ a constant, by differentiation, we obtain}$$

$$v^n \cdot \frac{dp}{dv} + np \cdot v^{n-1} = 0,$$

hence $\dfrac{dp}{dv} = -\dfrac{np}{v}$, or $v \cdot \dfrac{dp}{dv} = -np$,

and this value reduces equation (12) to the form

$$\frac{dQ}{dv} = \frac{1}{\gamma - 1} \, (\gamma p - np) = \frac{\gamma - n}{\gamma - 1} \cdot p \qquad . \qquad . \qquad . \quad (13)$$

or, $dQ = \dfrac{\gamma - n}{\gamma - 1} \, p \cdot dv = \dfrac{\gamma - n}{\gamma - 1} \, dW$

That is, for a very small addition of heat, the increase in the internal energy of gas is $\dfrac{\gamma - n}{\gamma - 1}$, work done on the gas; and for adiabatic expansion or compression $n = \gamma$, and $\dfrac{dQ}{dv} = 0$, which is obvious.

Alternative. To find the total heat, Q, received or rejected by 1 lb. weight of gas during an operation in which the change of state is from p_1, v_1, T_1, to p_2, v_2, T_2, by expansion or compression, according to the law $p \cdot v^n = $ a constant

Let $Q = $ external work done + change of internal energy For a very small change,

$$dQ = d \cdot W + dE$$
$$= p \cdot dv + C_v \cdot dT$$

The total heat change is

$$Q = \int_{v_1}^{v_2} p \ dv + C_v \int_{T_1}^{T_2} dT,$$

and becomes

$$Q = \frac{p_1 v_1 - p_2 v_2}{n - 1} + C_v \, (T_2 - T_1)$$

For a perfect gas.

$$C_v = \frac{R}{\gamma - 1}; \text{ and } T_1 = \frac{p_1 v_1}{R},$$

so that $T_2 - T_1 = \dfrac{1}{R} \, (p_2 v_2 - p_1 v_1)$.

Substituting, we have

$$Q = \frac{p_1 v_1 - p_2 v_2}{n - 1} + \frac{R}{\gamma - 1} \times \frac{1}{R} (p_2 v_2 - p_1 v_1).$$
$$= (p_1 v_1 - p_2 v_2)\left(\frac{1}{n - 1} - \frac{1}{\gamma - 1}\right),$$

hence $Q = \dfrac{p_1 v_1 - p_2 v_2}{n-1} \times \dfrac{\gamma - n}{\gamma - 1}$ (14)

or $Q = \dfrac{\gamma - n}{\gamma - 1} \times$ work done during the change.

For an infinitely small change, we obtain as above (13)

$$dQ = \frac{\gamma - n}{\gamma - 1} \times p \cdot dv, \text{ or } \frac{dQ}{dv} = \frac{\gamma - n}{\gamma - 1} \cdot p.$$

The latter expression gives the rate of heat reception or rejection per unit change of volume at any point on a pv diagram. Thus, for part of the expansion curve *after* the steep slope of the explosion peak of a gas engine indicator diagram, by plotting log p and log v, the value of n is obtained (p. 8).

The rate of heat reception *per second*

$$\frac{dQ}{dt} = \frac{dQ}{dv} \times \frac{dv}{dt}$$

becomes $\dfrac{dQ}{dt} = \dfrac{\gamma - n}{\gamma - 1} p \times$ volume swept by the piston per second.

Example 9. Suppose at part of the expansion curve of an indicator diagram from a gas engine the law is found to be $p v^{1 \cdot 3} =$ a constant, and the ratio of the specific heats of the mixture is $\gamma = 1 \cdot 37$, then by (13),

$$\frac{dQ}{dv} = \frac{1 \cdot 37 - 1 \cdot 3}{1 \cdot 37 - 1} \cdot p = \frac{7}{37} p$$

which is positive, and therefore the gas is receiving heat. Now, if the piston sweeps out 72 cub. ft. per min. when the absolute pressure on this expansion curve is 200 lb. per sq. in., what is the rate of heat reception at this instant ?

Here the change of volume per second is $\dfrac{72}{60} = 1 \cdot 2$ cub. ft. per sec. when $p = 200 \times 144$ lb. per sq. ft.

Hence $\dfrac{dQ}{dt} = \dfrac{\gamma - n}{\gamma - 1} p \cdot \dfrac{dv}{dt} = \dfrac{\cdot 37 - 1 \cdot 3}{1 \cdot 37 - 1} (200 \times 144) 1 \cdot 2$ ft.-lb.
per second

$$= \frac{7}{37} \times \frac{240 \times 144}{1400}, \text{ or } 4 \cdot 67 \text{ C.H.U per second.}$$

The relative slope of the isothermal and adiabatic lines affords guidance in each case.

If the index, n, for the **expansion curve** is less than γ, the slope of the curve is not so steep as the adiabatic, and the gas is receiving heat, $\dfrac{dQ}{dv}$ is positive ; but if n is greater than γ, the expansion curve has a steeper slope than the adiabatic curve, and the gas is losing heat, $\dfrac{dQ}{dv}$ is negative.

When $n = \gamma$, $\dfrac{dQ}{dv} = 0$, the curve is adiabatic, the gas neither gains

nor loses heat, and it does the work of expansion at the expense of its stock of internal energy. When $n = 1$, the expansion is isothermal, and the heat received is equal to the equivalent of the work done by the gas, while its internal energy remains the same. (See Fig. 26.) Operation from C. Whereas, during isothermal *compression*, the rate of heat *rejection* will be equivalent to the rate at which work is being done *upon* the gas.

Also when n for the **compression curve** is less than γ, heat is taken from the gas ; and if n is greater than γ, the heat *rejection* will be negative, and the gas will be receiving heat during its compression.

For example, if the law of the *compression curve* is $pv^{1 \cdot 25} = $ a constant, and $\gamma = 1 \cdot 4$ for the gas ; from which $\dfrac{dQ}{dv} = 0 \cdot 375p$. This is the rate at which the gas is giving out heat per unit decrease of volume.

Example 10.   An internal combustion engine has the following dimensions : diameter of cylinder, 22 in. ; stroke, 30 in ; compression ratio, 13·5. At the end of the suction stroke the pressure is 14 lb. per sq. in. and the temperature is 43° C.   Compression follows the law $pv^{1 \cdot 37} = C$.   Determine (a) the pressure and temperature at the end of compression ; (b) the weight of the charge ; (c) the work done ; and (d) the heat rejected during compression. Assume the specific heat at constant pressure to be 0·238 and at constant volume to be 0·169.   (*U.L., B.Sc. (Eng.)*, 1921.)

Given $\gamma = \dfrac{\cdot 238}{\cdot 169} = 1 \cdot 408$;

and $R = (C_p - C_v) J = (\cdot 238 - \cdot 169)\, 1400 = 96 \cdot 6$ ft.-lb.

The sectional area of cylinder $= \pi r^2 = 380 \cdot 1$ sq. in., and stroke volume $= \dfrac{380 \cdot 1}{144} \times \dfrac{30}{12} = 6 \cdot 6$ cub. ft. ; also clearance volume from compression ratio $= \dfrac{6 \cdot 6}{12 \cdot 5} = \cdot 528$ cub. ft., hence total volume of cylinder is $7 \cdot 128$ cub. ft. $=$ volume of charge.

$$T_1 = 273 + 43° \mathrm{C} = 316° \mathrm{C}. \text{ (absolute).}$$

(*b*) Weight of the charge

$$w = \frac{p_1 v_1}{R T_1} = \frac{(144 \times 14)\, 7 \cdot 128}{96 \cdot 6 \times 316} = 0 \cdot 4708 \text{ lb.}$$

(*a*) During compression,

$$p_1 \cdot v_1^{1 \cdot 37} = p_2 \cdot v_2^{1 \cdot 37}, \text{ or } p_2 = p_1 \left(\frac{v_1}{v_2}\right) = 14\,(13 \cdot 5)^{1 \cdot 37}$$

Taking logarithms,

$$\log p_2 = \log 14 + 1 \cdot 37 \log 13 \cdot 5 = 2 \cdot 69468,$$

hence $p_2 = 495 \cdot 1$ lb. per sq. in.

also $T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{1 \cdot 37 - 1} = 316 \times 13 \cdot 5^{0 \cdot 37}$

$$= 827 \cdot 77° \mathrm{C}. \text{ (absolute), or } 554 \cdot 8° \mathrm{C}.$$

(c) Work done during compression

$$= \frac{p_2 v_2 - p_1 v_1}{1} = \frac{144(495 \cdot 1 \times \cdot 528 - 14 \times 7 \cdot 128)}{1 \cdot 37 - 1}$$

$$= 62,900 \text{ ft.-lb.,} \quad \text{or} \quad \frac{62,900}{1400} = 44 \cdot 93 \text{ C.H.U.}$$

(d) Since $\gamma > n$, heat is rejected, and

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{Heat equivalent of work done.}$$

Therefore, heat rejected during compression

$$\frac{1 \cdot 408 - 1 \cdot 37}{1 \cdot 408 - 1} \times 44 \cdot 93 = 4 \cdot 185 \text{ C.H.U.} \qquad \textit{Answer.}$$

Example 11. Air at a temperature of 59 F. (15 C.) is compressed in a cylinder from 15 lb. pressure (absolute) to 120 lb. pressure (absolute) per square inch. The equation of the compression curve is $pv^{1 \cdot 2^5} = $ constant. Find the work done in compressing a pound of air, and the heat that escapes through the cylinder walls. (*U.L., B.Sc. (Eng.)*.)

Here $T_1 = 15° + 273 = 288°$ C. (absolute),

and $p_1 = 15 \times 144$ lb. per sq. ft.

Now by the characteristic equation for 1 lb. of air, $\dfrac{pv}{T} = 96$,

and $v_1 = \dfrac{96 \times 288}{15 \times 144} = 12 \cdot 8$ cub. ft.

Work done in compression is

• $$W = \frac{p_1 v_1}{n - 1} \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n - 1}{n}} - 1 \right\}$$

$$\left(\frac{p_2}{p_1} \right)^{\frac{n - 1}{n}} = \left(\frac{120}{15} \right)^{\frac{1 \cdot 25 - 1}{1 \cdot 25}} = 8^{\frac{1}{5}} = 1 \cdot 51572$$

and $$W = \frac{144 \times 15 \times 12 \cdot 8}{1 \cdot 25 - 1} \{ 8^{\frac{1}{5}} - 1 \}$$

hence $W = 4 \times 144 \times 15 \times 12 \cdot 8 (1 \cdot 51572 - 1) = 57,034$ ft.-lb.

∴ Heat equivalent of work done on the air

$$= \frac{57,034}{1400} = 40 \cdot 74 \text{ C.H.U.}$$

By (14) (p. 87), the heat rejected by the air during compression is

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{work done.}$$

Substituting values,

• $$Q = \frac{1 \cdot 4 - 1 \cdot 25}{1 \cdot 4 - 1} \times 40 \cdot 74 \text{ C.H.U.}$$

∴ Heat rejected

$$= \frac{0 \cdot 15}{0 \cdot 4} \times 40 \cdot 74 = \frac{3}{8} \times 40 \cdot 74 = 15 \cdot 28 \text{ C.H.U.}$$

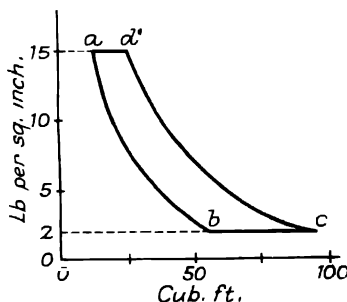or　　　$15 \cdot 28 \times 1 \cdot 8 = 27 \cdot 5$ B.Th.U.　　　　　　*Answer.*

Fig. 28

Otherwise, as a check, the volume after compression may be found

since　　$p_1 \cdot v_1{}^{1 \cdot 25} = p_2 \cdot v_2{}^{1 \cdot 25}, \quad \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{1 \cdot 25}} = 8^{\frac{4}{5}} = 5 \cdot 278$

and　　　$v_2 = \frac{12 \cdot 8}{5 \cdot 278} = 2 \cdot 425$

then, work done,

$$W = \frac{p_2 v_2 - p_1 v_1}{n - 1} = \frac{144(120 \times 2 \cdot 425 - 15 \times 12 \cdot 8)}{1 \cdot 25 - 1}$$

$$= 4 \times 144(291 - 192) = 57,024 \text{ ft.-lb.}$$

$$= 40 \cdot 73 \text{ C.H.U.}$$

and heat rejected

$$= \tfrac{3}{8} \times 40 \cdot 73 = 15 \cdot 274 \text{ C.H.U., or } 27 \cdot 49 \text{ B.Th.U.}$$

Example 12. Air at 60° F. and atmospheric pressure (15 lb. per sq. in.) is expanded adiabatically till the pressure is 2 lb. per sq. in. The air then receives heat at constant pressure until its temperature is again 60° F., and is afterwards compressed adiabatically to the atmospheric pressure, and exhausted at that pressure. Find the temperature at exhaust and the heat rejected in the exhaust per unit of work done.　　　(*U.L., B.Sc. (Eng.).*)

Given　$T_a = 60° + 460° = 520°$ F. (absolute),

take　　　$\gamma = 1 \cdot 4, \quad \frac{\gamma - 1}{\gamma} = \frac{1 \cdot 4 -}{1 \cdot 4} = \frac{2}{7}, \text{ and } C_p = 0 \cdot 238$

In adiabatic expansion, *ab,* Fig. 28,

$$\frac{T_b}{T_a} = \left(\frac{p_b}{p_a}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$T_b = T_a \left(\frac{p_b}{p_a}\right)^{\frac{\gamma - 1}{\gamma}} = 520 \left(\frac{2}{15}\right) = 292 \cdot 4° \text{ F. (abs.)}$$

Hence the work done *by* 1 lb. of air during adiabatic expansion *ab*, represented by the area under *ab* to zero pressure line,

is $\dfrac{144}{0\cdot4}(15 \times 12\cdot81 - 2 \times 54\cdot01) = 30{,}280$ ft.-lb.

also, during expansion *bc* at constant pressure, 2 lb. per sq. in., the area under *bc* represents the work done *by* the air

$$= 144 \times 2(96\cdot06 - 54\cdot01) = 12{,}110 \text{ ft.-lb.}$$

Again, the work done *on* the air during adiabatic compression *cd* is shown by the area under *cd* to zero pressure, and is equal to

$$\frac{p_c v_c}{\gamma - 1}\left\{\left(\frac{p_d}{p_c}\right)^{\gamma\,-\,1}\gamma - 1\right\} = \frac{144 \times 2 \times 96\cdot06}{1\cdot4 - 1}\left\{\left(\frac{15}{2}\right)^{\frac{2}{7}} - 1\right\}$$

$$= 360 \times 192(1\cdot7783 - 1) = 53{,}800 \text{ ft -lb.,}$$

and during exhaust *da* at constant pressure 15 lb. per sq. in., the work done *on* the air is represented by the rectangular area under *da*, between ordinates drawn from *d* and *a*, and is equal to pressure × change of volume,

$$144 \times 15(22\cdot77 - 12\cdot81) = 2160 \times 9\cdot96 = 21{,}513 \text{ ft.-lb.}$$

∴ net work done *on* the 1 lb. of air, represented by the area *abcd*, is

$$53{,}800 + 21{,}513 - (30{,}280 + 12{,}110) = 32{,}923$$

and equivalent to $\dfrac{32{,}923}{778} = 42\cdot32$ B.Th.U (nearly).

Hence the heat rejected in the exhaust, per unit of work done, is

$$\frac{96\cdot32}{32{,}923} = 0\cdot002926 \text{ B Th.U. per ft.-lb.} \qquad \textit{Answer.}$$

EXAMPLES III

1. Find the volume of 3 lb weight of gas at pressure 115 lb. per sq. in. absolute, and temperature 59 F. Take the specific heat at constant pressure as 0·2404 and at constant volume 0·172 per lb. weight.

2. A pound of dry air at 15·5 C. and 14·7 lb. per sq. in occupies 13·09 cub.ft. and is compressed adiabatically ($pv^{1\cdot1}$ = constant) to one-fifth of this volume. Find (*a*) its pressure and temperature at the end of compression, and (*b*) the pressure at the reduced volume when the air is cooled to 15·5° C.

3. One cub. ft. of gas, at 300 lb. per sq. in. absolute, expands to 5 cub. ft. according to the law $pv^{1\cdot2}$ = constant. Find the pressure at the end of expansion and the work done by the gas during expansion.

4. Find the temperature at the end of compression when air at 70° F. (21·1° C.) is compressed from 15 to 105 lb. per sq. in. absolute. Assume $n = 1\cdot35$.

5. A cubic foot of gas, at pressure 300 lb. per sq. in. absolute, expands until its pressure is 60 lb per sq. in. absolute, the law of expansion being $pv^{1\cdot2}$ = constant, find the work done by the gas during expansion.

6. In an air compressor the air is drawn in at a temperature of 60° F. and pressure 14·7 lb. per sq. in. absolute. The air drawn in per stroke is 12 cub. ft., and the final pressure is 90 lb. per sq. in. absolute. Find the work done during compression (*a*) if the air is compressed isothermally ; (*b*) if it is compressed

adiabatically. In the second case, find the temperature of the air when the pressure reaches 90 lb. absolute. (*U.L., B.Sc. (Eng.).*)

7. If $10 \cdot 2$ cub. ft. of air, at pressure 14 lb. per sq. in. absolute, be compressed isothermally to $1 \cdot 7$ cub. ft., what is (1) the pressure at the end of compression ; (2) the work done on the air ; and (3) the heat taken from it ?

8. Twelve cub ft. of air at 14 lb. per sq. in. absolute and $27°$ C. is compressed to 1 cub. ft. The compression law is $pv^{1 \cdot 3}$ · · constant. Find (*a*) the pressure and temperature at the end of compression ; (*b*) the work done in foot-pounds during compression ; and (*c*) the amount of heat received or rejected by the air to the cylinder walls when $\gamma = 1 \cdot 4$.

9 Ten cub. ft. of air at 90 lb. per sq. in. absolute and $65°$ F. are expanded to 4 times the original volume, the law for expansion being $pv^{1 \cdot 2}$ — a constant. Given that the specific heat of air at constant volume is $130 \cdot 3$ ft.-lb. per lb., and at constant pressure $183 \cdot 4$ ft.-lb. per lb., find (*a*) the temperature of the air at the end of expansion ; (*b*) the work done in foot-pounds ; (*c*) the amount of heat which must have been given by or been rejected to an external source during the expansion. (*U.L., B.Sc. (Eng.)*)

10. Air at $60°$ F. and pressure 15 lb. per sq. in., is compressed to 75 lb. per sq. in. absolute, and the compression curve is $pv^{1 \cdot 2}$ constant. Find the work done and heat rejected per pound of the air during compression.

•   (*U.L., B.Sc. (Eng.).*)

11. A cylinder contains $0 \cdot 5$ cub. ft. of gas at pressure 15 lb. per sq. in. absolute. What work in foot-pounds is expended in compression to 90 lb. per sq. in absolute if the compression curve follows the law $pv^{1 \cdot 35}$ constant ? What is the volume after compression ?

12. The pressure is 160 lb. per sq. in. absolute when gas occupies 1 ft. length of cylinder, 12 in. in diameter. What work is done by the gas kept at *constant temperature* in driving the piston through 2 ft. length of stroke, and what amount of heat must be given to the gas ?

13. One pound of air, at atmospheric pressure and $60°$ F., is compressed adiabatically to 6 atmospheres, find (*a*) its temperature and the work done by the pump. It is now allowed to cool at this pressure down to $60°$ F. ; (*b*) what amount of heat is given out, and what further work of compression is done ? If the final state had been brought about by isothermal compression, (*c*) what work would have been done by the pump ? Specific heat of air at constant pressure $0 \cdot 238$, at constant volume $0 \cdot 169$. (*U.L., B.Sc. (Eng.).*)

14 State and prove the relation existing between the specific heat of a gas at constant pressure and constant volume, and the characteristic constant for a gas.

If one-tenth of a pound of gas occupies $1 \cdot 345$ cub ft. at $15°$ C., and 15 lb. per sq. in., and if after adiabatic compression to $0 \cdot 5$ cub. ft. the pressure is $60 \cdot 9$ lb. per sq. in., what is the value of the specific heat at constant pressure of this gas ? (*U.L., B.Sc. (Eng.)*, 1925.)

Carnot's Reflections on the Motive Power of Heat. In 1824, Sadi Carnot published the extraordinary essay, of remarkable insight, entitled *Reflections on the Motive Power of Heat*, containing the fundamental principles from which thermodynamics has been developed.*

Carnot conceived a *closed, reversible cycle*, in which the working substance receives heat *only* at the higher constant temperature of the hot *source*, when expanding and doing external work, is cooled by adiabatic expansion, gives out heat only during isothermal compression at the lower constant temperature of the *condenser*; is then raised in temperature by adiabatic compression, and brought back again to its initial state of volume, temperature, pressure and physical properties, so that, he states as an *axiom*, the working substance must contain the same quantity of heat as it had originally.

In Carnot's *ideal engine* the piston and cylinder are supposed to be perfect non-conductors, absolutely impervious to heat, but the cylinder is fitted with a perfectly conducting end. The working substance is changed in temperature only by rapid adiabatic expansion and compression, with a perfect non-conducting cover on the end of the cylinder, so that there is no transfer of heat, except while heat is being received from the *source* or hot body at temperature T_1, and rejected to the refrigerator or cold *condenser* at T_2. Both of these bodies must have infinite capacity for heat, in order that each of them may remain at exactly the same constant temperature as the working substance during its isothermal expansion and compression while the transfer of heat is taking place through the perfectly conducting end cover of the cylinder.

In 1834, Clapeyron took up Carnot's work† and made the supposed changes and transformations during the cycle clearer, by an indicator diagram, Fig. 29.

Carnot's Cycle, with a Perfect Gas as the Working Substance. Take 1 lb. weight of a perfect gas as working substance in the cylinder, and let A, Fig. 29, represent the state as regards pressure p_1, and volume v_1, at absolute temperature T_1.

* Sadi Carnot, *Réflexions sur la puissance motrice du feu et sur les moyens propres à la développer* (Paris, 1824); or, "The Motive Power of Heat" (with a biographical sketch by a brother of Carnot), translated and edited by R. H. Thurston, 1890.

† See "*Memoire sur la puissance motrice du feu*," in the *Journal de l'école Polytechnique*, Tome XIV (1834), or Translation in Taylor's *Scientific Memoirs*, Part III (1837), p. 347.

(1) *Isothermal Expansion*, with the conducting end of the cylinder in contact with the source or hot body at the higher temperature T_1. Allow the gas to expand along the isothermal AB, Fig. 29, from volume v_1 to v_2 at B, receiving heat Q_1 at constant temperature. The external work done *by* the gas in driving the piston is represented by the area $ABba$, and equal to the heat received $Q_1 = RT_1 \log_e r$, while the temperature and internal energy of the gas remain the same.

(2) *Adiabatic Expansion*, BC, with the non-conducting cover on the cylinder end, the gas expands adiabatically, doing work on the
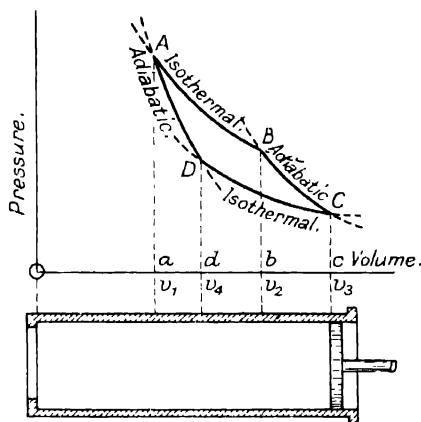
FIG. 29. CARNOT CYCLE WITH A PERFECT GAS

piston at the expense of the internal energy of the gas. Consequently the temperature falls from T_1 to the lower temperature T_2 at C. The pressure and volume are now p_3 and v_3, and the external work done *by* the gas is represented by the area $BCcb$, and is equal to $\dfrac{R(T_1 - T_2)}{\gamma - 1}$, which is also the change in internal energy of the gas.

(3) *Isothermal Compression*, CD Bring the conducting end of the cylinder into direct contact with the cold body or condenser at T_2. Force the piston slowly inwards until the state point D of the gas is on the adiabatic curve through A. The work done on the gas is $RT_2 \log_e r$, represented by area $CDdc$, and is equal to the heat rejected, $Q_2 = RT_2 \log_e r$.

The adiabatic drop in temperature from B to C must be equal to the rise in temperature from D to A ; that is,

$$\frac{T_1}{T_2} \quad \left(\frac{v_3}{v_2}\right)^{\gamma - 1} \quad \left(\frac{v_4}{v_1}\right)^{\gamma - 1}, \text{ hence } \frac{v_3}{v_2} \quad \frac{v_4}{v_1}, \text{ or, } \frac{v_3}{v_4} \quad \frac{v_2}{v_1} - r,$$

where r is the ratio of isothermal expansion and compression.

(4) *Adiabatic Compression, DA.* Again, put the non-conducting cover on the cylinder end, and force the piston rapidly inwards, compressing the gas adiabatically from D to A, until the temperature rises to the original T_1 of the hot body. The pressure and volume are now p_1 and v_1 ; and the additional work done *on* the gas by the piston is represented by the area $DAda$, equivalent to $\dfrac{R(T_1 - T_2)}{\gamma - 1}$ which is also the *gain* of internal energy of the gas.

During adiabatic expansion and compression, no heat is received or rejected, and, on the whole, there is no change of internal energy since the external work done *by* the gas, represented by the area under BC, and the work done *upon* it by the area under DA, are each equal to $\dfrac{R(T_1 - T_2)}{\gamma - 1}$ for the same change of temperature.

During the whole cycle, the heat received by the 1 lb. weight of gas at T_1 from the hot body is $Q_1 = RT_1 \log_e r$, and the heat rejected at T_2 to the cold body is $Q_2 = RT_2 \log_e r$.

There has been no other transfer, leakage, or waste of heat energy to or from the gas, which is brought to its original state, having the same temperature, internal energy, pressure and volume as at the start. Therefore, the difference

$$Q_1 - Q_2 = R(T_1 - T_2) \log_e r$$

is the net amount of heat that has disappeared as heat, and must have been converted into the net amount of external work done by the gas on the piston, and represented by the area $ABCD$ of the diagram.

The *thermal efficiency* of the Carnot ideal cycle is therefore—

$$\frac{\text{Work done } (W)}{\text{Heat received}} = \frac{Q_1 - Q_2}{Q_1} = \frac{R(T_1 - T_2) \log_e r}{RT_1 \log_e r} = \frac{T_1 - T_2}{T_1} \qquad . \quad (1)$$

that is, work done

$$W = Q_1 \left(\frac{T_1 - T_2}{T_1} \right) \quad \frac{Q_1}{T_1} (T_1 - T_2)$$

gives the largest fraction of the heat received from the hot body that can be converted into work, and this depends only upon the temperature limits at which the gas receives and rejects heat.

The heat is let down through an engine from a source at a high temperature, and some of it is rejected at a lower temperature ; the only change of temperature being adiabatic.

Carnot's cycle is **reversible,** that is, when the working substance is forced to perform the same operations in the opposite direction, i.e. counter clockwise, the same pv indicator diagram is traced, and, by the same net work expended, the same quantity of heat is taken in at the low temperature and rejected at the higher temperature.

Starting at state A, the gas is cooled by adiabatic expansion from T_1 to T_2 at D ; and, by isothermal expansion from D to C, the gas

takes heat Q_2 from the cold body at temperature T_2, while doing work represented by $RT_2 \log_\epsilon r$, or the area *DCcd*. Then adiabatic compression *CB* raises the temperature of the gas to T_1; and by isothermal compression *BA*, the gas rejects heat Q_1 at the higher temperature T_1 while *work is done upon* the gas equal to $RT_1 \log_\epsilon r$, or the area *BAab*, and the gas is restored to its original state at *A*.

In this reversed cycle, on the whole, there is no change of the internal energy of the working gas ; the area of the diagram *ADBC* indicates the net amount of

$$Work \; spent \; upon \; the \; gas = R(T_1 - T_2) \log_\epsilon r,$$

whilst heat extracted at T_2, from cold body, is $Q_2 = RT_2 \log_\epsilon r$, and rejected at higher temperature T_1, to hot body, is $Q_1 = RT_1 \log_\epsilon r$.

Hence the transfers of heat are exactly reversed, and, during the series of stages, the states of the gas are precisely the same in pressure, volume and temperature as when working direct. In the reversed engine the work spent upon the gas is equal in amount to the work which the gas does on the piston in direct working.

The *performance* of the ideal Carnot heat engine reversed is measured by

$$\frac{\text{Heat extracted from cold body}}{\text{Work expended}} = \frac{Q_2}{Q_1 - Q_2} = \frac{RT_2 \log_\epsilon r}{R(T_1 - T_2) \log_\epsilon r} ;$$

hence $\dfrac{Q_2}{\text{Work spent}} = \dfrac{Q_2}{Q_1 - Q_2} = \dfrac{T_2}{T_1 - T_2}$, that is

$$Q_2 = \text{Work done} \times \frac{T_2}{T_1 - T_2} \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

This reversed heat engine is also an ideal *refrigerating* machine, and, by the conservation of energy, $Q_1 = Q_2 +$ work done in driving the machine. In other words, the quantity of heat Q_1 rejected to the hot body is equal to the heat Q_2, received from the cold body, added to the work spent upon the gas.

Carnot's Principle. Reversibility is the sole test of perfection of a thermo-dynamic engine. No engine can perform more work from a given quantity of heat working between the same limits of temperature, or a reversible engine has the highest possible theoretical efficiency.

To prove this : Suppose, if possible, a heat engine, S, to be capable of doing more work from a given quantity of heat than a reversible engine, R, working between the same temperatures of source and condenser.

Couple the two engines together as a compound engine, S working direct and R reversed. R needs part w only of the work $w + w_1$ given out by S, to be able to restore to the source the heat received by S. Thus, during each cycle of the compound engine, the heat rejected to the source by R is equal to that supplied to S, while

work w_1 is done. This would go on continuously. Even if we suppose no mechanical friction, heat must have been spent to perform the work w_1, and this heat could only come from the condenser or cold body. The quantity of heat in the source or hot body remains the same, while that in the cold body must be diminished indefinitely, and work would be done continuously by the transfer of heat from the coldest body. This result is contrary to all experimental evidence, and to the axiom stated by Clausius : " It is impossible for a self-acting machine, unaided by any external agency, to convey heat from a body at a low temperature to one at a higher temperature." Or by Kelvin : " We cannot transform heat into work merely by cooling a body already below the temperature of the coldest surrounding objects."*

These statements of the second law of thermodynamics agree with the facts of experience, and apply to the performance of work continuously by heat engines working in complete cycles.

The *second law* governs the transformation of heat into work, and *gives the greatest fraction of the total heat supplied to any heat engine that can be converted into work done on the piston.*

Therefore no engine can have a higher thermal efficiency than a reversible one. It follows that all heat engines, with perfectly reversible cycles, receiving and rejecting heat at the same higher and lower temperatures, have the same efficiency, whether air or other gas be used in them. The efficiency of a reversible engine is independent of the nature or physical properties of the working substance employed, and depends solely on the temperatures between which the engine works.

The greatest amount of work which can be obtained from Q_1 units of heat received by the working substance at absolute temperature T_1, and rejecting heat at absolute temperature T_2, is

$$= JQ_1 \frac{(T_1 - T_2)}{T_1} \text{ ft.-lb.}$$

Carnot pointed out the **conditions** *necessary to attain perfection,* the criterion being complete reversibility in all the operations of an ideal heat engine—

(1) Heat is only taken in and given out by the working substance at the higher and lower temperatures between which the engine works. In this transfer of heat the working substance must be at the same temperature as the hot or cold body, and there must not be any direct interchange of heat between the fluid and bodies at sensibly different temperatures. There must not be any gain or loss of heat by contact with the cylinder walls, or due to internal friction or viscosity of the fluid.

(2) Change of temperature must only be due to change of volume,

and the pressure on the piston must be exactly balanced by resistance, so that there is to be no free expansion by sudden changes of pressure which produce drop of temperature, or throttling in expanding through a valve. Rushes and eddying motions, which are dissipated in heat and cause waste of energy, are not reversible.

Carnot's Cycle, using Vapour as the Working Substance. Consider the ideal perfect heat engine working on the Carnot cycle and using 1 lb. of saturated vapour, namely, vapour in contact with its own liquid. At A, Fig. 30, 1 lb. weight of water or other liquid occupies w cub. ft at the boiling point T_1 under the corresponding saturation pressure p_1. Heat is applied at this constant pressure until the whole pound of liquid is turned into vapour occupying the *volume* V cub. ft. at B. The heat received is the latent heat, L_1, of the vapour at this pressure. The iso-thermal expansion follows the horizontal line AB at constant pressure.

Then the dry saturated vapour is expanded adiabatically, reducing the pressure until the temperature falls to that of the condenser, T_2, that is, the lowest temperature of the available cooling water in a condenser. The vapour is wet along the adiabatic BC.

Fig. 30. Carnot Cycle with Vapour as the Working Substance

Next isothermal compression, CD, at constant lower pressure and temperature T_2. The vapour gradually condenses and its latent heat is rejected to the condenser until a point D is reached, such that adiabatic compression condenses all the vapour and restores the working substance in the liquid state to temperature T_1 at A. The adiabatic curves BC and DA of expansion and compression are usually plotted from tables giving the properties of vapours.

Since the cycle is reversible and heat L_1 is only taken in at the higher temperature T_1, and heat rejected only at T_2, the thermal efficiency is $\dfrac{T_1 - T_2}{T_1}$ and the work done per pound of the liquid is represented by the area $ABCD$, and equal to $JL_1 \cdot \dfrac{T_1 - T_2}{T_1}$ ft.-lb.

Suppose the difference between the higher and lower temperatures is very small, from T to $T - \delta T$, so that δT is the fall in temperature, and the corresponding difference in pressure δp. The indicator diagram is now reduced to the narrow parallelogram $ABcd$, of height δp, since the adiabatics are nearly straight lines. The length AB of the little diagram is the difference, $V - w$, between the volume of the saturated vapour V and that of the liquid w, of 1 lb. weight.

The work done in this Carnot cycle is equal to the area $ABcd$, and becomes nearly its height × length, or $\delta p(V - w)$. The
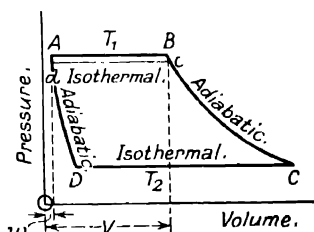
heat taken in during the cycle is L, the latent heat of evaporation, and the work done is $JL\dfrac{\delta T}{T}$ ft.-lb. per lb. of working substance; therefore

$$\delta p(V - w) = JL \cdot \frac{\delta T}{T}$$

In the limit, when δT is indefinitely small, the relation becomes

$$\frac{dp}{dT}(V - w) = \frac{JL}{T} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

or, $$V - w = \frac{JL}{T} \cdot \frac{dT}{dp} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (4)$$

known as **Clapeyron's Equation,** giving the relation between the latent heat of any vapour, the change in cubic feet of 1 lb. of the liquid when vaporized at absolute temperature T; and the rate at which the temperature of saturated vapour varies relatively to the pressure. The latter $\dfrac{dT}{dp}$ is given by the slope of the tangent to the saturation curve of temperature and pressure at the given absolute temperature, and from steam tables.

The student should plot the temperature-pressure curve of saturated steam or other vapour from tables, and find $\dfrac{dp}{dT}$ from the curve, also L and w, the results of experiments, from tables; then calculate the specific volume of 1 lb. of dry saturated steam at a few pressures by equation (3).

The accuracy of results for the saturation volumes may be compared with the tabulated values. Or, the temperatures of dry saturated steam may be taken, $2°$ C. apart, and half the difference of corresponding saturation pressures in pounds per square foot gives the rate at which p is changing per degree. The values of L, and w cub. ft. for the mean temperature, substituted in equation (3), give the saturation volume in cubic feet.

Since $(V - w)$ is positive for steam, it follows from equation (4) that $\dfrac{dT}{dp}$ is positive, that is, increase of pressure raises the boiling point.

The Clapeyron equation also applies to a change of state from solid to a liquid. If a solid contracts in melting, $\dfrac{dp}{dT}$ is negative, and the melting point is lowered by increase of pressure.

From the contraction of ice in melting, Professor James Thomson[*] calculated that the *melting point of ice would be lowered* about $0\cdot0074°$ C. for every atmosphere increase of pressure, and this was

[*] See Kelvin's *Mathematical and Physical Papers*, Vol. I, p. 156 and p. 165.

proved experimentally by his brother, Lord Kelvin. Thus, a pound of water at 0°C. changes its volume, in freezing, from 0·016 to 0·0174 cub. ft., and gives out 80 C.H.U., hence

$$\frac{dT}{dp} = \frac{(\cdot0174 - \cdot016)273}{1400 \times 80} = 0\cdot00000341,$$

and if dp be 1 atmosphere, 15 lb. per sq. in., or 2160 lb. per sq. ft.,

$$dT = 2160 \times \cdot00000341 = 0\cdot0074° \text{ C. (nearly)}.$$

Also, if the atmospheric pressure were reduced to a vacuum, the water vapour at the melting point would have a pressure of 0·09 lb. per sq. in., and ice would melt about 0·0074° C., instead of 0° C. When there is no air pressure, the temperature of melting ice is
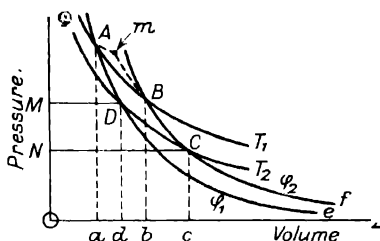

Fig. 31. Energy and Absolute Temperature

known as the *triple point*, because water substance may then be in the three states of melting ice, water, and vapour together in equilibrium.

Again, from equation (3),

$$\frac{\frac{\delta p}{\delta T}(V - w)}{JL} = \frac{1}{T}; \text{ or } \frac{\delta p(V - w)}{JL} = \frac{\delta T}{T} \qquad . \qquad . \qquad . \qquad (5)$$

taking δT as 1° C., and the work done* per unit of heat received at absolute temperature T, for 1 lb. of vapour, with the change of pressure in pounds per square foot corresponding to a drop of temperature 1° C. at that temperature, is simply $\frac{1}{T}$.

Graphic Representation of Energy. The *internal energy* of a pound of gas in the state A, Fig. 31, is represented by the area $ADeZaA$, under the adiabatic curve through A, indefinitely produced to cut

* See *Steam Power*, by Professor W. E. Dalby, p. 157. Examples of the physical properties of various working substances (steam, SO_2, NH_4, alcohol, acetone) gave the same result $\frac{1}{T} = \frac{1}{303}$, or 0·0033 ft.-lb. per unit of heat supplied at 30° C. (303° absolute) to 1 lb. of the substance for a fall of temperature from 30° to 29° C., illustrating clearly that work ·done does not depend on the physical properties of any particular working substance.

the volume axis at infinity, or absolute zero. Imagine the gas, originally in the state A as to pressure and volume, and temperature T, if it were possible, to be expanded adiabatically, doing external work at the expense of its own internal energy, without gain or loss of heat, until its pressure falls to zero. It can do no more external work, and contains no heat. The whole of the internal energy of the 1 lb. of gas is gradually spent in doing work on the piston, and the equations (p. 76) representing the area $ADeZaA$ under the adiabatic, become

$$\text{Work done} \qquad \frac{p_1 v_1}{\gamma - 1} = \frac{RT_1}{\gamma - 1} = K_v T. \tag{6}$$

which may be taken as the internal energy in 1 lb. of gas in the state A, at absolute temperature T_1. But we know that all gases would become solid and cease to behave like a perfect gas before reaching this low temperature.

We have no means of measuring the total amount of internal energy stored up in a given quantity of gas or other working substance. We can only measure or estimate the *change of internal energy* of a given mass of a substance in changing from one state to another.

Now allow the state point to move from A to B along any curve AmB. To find the change of internal energy during this operation, draw through B the adiabatic BCf. The internal energy of the gas in the state B is represented by the area $BCfZbB$, and the change of internal energy is the difference between the areas $BCfZbB$ and $ADeZaA$, which is independent of the shape of the curve AmB. The gas might have changed from A to B along the isothermal. It follows that the internal energy of a substance in a given state must always be the same, and that the change of internal energy in any transformation depends only on the initial and final positions of the state point, and *not* upon the path.

But, by Joule's energy equation, the heat absorbed $Q = E$ + external work done. The work done by the gas is represented by the area under the curve, either $AmBbaA$ or $ABbaA$, depending on the path between A and B. In this case the external work done along the isothermal will be less than that along AmB, by the area enclosed between these paths, in a clockwise direction. Therefore, the *heat received* by a given quantity of gas during a change of state from A to B is represented on the pv diagram by the area bounded by the actual curve AB or AmB and the two adiabatic curves through the initial and final points drawn to infinity.

The **Total Heat** of a substance in any state is defined by the equation $\qquad H = E + \dfrac{pv}{J}$

as the sum of the internal energy and the product of the pressure and volume, expressed in heat units, corresponding to the state.

Thus, on the pv diagram, Fig. 31, draw the constant pressure line DM, then the total heat of a gas in the state D, as regards pressure and volume, is shown by the sum of the rectangle $MDdO$ and the area under the adiabatic DeZ, or the total area $MDeZOM$. Along the isothermal DC, for a perfect gas, $p_1v_1 = p_2v_2$, and the heat energy received during expansion is equivalent to the external work done by the gas, $DCcdD$, while its internal energy remains constant.

For practical purposes, it is the difference between the energy in the two states that is considered, and the usual convention is to take an *arbitrary zero* from which the internal energy of a substance is reckoned. When the working substance is a perfect gas, the absolute zero of temperature is taken, and for calculations on internal combustion engines 100° C. or 0° C. When the working substance is a vapour, like steam, the zero temperature chosen corresponds to 0° C., and under pressure equal to its vapour pressure at that temperature. (See pp. 41 and 42.)

Absolute Temperature. In the Carnot reversible engine, as T_2 is lowered, the work area $ABCD$ of the diagram, Fig. 31 (p. 101), increases until the temperature is reduced to zero, when all the heat received at the higher constant temperature is converted into work. Since the work done can never exceed the heat received, there must be a zero limit of temperature at which the pressure of the working substance would vanish. This ideal limiting temperature is defined as the absolute zero.

By Carnot's principle, the efficiency of all perfectly reversible heat engines working between the same limits of temperature must be the same and equal to $(T_1 - T_2)/T_1$, and the work done

$$W = Q_1 \cdot \left(\frac{T_1 - T_2}{T_1} \right)$$

This relation, combined with the first law, forms the basis and expression of the absolute scale of temperature devised by Lord Kelvin and now universally adopted.

By the first law, $Q_1 - Q_2 = W$, and the heat rejected at the lower temperature

$$Q_2 = Q_1 - W = Q_1 \left(1 - \frac{T_1 - T_2}{T_1} \right) = Q_1 \left(\frac{T_2}{T_1} \right)$$

that is,
$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}, \text{ and } Q_2 = Q_1 \times \frac{T_2}{T_1},$$

also
$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \text{a constant}. \qquad (7)$$

Thus the ratio $\dfrac{T_1}{T}$ of the temperatures of the source and condenser

is the same as $\dfrac{Q_1}{Q_2}$ of the quantities of heat received and rejected at those temperatures in a Carnot cycle, and the *work done for a given drop of temperature is the same at any part of the scale*, which is absolute in the sense of being independent of the properties of any particular substance.

Rankine takes a series of reversible engines, working between equal intervals of temperature, and shows that these would do equal amounts of work when one receives the heat which another rejects.

Between a pair of adiabatic curves ADe and BCf, draw a series of isothermals for temperatures T_1, T_2, T_3, etc., at equal intervals of temperature $T_1 - T_2 = T_2 - T_3 = T_3 - T_4 = \delta T$. Each little enclosed area is an indicator diagram of one engine working on the Carnot cycle. As above, let $Q_1 =$ heat supplied during isothermal expansion at the highest temperature of the range T_1. The work done

$$= Q_1 \times \frac{T_1 - T_2}{T_1} = \frac{Q_1}{T_1} \cdot \delta T,$$

and the heat rejected at T_2

$$Q_2 = \frac{Q_1}{T_1} \times T_2$$

Then this heat Q_2, rejected by the first engine, is supplied to the second engine at T_2. Here work done $=$ heat supplied $\times$ efficiency

$$\left(\frac{Q_1}{T_1} \times T_2 \right) \times \left(\frac{T_2 - T_3}{T_2} \right) = \frac{Q_1}{T_1} \times T_2 \cdot \frac{\delta T}{T_2} = \frac{Q_1}{T_1} \cdot \delta T$$

and heat rejected $=$ heat supplied $-$ work done

$$= Q_1 \times \frac{T_2}{T_1} - Q_1 \quad \frac{\delta T}{T_1}$$

$$= Q_1 \quad \frac{T_2}{T_1} \quad \frac{\delta T}{T_1} \quad - Q_1 \quad \frac{T_2 - T_2}{T_1} \quad T_3$$

$$= Q_1 \times \frac{T_3}{T_1}$$

Similarly in the third engine, the efficiency is $\dfrac{\delta T}{T_2}$, and the work done $= Q_1 \times \dfrac{\delta T}{T_1}$, with heat rejected $= Q_1 \times \dfrac{T_4}{T_1}$.

Hence the work done by every engine during a cycle is the same $\dfrac{Q_1}{T_1} \cdot \delta T$, when each engine works through the same range of temperature δT on the perfect gas thermometer. Equal intervals of temperature may be defined as those intervals which give equal

amounts of work in a series of perfect heat engines This thermo-
dynamic scale is identical with that of the ideal perfect gas, but does
not agree exactly with the scales of actual gases because of the
variation in specific heat. (See corrections, p. 56.)

Again, suppose the engines in the series received quantities of
heat Q_1, Q_2, Q_3, etc., at absolute temperatures T_1, T_2, T_3, etc.,
and rejected heat at the lower temperatures. In each little rever-
sible cycle $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$, that is, $\dfrac{Q_1}{T_1} - \dfrac{Q_2}{T_2} = 0$, and when the summation
is taken all round the large reversible cycle, heat taken in being
reckoned as positive and heat rejected as negative $\Sigma \dfrac{Q}{T} = 0$.

By taking an indefinitely large number of steps and corre-
sponding reception and rejection at the different temperatures, we
come at last to a perfectly gradual or continuous change of tem-
perature. •If dQ represents the small quantity of heat taken in or
given out by a perfect reversible engine at the absolute temperature
T, then the sum of all the quantities $\dfrac{dQ}{T}$ is zero for a reversible
cycle, that is, the integration round the complete cycle

$$\int \frac{dQ}{T} = 0.$$

Entropy. The entropy, ϕ, is that characteristic or function of
the state of a substance which does not change in a reversible
adiabatic expansion or compression, and which is increased or
diminished when heat is taken in or given out in a reversible manner.
Since there is neither gain nor loss of heat along adiabatics in a
reversible process, these are lines of *constant entropy*, and called
isentropics. When a pound of any substance takes in or gives out
a quantity of heat Q at the *absolute* temperature T, its gain or loss
of entropy ϕ is measured by the quotient $\dfrac{Q}{T}$; so that the quantity, Q,
of *heat energy*, absorbed or rejected at the absolute temperature, is
the product of the two factors, *absolute* temperature T and *change
of entropy* ϕ; that is, $Q = T \cdot \phi$.

In the change of state point A to B, Fig. 31, Q_1 is the heat taken
in from any point A on the adiabatic ADe, ϕ_1, to B on the adiabatic
ϕ_2, at constant absolute temperature T_1, the quotient $\dfrac{Q_1}{T_1}$ is called the
increase of entropy, $\phi_2 - \phi_1$. There is no change of entropy when
the state point moves reversibly along an adiabatic : it follows that
the change of entropy does not depend upon the paths AB or AmB,
although the heat received does.

Temperature Entropy Diagram. On a temperature entropy
diagram the state of a substance is given by vertical ordinates to

represent *absolute temperature*, and horizontal distances its *change of entropy*, and an area represents *quantity of heat*. An adiabatic, or constant entropy line, will be shown by a vertical straight line AM, Fig. 32, parallel to the axis of temperature ; and an isothermal will be a horizontal straight line, AB, parallel to the axis of entropy.

Carnot's cycle on a $T\phi$ diagram. Suppose a pound weight of working substance, kept at constant temperature $T_1 = 1000°$ C. (absolute), expands and takes in heat from the source in amount $Q_1 = 150$ C.H.U. Its *gain* of entropy is $\dfrac{Q_1}{T_1} = \dfrac{150}{1000}$, or 0·15, which the source has lost This isothermal is represented on the $T\phi$ diagram, Fig. 32, by the horizontal line $AB = 0·15$, the increase of entropy of the working substance is $\phi_2 - \phi_1 = ON - OM$. The area $MABN = T_1(\phi_2 - \phi_1)$, represents the heat Q_1 received by the working substance. During adiabatic expansion BC, the temperature falls to 600° C. (absolute), say, but the entropy does not change. Next, isothermal compression CD at tempera ture $T_2 = 600°$ C., during which heat is rejected by the working substance, and its loss of entropy $\dfrac{Q_2}{T_2}$ must equal the gain $\dfrac{Q_1}{T_1}$, since the cycle is reversible, and $DC = AB$, or $\phi_2 - \phi_1 = 0·15$. The quantity of heat rejected is

$$Q_2 = \text{area } CDMN = T_2(\phi_2 - \phi_1),$$

that is, $600 \times 0·15$, or 90 C.H.U.

In adiabatic compression DA, the entropy remains constant, while the temperature rises to the initial value T_1, or 1000° C (absolute).

The heat turned into work during the cycle is

$$Q_1 - Q_2 = \text{area } ABCD = (T_1 - T_2)(\phi_2 - \phi_1),$$

that is, $150 - 90 = (1000 - 600) \, 0·15$, or 60 C H.U.

and the thermal efficiency

$$= \frac{60}{150} = \frac{1000 - 600}{1000}, \text{ or } 40 \text{ per cent.}$$

Here, heat converted into work

$$= \frac{Q_1}{T_1} \cdot (T_1 - T_2)$$

$$= \text{entropy} \times \text{drop of temperature.}$$

Carnot's assumption was that heat fell in temperature and work was done while the heat rejected was equal in amount to the heat received ; like water falling down a height in a water-wheel, and
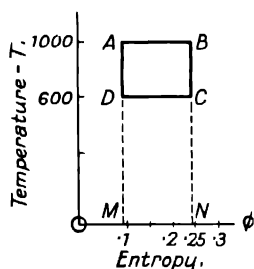
FIG. 32. CARNOT CYCLE

that the most efficient water-wheel was reversible and could lift the same weight of water through the same height, was based on the fallacy of the conservation of *heat*, which was changed to the conservation of *energy* by Joule, Kelvin, and others. Take $\frac{Q_1}{T_1}$, and entropy × drop of temperature, like weight of water × fall in height, and the analogy holds. The source has *lost* entropy equal to $\frac{Q_1}{T_1}$, and the condenser has *gained an equal amount of entropy* Q_2/T_2, but the amount of *heat*, Q_1, received from the source at T_1 is greater than that rejected, Q_2, at the lower temperature T_2; and the difference $Q_1 - Q_2$ is the amount of heat converted into work during the cycle.

Stirling's Air Engine with Regenerator. In 1827, Dr. Robert Stirling, and his brother James Stirling, patented and made the first hot-air engine to work on a perfect reversible cycle. This appears to be the first application of the *regenerator*, also invented by Stirling, in which heat is stored while the air falls from T_1 to T_2, the higher to the lower temperature, at constant volume before heat is rejected at the lower temperature. This transfer of heat is reversible, and the regenerator gives up the same heat that it took in, so that the air is gradually heated from T_2 to T_1 in its passage back through the regenerator, before receiving heat from the furnace at T_1. To be perfect, the regenerator must have infinite or very large capacity for heat compared with that of the air used, so that there

FIG. 33   STIRLING AIR ENGINE REGENERATOR AND DISPLACER

is no sensible rise or fall of temperature when storing and re-storing the heat; and further, the regenerator must be at the same temperature as the air at every part in its passage.

In one design, Fig. 33, a displacer D forces the working air up and down through the regenerator R, which is constructed of sheet iron plates $\frac{1}{40}$ in. thick, and kept $\frac{1}{50}$ in. apart by ridges to offer a large surface. The cooler or refrigerator W consists of copper pipes $\frac{1}{8}$ in. bore, about $\frac{1}{20}$ in. apart, and through these cold water circulates. The displacer D is lined on the lower end with brick dust B.
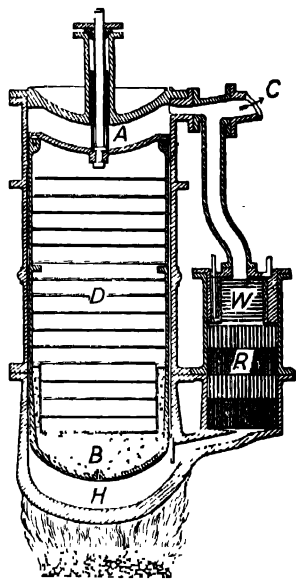
Compressed air at 150 lb. per sq. in is delivered into the air vessel A to keep up the average pressure, and supply any waste by leakage, the same air being used over again continuously as working substance. When the plunger D is raised to the top of its stroke, the air passes from the top end A down through the regenerator, taking in heat and rising to the higher temperature T_1 of the furnace.

Then 1 lb. of air receives heat from the furnace at constant temperature T_1, and expands from volume v_1 to v_2, along the isothermal AB on the ideal indicator diagram, Fig 34. The pressure drives
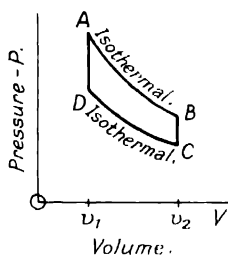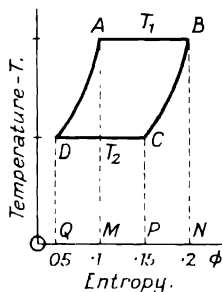

Fig. 34   Fig. 35

STIRLING CYCLE

up the piston in the motor cylinder at C, Fig 33 (not shown), and the heat received is equal to the work done, that is, $Q_1 - RT_1 \log_t r$, where the ratio of expansion $r - \dfrac{v_2}{v_1}$.

During operation BC, the plunger D, in its downstroke, passes the hot air out of H up through the regenerator to the cold end, W, and the top of the air vessel. The air leaves heat stored up in the regenerator, and falls in temperature to T_2 at constant volume, the pressure falling with the temperature. The heat stored in the regenerator is $C_t(T_1 - T_2)$ calories per lb

During stroke CD, the air is compressed from volume v_2 to v_1, at constant temperature T_2, being in contact with the cooler W, Fig. 33. The work done on the air is equal to the heat rejected,

$$Q_2 = RT_2 \log_t r.$$

During DA, the plunger D is raised and the air passed down through the regenerator, at constant volume, picking up the same quantity of heat previously stored there, $C_v(T_1 - T_2)$, and having its temperature raised to T_1 in its initial condition of volume, pressure, and temperature.

During the cycle, the work done = heat supplied – heat rejected,
that is, $Q_1 - Q_2 = RT_1 \log_\epsilon r - RT_2 \log_\epsilon r$,
and the efficiency is

$$\frac{\text{Heat converted into work}}{\text{Heat supplied}},$$

or,

$$\frac{Q_1 - Q_2}{Q_1} = \frac{RT_1 \log_\epsilon r - RT_2 \log_\epsilon r}{RT_1 \log_\epsilon r} = \frac{T_1 - T_2}{T_1} \tag{8}$$

Al

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}.$$

If the efficiency of the regenerator is e, the heat received by the air from the regenerator during the operation DA is $e\, C_v(T_1 - T_2)$.

Now total heat supplied is $RT_1 \log_\epsilon r + (1 - e)\, C_v(T_1 - T_2)$

and efficiency

$$\frac{R(T_1 - T_2)\log_\epsilon r}{RT_1 \log_\epsilon r + (1 - e)\, C_v(T_1 - T_2)} \tag{9}$$

In practice, e was about 0·8 to 0·9.

The *ideal temperature entropy diagram* of the Stirling regenerative engine is $ABCD$, Fig. 35. In the isothermal operation, AB, suppose 1 lb. of air takes in $Q_1 = 100$ heat units at $T_1 = 1000°$ C. (absolute) the gain of entropy is $\dfrac{Q_1}{T_1} = \dfrac{100}{1000} = 0\cdot 1$. The heat received is represented by the area $ABNM$.

During BC the air is gradually cooled from T_1 to T_2 in passing through the regenerator where heat is stored at constant volume, measured by the area $NBCP$. If the regenerator action is perfect, it restores the same amount at every point in the passage of the air through it in the opposite direction DA, also at constant volume. Hence the curves BC and DA are precisely alike, and have equal areas $DAMQ$ and $CBNP$ under them. The loss of entropy NP, and gain of entropy QM, is $C_v \log_\epsilon \dfrac{T_1}{T_2}$, since the fall and rise of temperature follow the logarithmic curve.

During the isothermal compression CD at T_2, the heat rejected is measured by the area $PCDQ$, which is $Q_2 = 600° \times 0\cdot1$, or 60 heat units.

The heat converted into work is $Q_1 - Q_2 = 100 - 60$, or 40 heat units.

Hence the thermal efficiency is 40 per cent.

Since $AB = DC$,

$$\frac{Q_1}{Q_2} = \frac{\text{area } MABN}{\text{area } PCDQ} = \frac{T_1}{T_2},$$

and the heat converted into work $= \dfrac{Q_1}{T_1}(T_1 - T_2)$.

Stirling's cycle is also perfect in the thermodynamic sense, and would be *reversible* if the regenerator had infinitely large capacity for heat.

The devise of the regenerator for storing and restoring heat directly, as heat, effects the change from T_1 to T_2, and T_2 to T_1, instead of the adiabatic expansion and compression in the Carnot cycle, and greatly reduces the length of the piston stroke.

Stirling's engine appears to be the *earliest perfect heat engine* constructed, but air is a bad conductor of heat, and there was loss of heat by the *external furnace*.

In 1845, at a Dundee foundry, a double-acting Stirling engine, having cylinder 16 in. diameter by 4 ft. stroke at 28 r.p.m., gave 45 B.H.P. The high ideal thermal efficiency was reduced by practical imperfections. The heating surface was about 75 sq. ft., and the efficiency of the furnace 0·44. After working three years, the engine was abandoned owing to the burning out of the heater. Air is such a bad conductor of heat that it could not be quickly and effectively heated without making the engine very bulky.

The Robinson Air Engine works on the Stirling cycle and is made for very small powers. The motor piston-rod acts on a crank-pin or disc, which is also connected by a short link to a rocking lever driving the air *displacer* plunger, 90° in advance of the motor piston. The air displacer cylinder is heated externally at the bottom by a coke fire, gas flame, or oil burner in small motors. The top of the displacer cylinder is cooled by water jacket. The air passes up and down through the displacer, to and from the motor cylinder, and the speed of the engine is regulated by a valve throttling the air in the passage leading to the motor cylinder. The same air is used over again in low pressure, and only small quantities of heat are dealt with in each cycle.

An engine having cylinder 10 in. diameter at 170 r.p.m. gives only $\frac{2}{3}$ H.P., and cannot compete with other modern heat engines.

The Ericsson Regenerative Air Engine,[*] receives and rejects heat at constant temperature, and the temperature of the air is changed in its passage through the regenerator at *constant pressure*. The *pv* indicator diagram is formed by two isothermals and two lines of constant pressure at the higher and lower limits. The $T\phi$ diagram is similar to that of the Stirling engine.

The working cylinder was heated directly at one end, and the air was compressed by a separate pump driven off the engine shaft.

The ship *Ericsson* had a set of four cylinders, each 14 ft. diameter by 6 ft. stroke, each with a compression pump, and all having the same receiver, making their strokes in succession at intervals of a quarter of a revolution, and all driving the same shaft. The regenerator was a box containing several layers of wire gauze, and

the heat lost in the regenerator was one-tenth of the whole heat stored. This air engine, running at 9 r.p.m., gave about 300 I.H.P., the temperature limits 414° F. and 122° F , and the mean effective pressure was about 2 lb. per sq. in. The heating surface was insufficient and consisted of the bottoms of the cylinders, about 700 sq. ft. According to Professor Norton, the fuel consumed was 560 lb. of anthracite per hour. or 1·87 lb. per I.H.P. hour, an efficiency higher than that of most steam engines at that time.

In 1898 at Boston, America, an Ericsson regenerative air engine, having cylinder 8 in. diameter by 3·9 in. stroke, running at a speed of 110 r.p.m., gave 0·27 indicated, and only 0·08 B H.P., and failed because the heating surface was burnt away in a short time.

In **Joule's Air Engine** the air receives heat at the higher constant pressure in a large heated chamber *H*, after adiabatic compression, and rejects heat into a cold chamber *C*, at a lower constant pressure. Fig. 36 is a diagram of the essential parts.
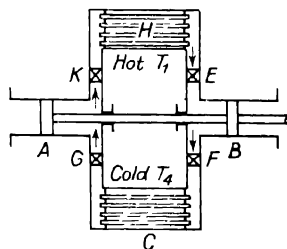
Air is kept cool at absolute temperature, T_4, in a large chamber. *C*, by circulating water. The pump *A* takes in a little of this air from *C*, and com-

Fig 36. Joule's Air Engine

presses it adiabatically until its pressure is the same as in the hot chamber *H*, where it is further heated from T_2 to T_1 at constant pressure in tubes, by burning fuel. A little of the air then passes at constant pressure into the working cylinder *B*, until the valve *E* closes, and adiabatic expansion follows to atmospheric pressure. The air, at absolute temperature T_3, which is above the atmospheric temperature T_4, is discharged, through valve *F*, at the lower constant pressure, back again into the cold chamber. The supply of air from the pump to the hot chamber is so small that it does not sensibly alter the higher constant pressure. Fig. 37 is the pressure-volume diagram of this cycle. The area *FDAE* represents the work done *on* each pound of air by the pump ; and area *EBCF* the work done *by* the air. The difference, *ABCD*, is the net amount of work done by each pound of air during one complete cycle. The numbers denote absolute temperatures. At the higher constant pressure *AB*, the ratio

$$\frac{T_2}{T_1} = \frac{\text{Volume } EA \text{ discharge by the pump}}{\text{Volume } EB \text{ admitted to the working cylinder}},$$

and at the lower pressure, *CF* is the volume of air discharged, while *FD* is the volume taken in by the pump. The volume swept by the pump piston is smaller than that of the working cylinder in this proportion.

absolute temperature remaining *constant* at T, during the small change, its gain or loss of entropy, $d\phi$, is measured by the ratio $\dfrac{dQ}{T}$, so that the change in heat energy is $dQ = T \cdot d\phi$.

Also, $dQ = C \cdot dT$, where C is the *constant specific heat* of the substance at that temperature ; and for unit weight of a substance changing from T_2 up to T_1, the total change of heat energy

$$\int_{T_2}^{T_1} dQ = C(T_1 - T_2), \text{ since } dQ = C \cdot dT.$$

If 1 lb. of a substance is changing continuously and reversibly in temperature from T_2 up to T_1, while heat is being taken in, the total gain of entropy is

$$\phi_1 - \phi_2 = \int_{T_2}^{T_1} \frac{dQ}{T} \qquad \int_{T_2}^{T_1} C \cdot \frac{dT}{T} = C \log_e \frac{T_1}{T_2} \tag{11}$$

At constant volume $C = C_v$, and at constant pressure $C = C_p$, in this expression.

Example 1. Assume that the entropy of 1 lb. of water at 0 C. is taken as zero. Calculate the entropy gained when the water is warmed to 100 C., and converted into dry saturated steam at 100 C.

In case steam tables are available, the entropy of dry saturated steam is given as $\phi = \phi_w + \dfrac{L}{T}$; that is, by adding the entropy of water at 100° C., and that due to the addition of the latent heat, L, at the constant absolute temperature of evaporation.

As a first approximation, suppose the heating is under the constant pressure of 1 atmosphere ; and the specific heat of water is the mean thermal unit. Callendar takes the varying pressure of saturation of water vapour from 0° to 100° C. in his steam tables.

(1) Gain of entropy of water, $\phi_w = 1 \times 2 \cdot 3026 \log_{10} \dfrac{373 \cdot 1}{273 \cdot 1} = 0 \cdot 312$

(2) The latent heat is 539·3 lb calories, and since the temperature remains constant until all the water is converted into steam under the pressure of 1 atmosphere, gain of entropy is

$$\frac{L}{T} = \frac{539 \cdot 3}{373 \cdot 1} = 1 \cdot 4454$$

Hence the total entropy gained is $\quad \phi_s = 0 \cdot 3120 + 1 \cdot 4454 = 1 \cdot 7574$

The accurate value by steam tables $\quad \phi_s = 0 \cdot 31186 + 1 \cdot 44546 = 1 \cdot 7573$

The difference is within the limits of error of experiment.

If more heat is given, after all the water is converted into dry steam, the temperature of the steam would rise above saturation. The steam is then *superheated*, and the additional gain of entropy

may be calculated by the expression $0 \cdot 48 \log_\varepsilon \dfrac{T}{T_1}$, where $0 \cdot 48$ is the specific heat of steam, for which Callendar gives $0 \cdot 47719$ in his tables.

Change of Entropy of a Perfect Gas when changing from state p_1, v_1, T_1 to the state p_2, v_2, T_2. The energy equation of 1 lb. of a perfect gas is—

$$Q = C_v(T_1 - T_2) + p \cdot (v_2 - v_1),$$

and for a small change in the limit,

$$dQ = C_v \cdot dT + p \cdot dv, \text{ divide across by } T,$$

$$\frac{dQ}{T} = C_v \cdot \frac{dT}{T} + \frac{p}{T} \cdot dv, \text{ and } pv = RT, \text{ or } \frac{p}{T} = \frac{R}{v}$$

Substituting this value gives

$$\frac{dQ}{T} = C_v \cdot \frac{dT}{T} + R \cdot \frac{dv}{v}, \text{ the expression for } d\phi$$

Integrating, we have,

$$\int_{T_1}^{T_2} \frac{dQ}{T} = C_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{T_1}^{T_2} \frac{dv}{v},$$

and change of entropy,

$$\phi_2 - \phi_1 = C_v \log_\varepsilon \frac{T_2}{T_1} + R \cdot \log_\varepsilon \frac{v_2}{v_1} \qquad . \qquad . \qquad . \qquad . \quad (12)$$

Again, since $R = C_p - C_v$, substituting for R,

$$\phi_2 - \phi_1 = C_v \cdot \log_\varepsilon \frac{T_2}{T_1} + (C_p - C_v) \log_\varepsilon \frac{v_2}{v_1}$$

$$= C_v \left(\log_\varepsilon \frac{T_2}{T_1} - \log_\varepsilon \frac{v_2}{v_1} \right) + C_p \cdot \log_\varepsilon \frac{v_2}{v_1}$$

$$= C_v \left(\log_\varepsilon \frac{T_2}{T_1} \times \frac{v_1}{v_2} \right) + C_p \cdot \log_\varepsilon \frac{v_2}{v_1}$$

But $\dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2}$, and $\dfrac{p_2}{p_1} = \dfrac{T_2}{T_1} \times \dfrac{v_1}{v_2}$, hence we obtain

$$\phi_2 - \phi_1 = C_v \log_\varepsilon \frac{p_2}{p_1} + C_p \log_\varepsilon \frac{v_2}{v_1} \qquad . \qquad . \qquad . \qquad . \quad (13)$$

Similarly, by substituting for $C_v = C_p - R$ in equation (12), we obtain

$$\phi_2 - \phi_1 = C_p \log_\varepsilon \frac{T_2}{T_1} - R \cdot \log_\varepsilon \frac{p_2}{p_1} \qquad . \qquad . \qquad . \qquad . \quad (14)$$

In calculating the change of entropy for 1 lb. of a perfect gas, we may use either (12), (13), or (14), where R, C_p and C_v are

expressed in heat units, v_1 and v_2 in cubic feet, p_1 and p_2 in pounds per square foot

During an *isothermal change* of state, $T_1 = T_2$, and equation (12) becomes

$$\phi_2 - \phi_1 = R \cdot \log_t \left(\frac{v_2}{v_1}\right), = (C_p - C_t) \log_t \frac{v_2}{v_1} \qquad \qquad . \quad (15)$$

In a *constant volume*, change $v_1 = v_2$, and equations (12) and (13) become

$$\phi_2 - \phi_1 = C_t \log_t \cdot \frac{T_2}{T_1}, \qquad C_t \cdot \log_t \frac{p_2}{p_1} . \qquad . \qquad . \qquad . \quad (16)$$

In a *constant pressure* change, $p_1 = p_2$, and equations (13) and (14) become

$$\phi_2 - \phi_1 = C_p \cdot \log_t \cdot \frac{v_2}{v_1} = C_p \cdot \log_t \frac{T_2}{T_1}. \qquad . \qquad . \qquad . \quad (17)$$

Change of entropy of 1 lb. of gas during expansion or compression according to the general law $pv^n = $ a constant.

By equation (13) (p. 86), $dQ = \frac{\gamma - n}{\gamma - 1} \cdot p \, dv$. Divide by T, we have

$$\frac{dQ}{T} = \frac{\gamma - n}{\gamma - 1} \times \frac{p}{T} \cdot dv, \text{ and since } \frac{p}{T} = \frac{R}{v}, \text{ we obtain}$$

$$\frac{dQ}{T} = R \cdot \frac{\gamma - n}{\gamma - 1} \cdot \frac{dv}{v}.$$

Integrating, we have

$$\int_{T_1}^{T_2} \frac{dQ}{T} = R \cdot \frac{\gamma - n}{\gamma - 1} \cdot \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\phi_2 - \phi_1 = R \cdot \frac{\gamma - n}{\gamma - 1} \cdot \log_t \frac{v_2}{v_1}$$

Now $\dfrac{v_2}{v_1} = \left(\dfrac{T_1}{T_2}\right)^{\frac{1}{n-1}}$, and $R = C_v(\gamma - 1)$

hence $\phi_2 - \phi_1 = C_t(\gamma - 1) \dfrac{\gamma - n}{\gamma - 1} \times \log_t \left(\dfrac{T_1}{T_2}\right)^{\frac{1}{n-1}}$

$$= C_v(\gamma - 1) \times \frac{\gamma - n}{\gamma - 1} \times \frac{1}{n-1} \cdot \log_\varepsilon \left(\frac{T_1}{T_2}\right), \text{ and}$$

$$\phi_2 - \phi_1 = C_v \cdot \frac{\gamma - n}{n - 1} \cdot \log_\varepsilon \frac{T_1}{T_2} \qquad . \qquad . \qquad . \qquad . \quad (18)$$

Hence from measurements of pressure and volume on the indicator diagram, $\phi_2 - \phi_1$ is found, also the corresponding temperature calculated, and plotted on the temperature entropy, $T\phi$ diagram.

Example 2. Ten cub. ft. of air, at 65 F. and 90 lb. per sq. in. absolute, are expanded to 4 times the original volume, the law of expansion being $pv^{1\cdot25}$ = constant. Find the change of entropy. Given K_v = 130·2 ft.-lb., and γ = 1·4.

Here $T_1 = 65° + 460 = 525°$ F. (absolute)

and $\dfrac{T_2}{T_1}$ $\left(\dfrac{1}{4}\right)^{1\cdot25-1}$ $\left(\dfrac{1}{4}\right)^{\frac{1}{4}}$ · $\left(\dfrac{1}{2}\right)^{\frac{1}{2}}$ $\dfrac{1}{2}$ $\sqrt{2}$ $0\cdot7071$

hence $T_2 = 525 \times \cdot7071 = 371\cdot2°$ F. (abs.), or $371\cdot2 - 460$
      $= -89°$ F.

Change of entropy

$$= C_v \cdot \frac{\gamma - n}{n - 1} \cdot \log_\varepsilon \frac{T_1}{T_2}, \text{ by equation (18)}$$

$$\frac{130\cdot2}{778} \times \frac{1\cdot4 - 1\cdot25}{1\cdot25 - 1} \times 2\cdot3026 \log_{10} \frac{525}{371}$$

$$\frac{130\cdot2}{778} \times 0\cdot15 \; \div 4 \; \times 2\cdot3026 \times 0\cdot1508 = 0\cdot0348$$

Answer.

Check by $\phi_2 - \phi_1 = \dfrac{R}{J} \cdot \dfrac{\gamma - n}{\gamma - 1} \cdot \log_\varepsilon \left(\dfrac{v_2}{v_1}\right)$

We have $R = K_v(\gamma - 1) = 130\cdot2 \times 0\cdot4 = 52\cdot08$

and change of entropy

$$= \frac{52\cdot08}{778} \times \frac{1\cdot4 - 1\cdot25}{1\cdot4 - 1} \times 2\cdot3026 \log_{10}\left(\frac{4}{1}\right) = \underline{0\cdot0348}$$

Answer.

Example 3. If 1 lb. of air occupying 3 cub. ft. at 180 lb. per sq. in. absolute, and 537° C., expands at constant temperature to 12 cub. ft., what is its pressure after expansion, the work done, heat reception, and change of entropy ?

Here the temperature is kept constant and, by Boyle's law,

$$p_2v_2 = p_1v_1, \text{ or } p_2 = p_1 \times \frac{v_1}{v_0} \quad \frac{180 \times 3}{12} = 45 \text{ lb. per sq. in.}$$

Work done,

$$W = p_1v_1 \log_\varepsilon r = 144 \times 180 \times 3 \log_\varepsilon \left(\frac{12}{3}\right)$$

but $\log_\varepsilon 4 = 2\cdot3026 \times 0\cdot60206 = 1\cdot3863.$

   $\therefore \; W = 144 \times 540 \times 1\cdot3863 = 107{,}800 \text{ ft.-lb.}$

Since the temperature, and consequently the internal energy, are kept constant, the heat taken in by the air is equivalent to the work done by it, and is

$$\frac{107,800}{1400} = 77 \text{ C.H.U., at } 537^\circ + 273 = 810^\circ \text{ C. (abs.)}$$

∴ gain of entropy

$$= \frac{\cdot\cdot}{810} = 0\cdot095 \text{ units.}$$

or, by equation (15), gain of entropy

$$= (C_p - C_1) \log \frac{v_2}{v} = \cdot0685 \log_e 4$$

$$\cdot0685 \times 1\cdot3863 = 0\cdot095 \text{ units.} \qquad Answer.$$

Example 4. A quantity of air having a volume of 2 cub. ft. at atmospheric conditions of 14·7 lb. per sq. in. and 15 C. is compressed according to the law $pv^{1\cdot15}$ a constant, until its pressure is 120 lb. per sq. in. Find the change in the internal energy of the air and also the change in entropy. The specific heats for air at constant pressure and constant volume are 0·238 and 0·169.   (*U.L., B.Sc. (Eng.), 1925.*)

Given $v_1 = 2$ cub. ft.

$p_1 = 14\cdot7 \times 144$ lb. per sq. ft.

$T_1 = 15^\circ + 273^\circ = 288^\circ$ C. (abs.)

$\dfrac{R}{J} = 0\cdot238 - 0\cdot169 = 0\cdot069$ C.H.U. per lb.

$\gamma = \dfrac{C_p}{C_1} = \dfrac{\cdot238}{\cdot169} = 1\cdot408$

Let $w = $ lb. weight of air

then $w = \dfrac{p_1 v_1}{RT_1} = \dfrac{14\cdot7 \times 144 \times 2}{0\cdot069 \times 1400 \times 288} = \dfrac{7}{46}$

Now, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\dfrac{120}{14\cdot7}\right)^{\frac{1\cdot15-1}{1\cdot15}}$

∴ $\log T_2 = \log 288 + \dfrac{\cdot\cdot}{23} (\log 120 - \log 14\cdot7)$

$= 2\cdot45939 + \dfrac{3}{23} (2\cdot07918 - 1\cdot16732)$

$= 2\cdot45939 + 0\cdot11894 = 2\cdot57833$

∴ $T_2 = 378\cdot7$ C. (abs.)

and the air gains internal energy $= w \cdot C_1 (T_2 - T_1)$

$= \dfrac{\cdot}{46} \times 0\cdot169(378\cdot7 - 288) = 2\cdot33$ C.H.U.

A!so, by equation (18), the change of entropy,

$$\phi_2 - \phi_1 = C_v \left(\frac{\gamma - n}{n - 1} \right) \log_\varepsilon \frac{T_1}{T_2}$$

$$= 0.169 \left\{ \frac{1.408 - 1.15}{1.15 - 1} \right\} \times 2.3026 \log \left(\frac{288}{378.7} \right)$$

$$= 0.169 \times \frac{.258}{.15} \times 2.3026(-0.11894) = -0.0797$$

The negative sign means that during the change of state the air loses entropy in amount 0·0797 units. **Answer.**

The student should check these results by different methods—

We have $p_1 v_1^{1\cdot15} = p_2 v_2^{1\cdot15}$, or

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1} \right)_{1\cdot15} = \frac{120}{14.7} \bigg)^{\frac{20}{23}}$$

$$\log \qquad \frac{20}{23} (\log 120 - \log 14.7) = \frac{20}{23} \times 0.91186 = 0.79293$$

$$\therefore \qquad \frac{v_1}{v_2} = 6.208, \text{ and } v_2 = \frac{2}{6.208} = 0.3222 \text{ cub. ft.}$$

or by $T_1 \cdot v_1^{n-1} = T_2 \cdot v_2^{n-1} \quad \therefore \frac{v_1}{v_2} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{n-1}} = \left(\frac{378.7}{288} \right)^{\frac{1}{0.15}}$

and $\quad \log \frac{v_1}{v_2} = \frac{20}{3} (\log 378.7 - \log 288) = \frac{20}{3} \times 0.11894 = 0.79293$

$$\log v_2 = \log 2 - 0.79293 = 0.30103 - 0.79293 = \bar{1}.5081$$
$$\therefore v_2 = 0.3222 \text{ cub. ft.}$$

Also (p. 116), change of entropy,

$$\phi_2 - \phi_1 = \frac{R}{J} \cdot \frac{\gamma - n}{\gamma - 1} \cdot \log_\varepsilon \frac{v_2}{v_1}$$

Substitute values,

$$\phi_2 - \phi_1 = 0.069 \left(\frac{1.408 - 1.15}{1.408 - 1} \right) 2.3026 \log \left(\frac{v_2}{v_1} \right)$$

$$\therefore \phi_2 - \phi_1 = -0.069 \times \frac{.258}{.408} \times 2.3026 \times 0.79293 = -0.0797$$

Alternative Method, useful for students.   Work done on the air during compression

$$= \frac{p_1 v_1}{1.15 - 1} \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{1.15-1}{1.15}} - 1 \right\} = \frac{144 \times 14.7 \times 2}{0.15} \left\{ \left(\frac{120}{14.7} \right)^{\frac{3}{23}} - 1 \right\},$$

$$= \frac{144 \times 14.7 \times 2}{0.15} (1.315 - 1) = 8891 \text{ ft.-lb.}$$

Otherwise,

$$\text{Work done} = \frac{p_2v_2 - p_1v_1}{n-1} = \frac{144}{0.15}(120 \times 0.32218 - 14.7 \times 2)$$

$$= 960(38.662 - 29.4) = 8891 \text{ ft.-lb.}$$

The heat equivalent of this work $= \dfrac{8891}{1400} = 6.35$ C.H.U.

Since $n < \gamma$, heat is given out by the air or rejected during compression, in amount

$$= \frac{\gamma - n}{\gamma - 1} \times \frac{\text{Work done (ft.-lb.)}}{J} = \frac{1.408 - 1.15}{1.408 - 1} \times 6.35 \text{ C.H.U.}$$

$$= 4.02 \text{ C.H.U.}$$

During compression of the air,

Increase of internal energy = Heat received − Heat rejected

$$= 6.35 - 4.02 = 2.33 \text{ C.H.U.}$$

Again, the change in entropy may be calculated by either of the equations (12), (13) or (14) (p. 115).

Thus, by (12),

$$\phi_2 - \phi_1 = C_v \cdot \log_\varepsilon \frac{T_2}{T_1} + \frac{R}{J} \log_\varepsilon \frac{v_2}{v_1}$$

$$= 0.169 \log_\varepsilon \frac{378.7}{288} + 0.069 \log_\varepsilon \frac{0.3222}{2}$$

$$= 0.169 \times 2.3026 \times 0.11894 - 0.069 \times 2.3026 \times 0.79293$$

$\therefore$ change in entropy $= 0.04628 - 0.12598 = -0.0797$

Also by (13), when the pressure and volume are known,

$$\phi_2 - \phi_1 = C_v \cdot \log_\varepsilon \left(\frac{p_2}{p_1}\right) + C_p \cdot \log_\varepsilon \left(\frac{v_2}{v_1}\right).$$

Substitute as above,

$$= 0.169 \times 2.3026 \times 0.91186 - 0.238 \times 2.3026 \times 0.79293$$

$\therefore$ Change in entropy $= 0.35481 - 0.43454 = -0.0797$

or (by 14),

$$\phi_2 - \phi_1 = C_p \cdot \log_\varepsilon \left(\frac{T_2}{T_1}\right) - \frac{R}{J} \cdot \log_\varepsilon \left(\frac{p_2}{p_1}\right),$$

$$= 0.238 \times 2.3026 \times 0.11894 - 0.069 \times 2.3026 \times 0.91186$$

$\therefore$ change in entropy $= 0.06518 - 0.14488 = \underline{-0.0797}$

<div align="right">Answer.</div>

Example 5. A Stirling regenerative air engine works between temperatures of 700° F. and 80° F., the ratio of isothermal expansion is 2. Calculate the ideal efficiency when (*a*) the engine is fitted with a perfect regenerator ;

(b) when the efficiency of the regnerator is 0·9. Take $Cp = 0·2375$ and $C_v = 0·1691$.

Given $T_1 = 700 + 460 = 1160°$ F. (abs.),

and $T_2 = 80 + 460 = 540°$ F. (abs.),

and $T_1 - T_2 = 620°$ F.

Also thermal equivalent of $R = C_{p}·- C_v = ·2375 - ·1691 = ·0684$

(a) Efficiency $= \dfrac{T_1 - T_2}{T_1} = \dfrac{1160 - 540}{1160} = \dfrac{620}{1160} = 0·5345$,

or 53·45 per cent.   *Answer.*

(b) By equation (9), (p. 109),

Heat supplied, $Q_1 = R . T_1 \log_\varepsilon r + (1 - e) C_v(T_1 - T_2)$

Heat rejected, $Q_2 = R . T_2 \log_\varepsilon r + (1 - e) C_v(T_1 - T_2)$

Efficiency· $\dfrac{R(T_1 - T_2) \log_\varepsilon r}{R . T_1 \log_\varepsilon r + (1 - e) C_v(T_1 - T_2)}$ and $\log_\varepsilon 2 = ·69315$

$\qquad = \dfrac{0·0684 \times 620 \times ·69315}{0·0684 \times 1160 \times ·69315 + 0·1 \times 0·1691 \times 620}$

$\qquad = \dfrac{29·4}{55 + 10·48} = ·449$, or ·45 per cent.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ *Answer.*

Example 6. In a double-acting Stirling engine working between the temperatures of 650° F. and 150° F., diameter of cylinder 16 in. by 4 ft. stroke, revolutions per minute 28, ratio of expansion 1·24, piston displacement per pound of air per stroke 1·06 cub. ft., brake horse-power 45·45. Estimate (a) the work done per pound of air per stroke ; (b) thermal efficiency of the engine, assuming the efficiency of regenerator 0·9 ; (c) the mean effective pressure ; and (d) indicated horse-power and mechanical efficiency.

Take Cp ·2375, $Cv - 0·1691$.

Given $T_1 = 650° + 460 = 1110°$ F. (abs.),

and $T = 150° + 460 = 610°$ F. (abs.).

Also $R = 778(·2375 - ·1691) = 778 \times ·0684$

$\qquad\qquad = 53·2$ ft.-lb. per lb.

(a) Work done per pound of air per stroke

$\qquad\qquad = R(T_1 - T_2) \log_\varepsilon r$, and $\log_\varepsilon 1·24 = ·2151$

$\qquad\qquad = 53·2 \times 500 \times 0·2151 = 5722$ ft.-lb.

(b) Heat supplied

$\qquad\qquad = RT_1 \log_\varepsilon r + (1 - e) C_v T_1 - T_2).$

Substituting values,

$\qquad\qquad = 53·2 \times 1110 \times 0·2151 + 0·1 \times 131·6 \times 500$

$\qquad\qquad = 12,700 + 6580 = 19,280$ ft.-lb.

Thus the heat energy wasted in the generator, due to imperfections, is 6,580 ft.-lb.

Hence the actual thermal efficiency of the engine

$$= \frac{\text{Work done by air on piston}}{\text{Heat supplied to engine}} = \frac{5722}{19,280} = 0.3 \text{ (nearly)}.$$

The ideal efficiency with perfect regenerator having efficiency unity would be

$$\frac{T_1 - T_2}{T_1} = \frac{1110° - 610°}{1110} = \frac{500}{1110} = 0.45,$$

or, neglecting waste in regenerator,

$$\frac{5722 \text{ ft.-lb.}}{12,700} = 0.45.$$

(*c*) Mean effective pressure

$$= \frac{\text{Work done per pound of air per stroke}}{\text{Volume swept by piston}} \quad \frac{5722}{1.06}$$

$$= 5,398 \text{ lb. per sq. ft.} = 37.5 \text{ lb. per sq. in.}$$

(*d*) Sectional area of piston $= \pi(8)^2 = 201$ sq. in.

Work done by air on the piston per minute (two strokes per revolution) $= 37.5 \times 201 \times 4 \times 28 \times 2$ ft.-lb.

Hence indicated horse-power

$$= \frac{37.5 \times 201 \times 4 \times 28 \times 2}{33,000} = 51.2 \text{ I.H.P.}$$

Mechanical efficiency

$$= \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{45.45}{51.2} = .888, \text{ or nearly 89 per cent}$$

and efficiency ratio

$$= \frac{\text{Thermal efficiency}}{\text{Ideal efficiency}} = \frac{0.3}{0.45} = .667$$

Example 7. The air engines of the ship *Ericsson* (p. 110) worked between the temperature limits 122 F. and 414° F. Piston displacement per pound of air, 22 cub. ft. ; ratio of expansion, 1·5 ; revolutions per minute, 9. Diameters of the four cylinders, each 14 ft., stroke 6 ft. Calculate (*a*) work done per pound of air per stroke ; (*b*) thermal efficiency of engines ; (*c*) heat energy wasted in the regenerator assuming its efficiency *e*   0·9 ; (*d*) mean effective pressure ; (*e*) indicated horse-power. Take K_p · 184·8 and K_v 131·6 ft.-lb per pound of air.

Given $T_1 = 414° + 460° = 874°$ F. (abs.),

$T_2 = 122° + 460° = 582°$ F. (abs.).

and $R = 184.8 - 131.6 = 53.2$ ft.-lb.

(*a*) Work done per pound of air per stroke

$$= R(T_1 - T_2) \log_\varepsilon r$$
$$= 53 \cdot 2(874 - 582) \log_\varepsilon 1 \cdot 5$$

$\therefore$ Work done $= 53 \cdot 2 \times 292 \times 0 \cdot 4055 = 6300$ ft.-lb.

(*b*) Heat supplied

$$= RT_1 \log_\varepsilon r + (1 - e) K_p (T_1 - T_2)$$
$$= 53 \cdot 2 \times 874 \times 0 \cdot 4055 + 0 \cdot 1 \times 184 \cdot 8 \times 292$$
$$= 18,860 + 5396 = 24,256 \text{ ft.-lb.}$$

$\therefore$ Thermal efficiency of engine

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{6300}{24,256} = 0 \cdot 26.$$

(*c*) Here, heat wasted in the regenerator $= 5396$ ft.-lb.
With a perfect regenerator, thermal efficiency

$$= \frac{6300}{18,860} = 0 \cdot 334, \text{ or } = \frac{874° - 582°}{874°} = 0 \cdot 334.$$

(*d*) Mean effective pressure

$$= \frac{\text{Work done per pound of air per stroke}}{\text{Volume swept by piston}} = \frac{6300}{22}$$
$$= 286 \text{ lb. per sq. ft.} = 2 \text{ lb. per sq. in. (nearly).}$$

(*e*) Sectional area of each cylinder

$$= \pi r^2 = \pi \times 7^2 = 154 \text{ sq. ft.}$$

Joint area of the four pistons

$$= 154 \times 4 = 616 \text{ sq. ft.}$$

$\therefore$ Work done per minute

$$= 286 \times 616 \times 6 \times 9 \text{ ft.-lb.}$$

and therefore indicated horse-power

$$= \frac{286 \times 616 \times 54}{33,000} = 288 \qquad \textit{Answer.}$$

EXAMPLES IV

1. Ten cub. ft. of dry air at $18°$ C. and 90 lb. per sq. in. absolute pressure, are expanded to 4 times the original volume, the law of expansion being $pv^{1 \cdot 25} = $ constant. Given that the specific heat of air at constant volume is $0 \cdot 169$, and at constant pressure $0 \cdot 238$, find the change in entropy.

2. One lb. weight of dry air occupies 3 cub. ft. at 110 lb. per sq. in. absolute pressure, and $227°$ C., and is expanded at this constant temperature to 12 cub. ft. Find the heat taken in and the change in entropy.

3. One lb. of dry air occupies $12 \cdot 39$ cub. ft. at $0°$ C. and $14 \cdot 7$ lb. per sq. in. absolute, and is compressed to 200 lb. per sq. in. absolute, (*a*) isothermally, (*b*) when the compression is $pv^{1 \cdot 2} = $ constant. What is (1) the work done on

the air in foot-pounds ; (2) the amount of heat taken from the air during compression ; and (3) the change of entropy in each case ?   Take the specific heat of air at constant pressure, 0·238, and at constant volume 0·17.

4. The lower limit of temperature of a Stirling engine is 1,000° F. (538° C.). The maximum and minimum pressures are 180 lb. per sq. in. absolute, and 30 lb. per sq. in. absolute, and the expansion ratio is 3.   Calculate the ideal efficiency.   (*U.L.*, *B.Sc.* (*Eng.*).)

5. A Stirling engine, with perfect regenerator, works between pressures of 135 lb. per sq. in. absolute and 15 lb. per sq. in. absolute, and temperatures 550° F. and 50° F. respectively.   Calculate the mean effective pressure on the piston.

6. In a Stirling engine working between the temperatures of 700° F. and 80° F., the ratio of isothermal expansion is 2.   Calculate the ideal efficiency when (*a*) the engine is fitted with a perfect regenerator ; (*b*) when the efficiency of the regenerator is 0·9.   Take $C_p = 0·2375$ and $C_v = 0·1691$.

CHAPTER V

AIR COMPRESSORS

AN air compressor is any machine which takes in air, compresses, and delivers it to a receiver at a higher pressure ; and may be of (1) the ordinary cylinder and reciprocating piston type, or (2) the rotary type, including turbo-compressors.

The ideal indicator diagram, $ABCD$, Fig. 39, shows the action in the common piston compressor.

During the *suction* stroke, AB, volume v_1 cub. ft. of free air flows into the cylinder at atmospheric pressure p_1 lb. per sq. ft. absolute, and the work done *on* the piston is p_1v_1 ft.-lb.

BC is the *compression* curve, $pv^n = $ a constant, the slope lying between the adiabatic BF and isothermal BE. The work done *on* the air during compression to absolute pressure p_2, and volume v_2, is $\dfrac{p_2v_2 - p_1v_1}{1}$ where $n = 1\cdot4$ for adiabatic compression.
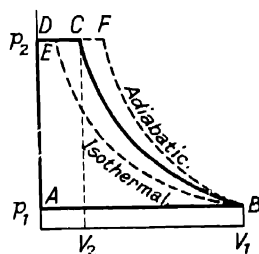
Fig 39

When there is a water-jacket around the cylinder, n may vary from $1\cdot35$ to $1\cdot25$ for thoroughly efficient cooling ; and in the special case of cooling the air during compression, by spraying water into the cylinder, which is drained away from the discharged air, n may be $1\cdot2$. Lower values of n are probably due to leakage past the piston or valves.

CD is the delivery of the air from the cylinder to a receiver at constant pressure p_2, and the work done during this stroke is p_2v_2.

The total effective work of suction, compression, and discharge in foot-pounds per pound of free air compressed per cycle is represented by the area $ABCD$, and is

$$W = \frac{p_2v_2 - p_1v_1}{n-1} + p_2v_2 - p_1v_1 = \frac{n}{n-1}(p_2v_2 - p_1v_1) \quad (1)$$

and since, for one lb. weight of air,

$$pv = RT, \quad W = \frac{n}{n-1} \cdot R(T_2 - T_1) \quad . \qquad . \qquad . \qquad . \quad (2)$$

Also during compression, BC, the temperature is raised from T_1 to T_2 absolute, and

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n}{n}}$$

Substitute this value of T_2 in (2), gives

$$W = \frac{n}{n-1} \cdot RT_1 \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\} \quad . \tag{3}$$

It is obvious that for least loss by rise of temperature, *isothermal compression* is the ideal to be aimed at, when the temperature T_1 would remain constant, and no energy wasted in heating the air while its pressure is raised. Then the work done in suction, isothermal compression, BE, to pressure p_2 and volume v at E, and delivery of the air at constant pressure p_2, would be given by the area $ABED$,

$$= p_1 v_1 \cdot \log_\epsilon \left(\frac{v_1}{v}\right) + p_2 v - p_1 v_1$$

$$= p_1 v_1 \log_\epsilon \left(\frac{p_2}{p_1}\right) = R \cdot T_1 \log_\epsilon \left(\frac{p_2}{p_1}\right) . \qquad . \qquad . \tag{4}$$

Since isothermal compression is the ideal standard to be attained, the *ideal thermal efficiency* of an air compressor is the work done as calculated by either of the expressions (4), divided by that found from equations (1), (2), or (3).

This " efficiency " is based on piston displacement, neglecting all losses by leakage, cooling, and clearance ; but a closer practical estimate of the actual performance is obtained by measurement of the volume of air delivered per minute by the compressor, expressed in terms of " free air," that is, reduced to the volume it would occupy under ordinary atmospheric conditions of pressure 14·7 lb. per sq. in. (abs.), and temperature 60° F. ; then

$$\text{Thermal efficiency} = \frac{\text{Isothermal horse-power on air delivered}}{\text{Indicated air horse-power}}$$

For practical purposes it is necessary to ascertain the brake horse-power applied at the compressor shaft, and to make careful measurement of the volume of air actually delivered at a steady flow through a receiver, and when expanded down to atmospheric pressure and temperature (see p. 148). Then the

$$\text{Overall efficiency} = \frac{\text{Isothermal horse-power based on air delivered}}{\text{Brake horse-power at compressor shaft}}$$

Cooling. When the slope of the compression curve is steeper than the isothermal, the temperature of the air is raised, and extra work, BCE, Fig. 39, is required. Not only is the heat lost by subsequent *cooling* in the air receiver and air mains, but the volume of air shrinks at constant pressure and more work has to be done to keep up the pressure, represented by the rectangular area under CE, bounded by the ordinates at C and E, and equal to $p_2(v_2 - v)$ ft.-lb. Moreover, heating of the valves to a high temperature,

combined with excessive lubrication of the compressor cylinders, may cause carbonization of the lubricating oil, gumming, and other serious troubles. It is, therefore, essential to have thoroughly efficient cooling of the air during compression by water-jacketing the cylinder walls and head, especially the valves and covers.

Multi-stage Compressors. In order to reduce the losses during the compression of air to high pressures, above, say, 120 lb. per sq. in. (gauge), the compression is usually carried out by two or three stages in separate cylinders ; and after each stage the air is cooled, at constant pressure, down to the initial temperature, and thus reduced in volume, by passing through an *inter-cooler*, with circulation of cold water.

In modern practice, the copper coils used in small machines are being replaced by straight tube inter-coolers, on the counter current principle, with air passing through the tubes while the cooling water surrounds them and flows in the opposite direction, so that the air leaves the cooler almost at the temperature of the inlet water.

Fig. 40 shows the saving of work, by the shaded area *CEFL*, in a two-stage compressor. *AB* represents the volume, v_1, of air at pressure, p_1, taken into the low pressure cylinder, and *BC* is the compression curve according to some law pv^n = constant.

The air is then cooled at constant pressure p_2 down, as near as possible, to the initial temperature, at E, and its volume reduced to v_2. The air is then delivered to the high pressure cylinder where compression follows the curve *EF*, having pv^n = constant, until the pressure is p_3 at F and the volume HF is v_3.

FIG. 40. TWO-STAGE COMPRESSION

The work done in foot-pounds per pound of air per cycle, in the two cylinders, is

$$W = \frac{n}{n-1} . RT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} . RT_1 \left[\left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1} . RT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 2 \right] \qquad . \qquad . \quad (5)$$

The total work expended will be the least possible when

$$y = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}}$$

is a minimum. Now, since p_1 and p_3 are constants, differentiate with respect to p_2, and we have

$$\frac{dy}{dp_2} = \frac{n-1}{n} \cdot p_2^{-\frac{1}{n}} \cdot \frac{p_1^{\frac{1}{n}}}{p_1} + \frac{1-n}{n} p_3^{\frac{1}{n}} \cdot p_2^{\frac{1}{n}-2} = 0$$

for a minimum.

Divide by $p_2^{\frac{1}{n}}$, and we have

$$\frac{n-1}{n} \cdot p_1^{\frac{1}{n}} = \frac{n-1}{n} \cdot p_3^{\frac{1}{n}} \cdot p_2^{\frac{2(1-n)}{n}}$$

from which

$$p_2^{\frac{2(1-n)}{n}} = \frac{p_1^{\frac{1}{n}}}{p_3^{\frac{n-1}{n}}}; \quad \therefore p_2^2 = p_1 \cdot p_3, \text{ or } \frac{p_2}{p_1} = \frac{p_3}{p_2} \quad . \quad . \quad (6)$$

Hence for the least work to be done, the ratios of compression in the two stages are equal, and $p_2 = \sqrt{p_1 \cdot p_3}$.

At the end of the first stage compression in the low pressure cylinder the air is cooled in the inter-cooler from T_2 to T_1, at E on the isothermal and the ratio of the volumes of the two cylinders is

$$\frac{v_1}{v_2} = \frac{p_2}{p_1} = \sqrt{\frac{p_3}{p_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Also, since

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}, \text{ and } \frac{T_3}{T_1} = \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}},$$

we have

$$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

showing that the ratio of the initial and final temperatures during compression is the same for each stage when the work of compression is a minimum.

Substitute (6) in (5) and put $RT_1 = p_1 v_1$, we have the minimum work done in ft.-lb. per lb. weight of air for two stage compression,

$$W = \frac{2n}{n-1} \cdot p_1 v_1 \left\{ \left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right\} \quad . \quad . \quad . \quad (9)$$

In *three-stage compression* the volume v_1 of free air at absolute pressure p_1 is compressed to pressure p_2 in the low pressure cylinder, and is then delivered at constant pressure through an inter-cooler to the intermediate pressure cylinder, its volume shrinking to v_2, and temperature from T_2 to T_1 on the isothermal. The volume of air v_2 at pressure p_2 is now compressed to pressure p_3 in the inter-mediate cylinder, and is then delivered through another inter-cooler to the high pressure cylinder, its volume being reduced to v_3, at constant pressure, and temperature T_3 to T_1.

In the high pressure cylinder the air is compressed to pressure p_4, and is then delivered to the receiver at this constant pressure p_4.

The work done in foot-pounds per pound of air per cycle in the three cylinders is

$$W = RT_1 \cdot \frac{n}{n-1} \cdot \left[\left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} + \left\{ \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}} - 1 \right. \right.$$

$$\left. \left. + \left\{ \left(\frac{p_4}{p_3} \right)^{\frac{n-1}{n}} - 1 \right\} \right] . \qquad . \qquad . \qquad . \qquad . \qquad (10)$$

since $RT_1 = p_1 v_1 = p_2 v_2 = p_3 v_3$.

The same conditions hold for least work to be done, as in two-stage compression, and

$$\frac{p_2}{p_1} \quad \frac{p_3}{p_2} \quad \frac{p_4}{p_3} \quad \left(\frac{p_4}{p_1} \right)^{\frac{1}{3}} \qquad . \qquad . \qquad . \qquad . \qquad (11)$$

Substitute (11) in (10), we have the least work done in foot-pounds per pound of air,

$$W = \frac{3n}{n-1} \cdot RT_1 \left\{ \left(\frac{p_4}{p_1} \right)^{\frac{n-1}{3n}} - 1 \right\} . \qquad . \qquad . \qquad . \qquad (12)$$

The ratio of the volumes of the three cylinders will be

$$\frac{v_1}{v_2} = \frac{v_2}{v_3} = \left(\frac{p_4}{p_1} \right)^{\frac{1}{3}} .$$

Volumetric Efficiency is the ratio of the volume of air actually delivered, when reduced to " free air," at normal temperature and pressure, to the volume swept by the piston. The typical indicator diagrams, Figs. 41 and 42, show the *clearance* and losses due to the resistance of the suction and delivery valves. The clearance volume includes not only the space between the cylinder end and piston at the end of its in-stroke, but also the valve pockets and ports between the valve and cylinder as in large vertical compressors, which have mechanically operated piston valve or other type at the side of the cylinder and connected to it by ports.

In the best modern practice the clearance is from 1·5 to 2 per cent of the cylinder volume in the low pressure cylinders, and about 2·5 to 3 per cent in the intermediate and high pressure cylinders, which are usually fitted with automatic valves both for suction and delivery.

In Fig. 40 the ratio MB/AB is the *apparent* volumetric efficiency of the compressor reckoned on piston displacement, neglecting slip and leakage past the piston and valves for high pressures, as well as the differences between the pressure and temperature of the air left in the cylinder and that sucked in.

With positively operated suction valves the maximum volumetric

efficiency can be attained, as in Fig. 41 ; and Fig. 42 also shows* that highly compressed air left in the cylinder must be reduced by expansion before the atmospheric pressure can overcome the resistance of the spring on the automatic suction valve.

It is obvious that the greater the clearance the longer will be the travel of the piston on the suction stroke before the air left at the higher pressure in the clearance, after discharge and leakage at the delivery valve, has expanded just below atmospheric pressure to allow the suction valve to open and admit the fresh charge. The
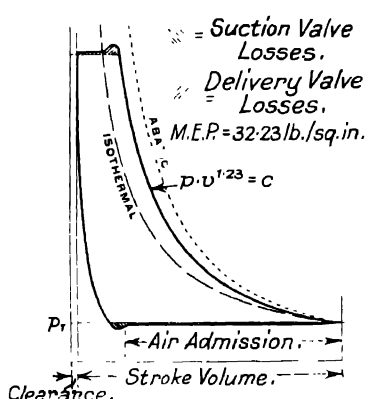
FIG. 41. SINGLE-STAGE COMPRESSION POSITIVELY OPERATED SUCTION VALVES. AUTOMATIC DELIVERY VALVES
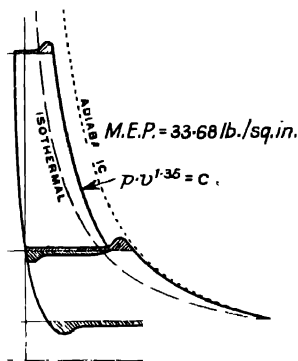
FIG. 42. TWO-STAGE COMPRESSION WITH INTERCOOLING AUTOMATIC SUCTION AND DELIVERY VALVES

Power per cub ft free air delivered   0·202 B H.P

Power per cub. ft free air delivered $\Big\{$ — 0·202 B.H.P.

result is that more piston strokes are required to deal with the same quantity of air, and with greater loss by friction

Properly designed mechanical inlet valves ensure the cylinder filling with air at atmospheric pressure, while the spring-loaded automatic valve inevitably causes attenuation due to the fact that the atmospheric pressure has to compress the spring of the suction valve so that the pressure inside the cylinder will be less than atmospheric. Consequently, the equivalent air delivered, measured in terms of free air, will be less, and the volumetric efficiency will be less. It will be noticed, in Fig. 42, that there is loss where the delivery pressure in the first stage is higher than the inter-cooler pressure ; and the suction pressure in the second stage is lower than the inter-cooler pressure, owing to the resistance of the spring-loaded valves. There is also considerable *inertia losses* due to

starting and stopping the air-flow at the suction, and in the air main on delivery of the air.

The volumetric efficiency is about 85 to 90 per cent in well-designed air compressors.

In the larger vertical double-acting compressors, with two or more stages, rated up to 5,000 cub. ft. per min. of free air delivered at 120 lb. per sq. in., and taking 1,000 B.H.P. at 240 r.p.m., the mechanically operated valves are usually fitted at the side of the cylinder, and the clearance may be 8 to 10 per cent of the cylinder volume, the actual figure depending to some extent on the stroke/bore ratio

Leakage past the piston-rings in three-stage compressors tends to increase as the pressure becomes higher, and increases the pressure at the lower stages. In modern practice, multi-stage air compressors, for pressures of 2,000 lb. per sq. in., fibre or similar packing is used as neck-rings for the final stage plunger, which is ground and accurately guided. For extremely high pressure the ground piston is packed with a hat leather, the plunger being made hollow and kept cool by circulating water through it as in large gas engine pistons.

The area of the indicator diagram, Fig. 41, from the single stage machine, is less than the combined areas of the two-stage indicator diagrams, Fig. 42, for the same delivery pressure at 100 lb. per sq. in. (gauge). The Reavell compressor of this single-stage type is fitted with cylinders of thin cast-iron liners inserted in the casing sealed against water pressure by a rubber ring, and the joint on the top face of the cylinder, which has to resist the air pressure, is made of metal to metal. Consequently, the heat transmission to the cooling water is very efficient, and the cylinder walls give up heat to the jacket water during the suction stroke ; so that the mean temperature is lower than in a double-acting compressor, with high piston speed and thick cylinder walls cast with jacket and flanges.

Regulation of Output. The three-stage air compressor, of the trunk piston type, works under best conditions at its normal capacity or rated load ; that is, the volume of free air per minute it is designed to compress and deliver at a given pressure, or the cubic feet of free air per minute displacement. In order to reduce the quantity of air delivered at the same pressure, one plan is to blow off the surplus air from the first stage inter-cooler drain by a regulating valve. Another way is to throttle or wire-draw the suction to the low pressure cylinder, but this leads to trouble : thereby the compression ratio of the high pressure cylinder is increased, for the same delivery pressure, causing a great rise of temperature ; and, further, there is the risk of the crank-case lubricant being sucked into the low pressure cylinder in excessive quantity by the reduced pressure. This lubricating oil may cause

carbonization and gumming, if carried into the high pressure cylinder, at the increased temperature, and may be ignited during compression. With leaking delivery valves there is the further danger of oil being carried over into the storage bottle or air receiver, and the highly heated air may pass back during the suction stroke and increase the compression temperature. Explosions in air receivers have been frequently traced to the ignition of the lubricating oil.

The third method is that of variable clearance control; namely, to retain a minimum clearance in the low pressure cylinder, and to greatly increase the clearance of the intermediate and high pressure cylinders, so as to vary the volume of air delivered without seriously altering the pressure and temperature.

As an example of modern practice, the quadruplex single-stage air compressor, made by Messrs. Reavell & Co., Ipswich, is single acting. Four cylinders deliver in one revolution into a common air belt. The connecting rods are forked and secured to the gudgeon. Admission is through ports in the gudgeon pin, and corresponding slots in the piston. Thus no suction inlet valves are required. There is practically no clearance, except the small port which is machined in the top of each piston. The volumetric efficiency is

Trials	1	2	3	4
Pressure in delivery main, lb. per sq. in. gauge.	120	100	75	50
Revolutions per minute . . .	309	301	306	303
Pressure in receiver and pipes, after cooling to suction temperature, lb. per sq. in. gauge .	107·5	86	64	43
Volume delivered, cubic feet free air . .	649·5	506·5	359·5	222
Volume swept by pistons, cubic feet . .	767·5	587·0	403·5	241
Volumetric efficiency, per cent . . .	84·6	86·3	89·1	92·1
Horse-power at compressor shaft . .	130	118·7	104·3	88·4
Indicated air horse-power . . .	100·4	90·7	82·1	66·8
Volume of free air actually delivered per minute at revolutions given above . . .	582·5	579	607·5	622·5
Brake horse-power per cubic foot delivered .	0·223	0·205	0·172	0·142
Weight of cooling water per min.-lb. . .	80	80	80	80
Temperature range of cooling water, Fahr. .	55·84	55·8	55·75	55·7
Horse-power at compressor shaft, B.Th.U. per min.	5510	5040	4420	3750
Heat extracted by cooling water, B.Th.U. per min.	2330	2000	1600	1200
Thermal efficiencies per cent.–				
Isothermal horse-power based on displacement / Indicated air horse-power	96·5	96	95·5	95
Isothermal horse-power based on delivered air / Indicated air horse-power	82·4	83·5	86	88·5
Overall efficiencies per cent.–				
Isothermal horse-power based on delivered air / Brake horse-power at compressor shaft	63·6	64	67·6	67

considerably higher than when automatic valves are used, because the absence of the latter enables the clearance to be lessened. The delivery plate-valve consists of ribbon steel strips seated in a circular plate having slotted ports.

Captain H. Riall Sankey made careful tests of a Reavell quad-ruplex compressor: cylinder 12·5 in. diameter by 8 in. stroke, single stage, rated at 672 cub. ft. of free air per min. displacement, and compressing to a delivery pressure of 120 lb. gauge pressure. Some of the results are as shown on previous page.

Example 1. The indicator card, Fig. 43, taken from this compressor at speed 309 r.p.m., during the first trial gives data for the calculation of indicated air horse-power at one instant. The student should plot the above results of trials as a graph on delivery pressure as base, with ordinates :

120·5 lbs $\frac{\square''}{-}$ $\mathcal{M}\mspace{1mu}\bigwedge\mspace{1mu}- ---$

Total Air Horse Power 4 Cyls = 99·8

M.E.P. 33·08 lbs$^{\square''}$
A.H.P. 24·95
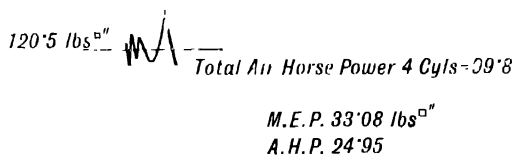
Fig. 43. Indicator Diagram from Air Compressor (*Reavell*)

(1) the overall isothermal efficiency ; (2) brake horse-power per cubic foot, and isothermal horse-power per cubic foot of free air delivered ; and (3) the volumetric efficiency.

Compressed Air Motors. Compressed air is used for a large variety of purposes, and on account of its safety, especially for the transmission of power in mines. After cooling in the compressor receiver, the compressed air flows through supply mains to the motor and, in the cylinder, does work by expansion. The cycle in the motor cylinder is like that of the air compressor reversed.

If the air after expansion is exhausted just above atmospheric pressure the temperature may be below the freezing point, then any moisture in the air would be deposited as snow, which would block the exhaust valves of the motor. In order to prevent snow and ice forming in the motor cylinder the air is warmed by steam and passed through a " preheater," or heating stove, at constant pressure, before admission to the motor. The volume of the air is thereby increased in proportion to the absolute temperature, and part of this heat energy is converted into additional work in the motor cylinder.

Let 1 lb. of compressed air from the mains in the state p_1, v_1, and T_1 (absolute) be heated at constant pressure to T, so that

$$\frac{v}{v_1} = \frac{T}{T_1}, \text{ or } v = v_1 \times \frac{T}{T_1}$$

When admitted into the motor cylinder at constant pressure p_1, Fig. 44, the work done is $p_1 v$. Next, during expansion from absolute pressure p_1 to p_2, according to the law $pv^n =$ constant, the work done $= \dfrac{p_1 v - p_2 v_2}{n - 1}$, and during exhaust at constant pressure p_2 the work done by the piston *on* the air $= - p_2 v_2$.

The total effective work done *by* the air during the cycle is

$$W = p_1 v + \frac{p_1 v - p_2 v_2}{n - 1} - p_2 v_2 - \frac{n}{n - 1}(p_1 v - p_2 v_2) ;$$

but $pv = RT$, and $W = \dfrac{n}{n - 1} . R(T - T_2) - \dfrac{n}{n - 1} . RT\left(1 - \dfrac{T_2}{T}\right)$,

also $\quad \dfrac{T_2}{T} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ Substituting this value, we have

$$W = \frac{n}{n - 1} . RT \left\{ 1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \right\} \qquad . \qquad . \quad (13)$$
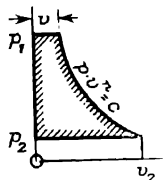
Fig. 44

Example 2. An engine is supplied with compressed air at 90 lb. per sq. in. (abs.) and 65° F. The air is expanded according to the low $pv^{1\cdot1} =$ constant, down to 15 lb., and then exhausted at that pressure. Determine the pounds of air that will be used per hour per indicated horse-power, and calculate the temperature of the air at the end of expansion. Neglect losses due to clearance, etc.

(U.L., B Sc. (Eng.).)

Take 1 lb. of air at

$T_1 = 65° + 460° = 525°$ F. (abs.) ; and $p_1 = 90 \times 144$ lb. per sq. ft.

$v_1 = \dfrac{RT_1}{p_1} = \dfrac{53 \cdot 2 \times 525}{90 \times 144} = 2 \cdot 1551$ cub. ft. per lb.

During expansion

$p_1 . v_1{}^{1\cdot3} = p_2 . v_2{}^{1\cdot3} \quad \therefore v_2 = v_1 \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{1\cdot3}} = 2 \cdot 1551 \times 6^{\frac{1}{1\cdot3}}$

Take logarithms,

$\log v_2 = \log 2 \cdot 1551 + \dfrac{1}{1 \cdot 3} \log 6 = 0 \cdot 33347 + \dfrac{0 \cdot 77815}{1 \cdot 3} = 0 \cdot 93204$

$\therefore v_2 = 8 \cdot 5515$ cub. ft.

Work done by 1 lb. of air in motor $= \dfrac{n}{n - 1}(p_1 v_1 - p_2 v_2)$

$= \dfrac{1 \cdot 3}{0 \cdot 3} \times 144(90 \times 2 \cdot 1551 - 15 \times 8 \cdot 5515)$

$= 624(193 \cdot 959 - 128 \cdot 273) = 624 \times 65 \cdot 686$ ft.-lb.

∴ Weight of air per indicated horse-power hour

$$= \frac{33{,}000 \times 60}{624 \times 65{\cdot}686} = 48{\cdot}31 \text{ lb.} \qquad \textit{Answer.}$$

Now $\dfrac{T_1}{T_2} = \left(\dfrac{v_2}{v_1}\right)^{1{\cdot}3 \; 1} = \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{1} \frac{3}{3}}$ ∴ $T_2 = T_1 \left(\dfrac{p_2}{p_1}\right)^{\frac{0}{1} \frac{3}{3}} = 525 \left(\dfrac{1}{6}\right)^{\frac{3}{13}}$

and $\log \; T_2 = \log 525 - \dfrac{3}{13} \log 6 = 2{\cdot}72016 - \dfrac{3}{13} \times 0{\cdot}77815 = 2{\cdot}540586$

∴ $T_2 = 347{\cdot}2°$ F. (abs), or $347{\cdot}2° - 460° = -112{\cdot}8°$ F.

Answer.

Example 3. What is the horse-power required to drive an air compressor which takes 20 lb. of air per min. at 60° F. and atmospheric pressure, compresses it in one stage to 6 atmospheres, and delivers the air to a receiver at this pressure. Compression curve, $pv^{1{\cdot}2}$ = constant, and mechanical efficiency of the compressor 80 per cent. Neglect losses due to clearance, cooling, leakage, etc.

Here $T_1 = 60° + 460° = 520°$ F. (abs.), and, as in Fig. 39, we have

Work done per pound of air $= \dfrac{n}{n-1} \cdot RT_1 \left[\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$

$= \dfrac{1{\cdot}25}{1{\cdot}25 - 1} \times 53{\cdot}2 \times 520 \left[\left(\dfrac{6}{1}\right)^{\frac{1{\cdot}25-1}{1{\cdot}25}} - 1 \right]$ and ∵ $\dfrac{1{\cdot}25 - 1}{1{\cdot}25} = \dfrac{1}{5}$

$6^{\frac{1}{5}} = 1{\cdot}431$

$= 5 \times 53{\cdot}2 \times 520(1{\cdot}431 - 1) = 59{,}617$ ft.-lb.

∴ Work for 20 lb. air per min. $= 59{,}617 \times 20$ ft.-lb. per min.

∴ Horse-power required $= \dfrac{59{,}617 \times 20}{33{,}000} \times \dfrac{100}{80} = 45{\cdot}16.$ *Answer.*

To check, take the volume of 1 lb. dry air at N. T. P.

$= 12{\cdot}39$ cub. ft. $= \dfrac{1}{{\cdot}0807}$

∴ volume of 1 lb. dry air at 1 atmosphere and 60° F.

$= 12{\cdot}39 \times \dfrac{520}{492} = 13{\cdot}09$ cub. ft.

∴ volume of 20 lb. of free air at 60° F. $= 13{\cdot}09 \times 20 = 261{\cdot}8$ cub. ft.

Now $p_1 \cdot v_1^{1{\cdot}25} = p_2 \cdot v_2^{1{\cdot}25}$, and $\left(\dfrac{v_2}{v_1}\right)^{\frac{5}{4}} = \dfrac{p_1}{p_2} = \dfrac{1}{6}$

∴ $v_2 = v_1 \times \left(\dfrac{1}{6}\right)^{\frac{4}{5}}$, and $\log v_2 = \log 261{\cdot}8 - \dfrac{4}{5} \log 6 = 1{\cdot}7954$

$v_2 = 62{\cdot}43$ cub. ft.

$$W = \frac{n}{n-1}(p_2v_2 - p_1v_1) = \frac{1 \cdot 25}{0 \cdot 25} \times 144 \times 14 \cdot 7(6 \times 62 \cdot 43 - 1 \times 261 \cdot 8)$$

$$= 720 \times 1657 \cdot 86 \text{ ft.-lb.}$$

and horse-power required $= \dfrac{720 \times 1657 \cdot 86}{33,000} \times \dfrac{100}{80} = 45 \cdot 2$ *Answer*.

The amount of heat that escapes through the cylinder walls to the cooling water may be estimated by equation (14) (p. 87),

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{work done} = \frac{1 \cdot 4 - 1 \cdot 25}{1 \cdot 4 - 1} \times \frac{59617 \times 20}{1400} \text{ C.H.U. per min.}$$

$$= 319 \cdot 38 \text{ C.H.U. per min., equivalent to} \frac{319 \cdot 38 \times 1400}{33,000} = 13 \cdot 54 \text{ H.P}$$

Example 4. Air under atmospheric conditions of 15 lb. per sq. in. and 10° C., is drawn into the low pressure cylinder of a two-stage air compressor, and is compressed adiabatically to 100 lb. per sq. in. The air is then cooled at constant pressure to 20° C., and is drawn into the high pressure cylinder, where it is compressed to 600 lb. per sq. in., at which pressure it is discharged into the reservoir. Calculate the horse-power of the compressor which thus deals with 800 cub. ft. of air per hour measured at atmospheric conditions.

(*U.L.*, *B.Sc.* (*Eng.*))

Let p_1, v_1, T_1 and p_2, v_2, T_2 be the pressure, volume, and temperature, respectively, at start and end of the first stage ; also p_2, v_2', T_2' and p_3, v_3, T_3 at the start and end of the second stage.

Neglecting clearance volume, the work done in compressing 1 lb of air under above conditions, by equation (3), is

$$W \quad \frac{\gamma}{\gamma - 1} \cdot p_1v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] + \frac{\gamma}{\gamma - 1} \cdot p_2v_2' \left[\left(\frac{p_3}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \text{ft.-lb.}$$

$$= \frac{\gamma}{\gamma - 1} \cdot RT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] + \frac{\gamma}{\gamma - 1} \cdot RT_2' \left[\left(\frac{p_3}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \text{ft.-lb.}$$

$$= \frac{1 \cdot 4}{0 \cdot 4} \times 96 \left\{ 283 \left[\left(\frac{100}{15}\right)^{\frac{0 \cdot 4}{1 \cdot 4}} - 1 \right] + 293 \left[\left(\frac{600}{100}\right)^{\frac{0 \cdot 4}{1 \cdot 4}} - 1 \right] \right\} \text{ft.-lb.}$$

$$= 336 \left\{ 283 \left(\frac{20^{\frac{3}{7}}}{3} - 1\right) + 293(6^{\frac{2}{7}} - 1) \right\} \text{ft.-lb.}$$

$$= 336 \left\{ 283(1 \cdot 7195 - 1) + 293(1 \cdot 6685 - 1) \right\}$$

$$= 336 \left\{ 203 \cdot 62 + 195 \cdot 87 \right\} = 336 \times 399 \cdot 49$$

$$= 134,230 \text{ ft.-lb. per lb. of air.}$$

The compressor has to deal with 800 cub. ft. of air per hour, measured under atmospheric conditions, i.e.

$v = 800$ cub. ft. when $p = 15$ lb. per sq. in., and $T = 283°$ C. (abs.).

Now $\quad pv = w \cdot RT$, where w = weight of air $= \dfrac{pv}{RT}$

$$w = \frac{144 \times 15 \times 800}{96 \times 283} = \frac{18,000}{283} = 63 \cdot 6 \text{ lb.}$$

∴ work in compressing 63·6 lb. of air per hour

$\quad = 63 \cdot 6 \times 134,230$ ft.-lb. per hour.

∴ horse-power of compressor $= \dfrac{63 \cdot 6 \times 134,230}{33,000 \times 60}$ 4·31 H.P.

Answer.

Students may check this result by calculating the changes in volume of air by adiabatic compression, i.e. from 800 to 206·34 cub. ft. in the low pressure cylinder, and from 124·24 to 34·548 cub. ft. in the high pressure cylinder ; also determine the amount of heat given per hour by the air to the water in the inter-cooler.

Example 5. Determine the size of cylinder for a double-acting air compressor of 50 I.H.P., in which the air is drawn in at 15 lb. pressure and 60° F., and compressed, according to the law $pv^{1\cdot2}$ = constant, to 90 lb. pressure. Revolutions per minute, 100 ; average piston speed, 500 ft. per min. Neglect clearance. (*U.L., B.Sc. (Eng.*).)

Given $p_1.v_1^{1\cdot2} = p_2.v_2^{1\cdot2}$, ∴ $\dfrac{v_1}{v_2} = \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{1\cdot2}} = \left(\dfrac{90}{15}\right)^{\cdot\dot{5}} = 6^{\cdot\dot{5}} = 4 \cdot 451$

where p_1 and p_2 are pounds per square inch, and v_1 and v_2 cubic feet.

Mean effective pressure p_m lb. per sq. in. $= \dfrac{\text{Work done per cycle}}{\text{Stroke volume}}$.

Then $144 \times p_m = \dfrac{1 \cdot 2}{0 \cdot 2}\left(\dfrac{p_2 v_2 - p_1 v_1}{\cdots}\right) \times 144$

and $\quad p_m = 6 \quad {}^{-p_1} = 6\left\}\dfrac{90}{4 \cdot 451} - 15\right.$

$\quad = 6(20 \cdot 2203 - 15) = 31 \cdot 3218$ lb. per sq. in.

Let A be the sectional area of cylinder in square inches,

$(A \times 31 \cdot 3218) \times 500 = 50 \cdot \times 33,000$ ft.-lb. per min.

$$A \quad \frac{3300}{31 \cdot 3218} = 105 \cdot 36 \text{ sq. in.}$$

Diameter of cylinder $= \sqrt{\dfrac{105 \cdot 3\dot{5}}{0 \cdot 7854}} = 11 \cdot 58$ in. *Answer*.

A double-acting compressor at speed 100 r.p.m. gives 200 piston strokes per min., and the average piston speed = 500 ft. per min.

$$\text{piston stroke} = \frac{500}{200} = 2\cdot5 \text{ ft.} \qquad \textit{Answer.}$$

Example 6. A two-stage air compressor is to deliver air at 800 lb. per sq. in. pressure. The cylinders have the same stroke and the air is cooled to atmospheric temperature, 15° C., in the inter-cooler. Determine the ratio of the cylinder diameters so that the power required to drive the compressor shall be a minimum. Find the work required to compress and deliver a pound of air. Take atmospheric pressure as 15 lb. per sq. in. Assume adiabatic compression.   (*U.L., B.Sc. (Eng.)*, 1924.)

In a two-stage compressor, assuming that air enters the second cylinder at atmospheric temperature, as in Fig. 40, and neglecting friction, leakage, etc., the work done in compressing 1 lb. of air from p_1 to p_2 lb. per sq. ft., by equation (5), is

$$W = \frac{n}{n-1} \cdot p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right]$$

It has been shown (p. 128), by differentiating and equating to zero, that the above expression for work is a minimum when

$$p_2 = \sqrt{p_1 p_3},$$

from which it follows, by equation (9), that

$$W = \frac{2n}{n-1} \cdot p_1 v_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right] \qquad \blacktriangleright$$

As air is at the same temperature when admitted to the high pressure cylinder as when admitted to the low pressure cylinder, then $p_1 v_1 = p_2 v_2$, also $p_2 = \sqrt{p_1 p_3} = \sqrt{15 \times 800} = 109\cdot544$

$$\therefore \frac{v_1}{v_2} = \frac{p_2}{p_1} = \frac{109\cdot544}{15} = 7\cdot3,$$

and the cylinders have same stroke.

$$\therefore \frac{d_1}{d_2} = \sqrt{7\cdot3} = 2\cdot7 \text{ — ratio of cylinder diameters.} \quad \textit{Answer}$$

The work done to compress and deliver 1 lb. of air, by equation (9), as above,

$$W = \frac{2n}{n-1} \cdot RT_1 \cdot \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right],$$

for air $n = \gamma = 1\cdot4$, and $\dfrac{n-1}{2n} = \dfrac{1\cdot4 - 1}{2\cdot8} = \dfrac{0\cdot4}{2\cdot8} = \dfrac{1}{7}$.

$$\therefore W = \frac{2\cdot8}{0\cdot4} \times 96 \times 288 \left[\left(\frac{800}{15}\right)^{1/7} - 1 \right] \text{ ft -lb}$$

$$= 193{,}536 \,|\, 1\cdot764875 - 1 \,| = 148{,}030 \text{ ft.-lb.} \quad \textit{Answer.}$$

Example 7. Prove that the ratio of throat pressure to initial pressure in a convergent-divergent nozzle is $\left(\dfrac{2}{n+1}\right)^{\frac{n}{n-1}}$, when n is the adiabatic index for expansion, and find an expression in terms of the initial pressure and specific volume for the discharge per square foot of throat area per second.

<div align="right">(<i>U.L., B.Sc. (Eng)., 1924.</i>)</div>

Suppose a gas to flow through a convergent-divergent nozzle, Fig. 45, from a chamber where the pressure is p_1 lb. per sq. ft. to a chamber where the pressure is p_3 lb. per sq. ft. Let p_3 be less than half p_1. Assume the pressure at the throat $= p_2$ lb. per sq. ft.

Let A = area of throat in square feet.

v_1 and v_2 = specific volumes of gas, at pressures p_1 and p_2, in cubic feet per pound.

V = velocity of gas at throat in feet per second.
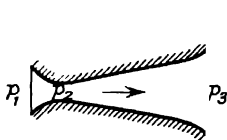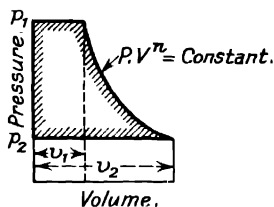
FIG. 45   FIG. 46

Then discharge

$$W = \frac{AV}{v_2} \text{ lb. per sec.} \qquad \frac{W}{A} \qquad \frac{V}{v_2} \tag{1}$$

It will be found, when we get an expression for V in terms of p_1, p_2, v_1, v_2, and n, that the ratio $\dfrac{V}{v_2}$ reaches a maximum and then diminishes indefinitely. This means that for any discharge the cross-section of the nozzle must have a " throat " or minimum area.

From the pv diagram, Fig. 46, we get

Heat drop = gain of kinetic energy = $\dfrac{V^2}{2g}$

$$= \frac{n}{n-1} [p_1 v_1 - p_2 v_2] \text{ ft.-lb. per lb. of gas}$$

$$\therefore V = \sqrt{2g \cdot \frac{n}{n-1} \cdot (p_1 v_1 - p_2 v_2)} \text{ ft. per sec}$$

As the expansion is adiabatic, we have

$$p_1 . v_1^n = p_2 . v_2^n \qquad v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

Hence $\dfrac{W}{A} = \dfrac{V}{v_2} = \dfrac{1}{v_1}\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}} \sqrt{2g \times \dfrac{n}{n-1}(p_1v_1 - p_2v_2)}$

$\qquad = \dfrac{1}{v_1}\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}} \sqrt{\dfrac{2gn}{n-1}\left[1 - \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]p_1v_1}$

$\qquad = \sqrt{\dfrac{2gn}{n-1} \cdot \dfrac{p_1}{v_1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\dfrac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$ . . . (2)

$\dfrac{W}{A}$ will be a maximum, when $\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\dfrac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]$ is a maximum

Let $\quad \dfrac{p_2}{p_1} = x$, then $y = x^{\frac{2}{n}} - x^{\frac{n+1}{n}}$,

differentiating, $\quad \therefore \dfrac{dy}{dx} = \dfrac{2}{n} \cdot x^{\frac{2}{n} - \frac{n}{n}} - \dfrac{n+1}{n} \cdot x^{\frac{1}{n}}$

For a maximum, $\dfrac{2}{n} \cdot x^{\frac{2}{n} - \frac{n}{n}} = \dfrac{n+1}{n} \cdot x^{\frac{1}{n}}$

$\dfrac{2}{n+1} = x^{\frac{n-1}{n}} \quad \therefore x = \left(\dfrac{2}{n+1}\right)^{\frac{n}{n-1}}$

$\therefore \dfrac{p_2}{p_1} = \left(\dfrac{2}{n+1}\right)^{\frac{n}{n-1}}$

Hence maximum discharge from equation (2),

$$W = A\sqrt{\dfrac{2gn}{n-1} \cdot \dfrac{p_1}{v_1}\left[\left(\dfrac{2}{n+1}\right)^{\frac{2}{n-1}} - \left(\dfrac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]} \quad Answer.$$

Example 8. A single-stage air compressor displaces 1 cub. ft. per working stroke. The clearance is to be taken as 0·15 cub. ft. (this is excessive) for the purposes of this question. During the compression the index n can be taken as 1·3, and at the end of the delivery stroke the air in the clearance space is at a temperature of 100° C. The law of the expansion of the clearance air is $n = 1·35$. Find the actual volume of air, at atmospheric pressure and temperature, sucked in per stroke. Assume suction pressure 14·7 lb. per sq. in. and temperature of admitted air 14° C. Delivery pressure at 80 lb. per sq. in.
(*U.L., B.Sc. (Eng.)*, 1920.)

Suppose the heat gained or lost to the cylinder walls is negligible, and the clearance air expands to 14·7 lb. per sq. in. before atmospheric air is freely admitted.

To find the temperature and volume at end of expansion of clearance air,

$p_1.v_e^{1·35} = p_2.v_c^{1·35}$, or $\left(\dfrac{v_e}{v_c}\right)^{1·35} = \dfrac{p_2}{p_1}$.

$v_e = v_c\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{1·35}} = 0·15\left(\dfrac{80}{14·7}\right)^{\frac{1}{1·35}} = ·15 \times 3·5076 = 0·526$ cub. ft.

On Fig. 47, ad = piston displacement or stroke volume = 1 cub ft. Weight of clearance air

$$= \frac{p_2 \cdot v_c}{R \cdot T_c} = \frac{80 \times 144 \times 0.15}{96 \times 373} = \frac{18}{373} = 0.04826 \text{ lb.}$$

Let temperature of clearance air at end of expansion be T_e (abs.)

then $\quad \dfrac{T_c}{T_e} = \left(\dfrac{p_2}{p_1}\right)^{\frac{1.35-1}{1.35}} = \left(\dfrac{80}{14.7}\right)^{\frac{7}{27}} = 1.5515$

$\therefore T_e = \dfrac{T_c}{1.5515} = \dfrac{373}{1.5515} = 240.4°\text{ C. (abs.}$
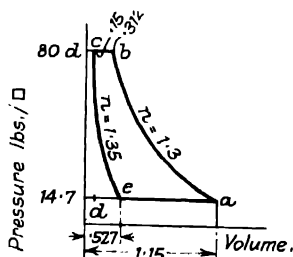
FIG. 47

Again,

let $\quad T_a$ = temperature (°C. abs.) of air in cylinder at end of suction stroke,

volume $\quad v_a = 1.15$ cub. ft., since piston displacement is *one* cub. ft.

and pressure $p_1 = 14.7$ lb. per sq. in.

$\therefore$ weight of air in cylinder $= \dfrac{p_1 \cdot v_a}{R \cdot T_a} = \dfrac{14.7 \times 144 \times 1.15}{96 \times T_a} = \dfrac{25.358}{T_a}$ lb

Neglecting work done during suction stroke, T_a is the resultant temperature obtained by mixing

$\left(\dfrac{25.358}{T_a} - 0.04826\right)$ lb. of air at 14° C. (287° C. abs.)

with 0.04826 lb. of air at 240.4° C. (abs.)

$\therefore T_a = \dfrac{287 \left(\dfrac{25.358}{T_c} - 0.04826\right) + 0.04826 \times 240.4}{\dfrac{25.358}{T_a}}$

$25.358 = \dfrac{7277.6}{T_a} - 13.85 + 11.6 = \dfrac{7277.6}{T_a} - 2.25$

$\therefore T_a = \dfrac{7277.6}{27.608} = 263.6°\text{ C. (abs.).}$

Hence weight of air drawn into cylinder

$$= \frac{25 \cdot 358}{263 \cdot 6} - 0 \cdot 0482 = 0 \cdot 0962 - 0 \cdot 0482 = 0 \cdot 048 \text{ lb.}$$

and the volume of 0·048 lb. of air at 14·7 lb. per sq. in. and 14° C

$$\frac{0 \cdot 048 \times 96 \times 287}{14 \cdot 7 \times 144} = \underline{0 \cdot 625 \text{ cub. ft.}} \qquad \textit{Answer}$$

= volume of free air sucked in per stroke at atmospheric pressure and temperature.
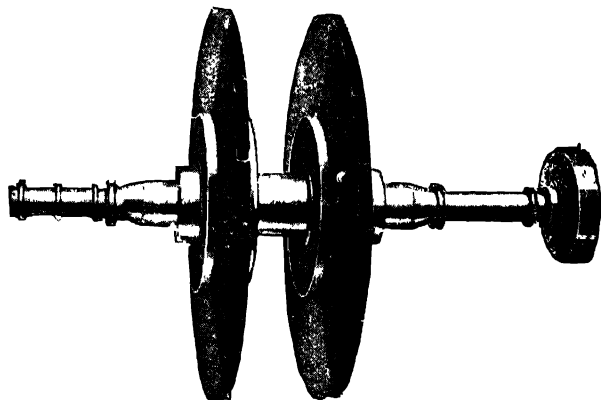
Fig. 48. Two-stage Rotor of a Turbo-Compressor

To check this result,

$$p_a \cdot v_a{}^{1 \cdot 3} = p_b \cdot v_b{}^{1 \cdot 3} \quad \therefore \frac{v_a}{v_b} = \left(\frac{80}{14 \cdot 7}\right)^{\frac{1}{1 \cdot 3}} = 3 \cdot 6811$$

$$\therefore v_b = \frac{v_a}{3 \cdot 6811} = \frac{1 \cdot 15}{3 \cdot 6811} = 0 \cdot 3124 \text{ cub. ft}$$

and $\qquad \dfrac{T_b}{T_a} = \left(\dfrac{p_b}{p_a}\right)^{\frac{1 \cdot 3 - 1}{1 \cdot 3}} = \left(\dfrac{80}{14 \cdot 7}\right)^{\frac{\cdot 3}{1 \cdot 3}} = 1 \cdot 48$

$$\therefore T_b = 263 \cdot 6 \times 1 \cdot 48 = 390° \text{ C. (abs.)}$$

$\therefore$ weight of air in cylinder at b

$$= \frac{p_2 v_b}{R T_b} = \frac{80 \times 144 \times 0 \cdot 3124}{96 \times 390} = 0 \cdot 096 \text{ lb}$$

But the weight of clearance air = 0·048 lb.

$\therefore$ weight of air delivered = 0·096 − ·048 = 0·048 lb. per cycle,

which is the weight of air sucked in per stroke, neglecting friction and leakage past the piston and at valves.

Turbo-Compressors are taking the place of the ordinary reciprocating piston type of compressor, especially when large volumes of air are dealt with, for varying loads of more than 4,000 cub. ft. per min. at delivery pressure of 80 to 100 lb. per sq. in. (gauge), as used in mining installations for the supply of air to drills, coal cutters, haulers, and other pneumatic appliances.

Turbo-Blowers, or compressors having low compression ratio, made of rated output from 500 cub. ft. per min. upwards, are suitable for many appliances, i.e. blast air to furnaces, boosters for delivery of gas from gas works, and for Diesel marine engines as scavenge blowers and superchargers. The same type of turbo
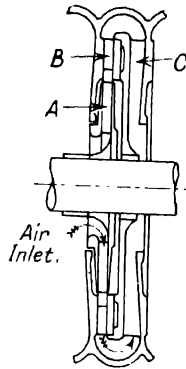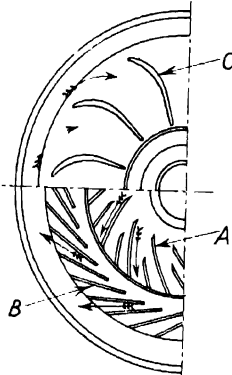
FIG. 49   FIG. 50

REAVELL TURBO-COMPRESSOR

machine is used for exhausting. both in gas works and in connection with the bulk method of unloading grain from ships by suction.

A steam turbine for direct coupling gives the most suitable drive for turbo-compressors on account of its great flexibility of speed between 3,000 and 10,000 r.p.m., and ease of control.

Fig. 48 shows a two-stage *rotor* of a turbo-compressor, consisting of built-up impellers, each having a main and a cover disc with the necessary blades riveted to each, and carried on a shaft which runs on roller bearings. Figs. 49 and 50 are sections through one stage of a Reavell high pressure turbo-compressor.

The three essential parts are : *A*, the rotor with impeller blades ; *B*, the *diffuser guide blades,* polished perfectly smooth, and fitted in the annular space of the stationary casing, or *stator,* surrounding the impeller disks. These diffuser blades are made from thin flexible strips of sheet steel or bronze, and held in position by distance pieces carefully shaped and fitted into a dove-tailed groove in the stator. The diffuser blades have a close pitch to ensure very fine adjustment of the small inlet angle, necessary for variation

of the volume of air to be delivered. *C*, the *guide blades or vanes* required to deliver the compressed air to the next stage with as little frictional loss as possible. The arrows, Figs. 49 and 50, show the direction of the flow of air into the impeller *A*, through the diffuser *B*, and between the guide blades *C*, so passing into the next stage, where the same process takes place.

The air, in passing through the diffuser, has its kinetic energy converted into pressure energy. The *conditions of air flow* for the efficient conversion of energy are : (1) In order to reduce frictional losses, the diffuser passages and surfaces over which the air passes must be polished as smooth as possible ; (2) the close pitch of the blades must ensure guidance of the air flow in stream-line formation, so as to avoid eddies which would dissipate the pressure energy in heat ; (3) the shape of the blades must provide a gradual tapering
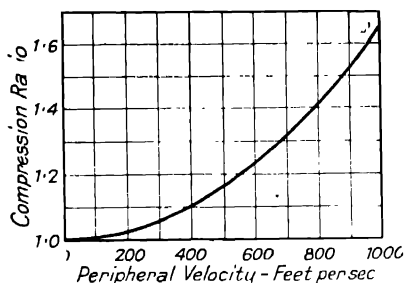
Fig. 51
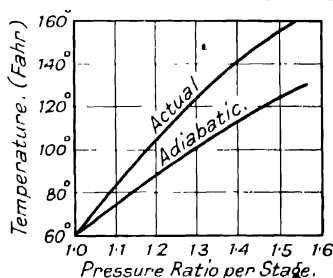
Fig. 52

of the sections of the guide passage, without sharp bends, and deflect the flow of air from a tangential into a radial direction, in the shortest path, *without shock*.

It is obvious that the higher the peripheral speed at the tip or rim of the rotor disks, the greater will be the velocity of the air projected into the diffuser. Peripheral speed is limited by the stresses in the metal due to centrifugal force, and special alloy steels, like nickel-chrome steel, allow of speeds up to 800 ft. per sec. in common practice. The curve, Fig. 51, shows that a velocity of 800 ft. per sec. produces a compression ratio of the outlet pressure (absolute) to inlet pressure (absolute) equal to 1·4, that is, 40 per cent rise of pressure in a single stage. A modern turbo-compressor will produce a pressure of 100 lb. per sq. in. (gauge) with 8 to 10 such stages.

Cooling. In the piston type of compressor the temperature at the end of compression is usually less than that due to adiabatic compression, by reason of heat transfer to the cylinder walls ; but in the turbo-compressor the maximum temperature attained at the outlet of the diffuser blades is generally above that of adiabatic compression with the same compression ratio, owing to the skin friction and scrubbing of the air at high velocity between the impeller

disks and the walls of the casing, since the resistance offered by friction to the motion of the air is proportional to the square of the velocity. The actual temperature rise, shown in Fig. 52, is from 60° F. to 140° F. per stage, under the above conditions of peripheral speed. The mean density of the air during its passage through the impeller is thereby reduced, as also the delivery pressure whilst the energy absorbed at the compressor shaft is increased. For instance, in a compressor of rated output, 10,000 cub. ft. of free air per min., this mass of air passing per minute weighs 765·2 lb. and the heat energy due to the rise of its temperature 80° F., amounts to

$$80 \times 765 \cdot 2 \times 0 \cdot 24 = 14,692 \text{ B.Th.U. per min.}$$

Now 1 H.P. $= \dfrac{33,000}{778}$, equivalent to 42·416 B.Th.U. per min.,

hence the heat energy generated per stage in the compressor is equivalent to

$$\frac{14,692}{42 \cdot 416} = 346 \cdot 38 \text{ H.P.},$$

and this heat energy must be abstracted by the cooling water in order to reduce the air to the original inlet temperature before it enters the next stage.
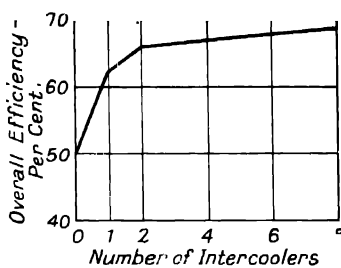
Fig. 53

The tendency in practice is to increase the disk speed in order to effect the total compression in fewer stages, but this means a still larger temperature rise per stage. Hence it is necessary to employ the most effective methods of cooling.

The casing of the turbo-compressor may have a water jacket, but the cooling surface obtainable is not sufficient to abstract all the heat, and in addition *inter-coolers*, like the condenser of a steam turbine, are usually placed below the compressor casing. The simplest construction appears to be not to jacket the casing, and to use an inter-cooler after the first and second stages, and every third stage. The diagram, Fig. 53, for a compressor delivering 10,000 cub. ft. of free air per min., compressed in nine stages to a gauge pressure of 100 lb. per sq. in., is based on the assumption that the temperature of the air is brought down to suction temperature after each inter-cooler. 'The greatest gain is from the first two coolers, and only a small gain from the cooler after every other stage. The over-all efficiency per cent is equal to the

$$\frac{\text{Ideal isothermal horse-power}}{\text{Brake horse-power}} \times 100.$$

Governing. In the earliest types of turbo-compressors, the diffuser guide blades were either omitted altogether or rough ribs were cast with the casing, and the loss by friction was excessive. To reduce this loss and obtain the exact inlet angle for all the blades, these were cast in rings and inserted in the stator. When the volume of air required exceeded the normal output, there was
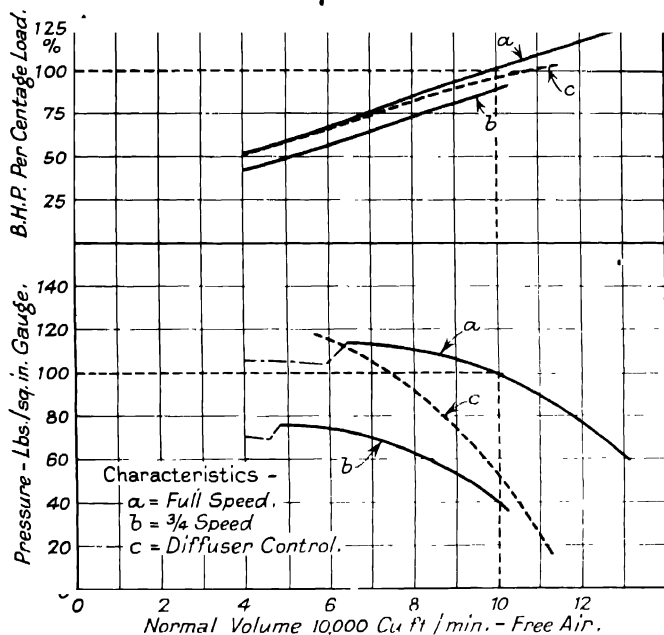
FIG. 54. CHARACTERISTIC CURVES OF HIGH PRESSURE
TURBO-COMPRESSOR

a gradual fall in the pressure of the delivered air, and much greater brake horse-power was required at the driving shaft.

A marked advance was made in the Reavell design, giving greater control and flexibility for considerable variations in volume.

The characteristic curves, Fig. 54, indicate the relation between the pressure and output volume of air, also the horse-power curves corresponding to the variation of output in terms of volume, under the different conditions.

For varying loads, the angle of the diffuser blades, *B*, Fig. 50, is altered by means of an external hand wheel. The effect is to alter the characteristic curve from an output corresponding to the pressure volume curve *a*, at normal full speed, to that of *c*, at one point of diffuser control and full speed. This method of control is of great advantage in colliery work, where practically constant

pressure is required, but considerable variations in volume are demanded.

Also, by reducing the entrance angle, much smaller volumes can be delivered at a lower pressure, as shown by the curve *b*, for the usual speed reduction anticipated. In both cases the overload or brake horse-power at the driving shaft is reduced.

Measurement of Output of Air Compressors. One of the largest compressed air installations known to the Author is that in connection with the Victoria Falls and Transvaal Power Scheme, in
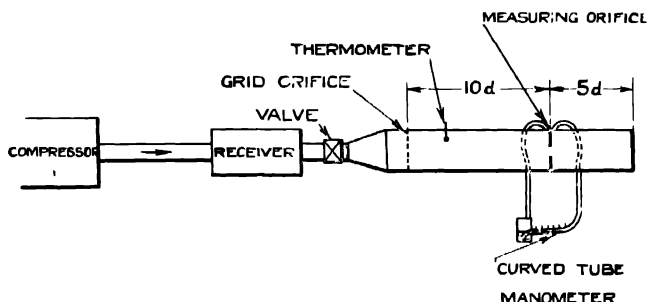
FIG. 55. MEASUREMENT OF AIR-COMPRESSOR DISCHARGE
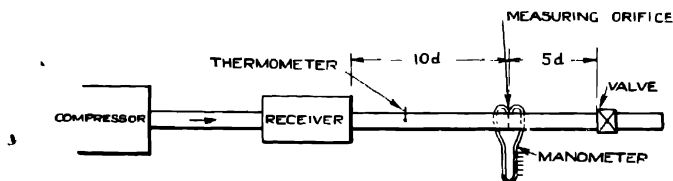
FIG. 55A MEASUREMENT OF COMPRESSED AIR

which the air is compressed to a pressure of 100 to 120 lb. per sq. in, (gauge) by multi-stage turbo-compressors, and distributed along mains over 32 miles in length to the mines on the Rand. The capacity of the air meters installed is over 300 million horse-power hours per annum. The meters register in kilowatt-hour units the energy necessary to compress the air supplied from the mean atmospheric pressure 12 lb. per sq. in., and the temperature at Johannesburg, to the pressure of delivery, on the assumption that the compression is isothermal, of a fixed overall efficiency. The maximum rate of supply of compressed air is 5,000 tons of free air, compressed to 120 lb. per sq. in. (gauge) and delivered each working day, or a yearly output of over 2,000,000 tons of compressed air.

Mr. John L. Hodgson carried out the scientific research work, invention, and design of the various appliances for metering this compressed air, and registering the power accurately to within

$\pm$ 1 per cent.   He found that one general formula (p. 153) and two curves of coefficients of discharge suffice to represent all types of flow for an orifice of given proportions for any one easily compressible, elastic fluid such as air, and for all relatively incompressible fluids such as water.

Air compressors are rated by the volume of air actually delivered at the given pressure, expressed in cubic feet of " free air " per minute, that is, the volume of air delivered when expanded down to atmospheric conditions at 14·7 lb. per sq. in. (abs.) and 60° F.

Fig. 55 shows the most accurate arrangement of measuring air compressor discharge by means of an orifice ; although Fig. 55a is the more usual way.   The compressor delivers air at the required
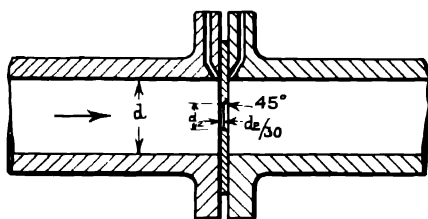
Fig. 56.   Square-edged Circular Orifice

pressure into a receiver of sufficient capacity, and a control valve regulates the flow from the receiver so as to maintain the full pressure in the receiver.   The air is expanded through the valve, down to nearly atmospheric pressure, along the necessary length of straight pipe, 10 and 5 times the diameter of the delivery pipe as shown in Fig. 55, free from valves, bends, tees, and obstructions on the up-stream and down-stream sides of the measuring orifice. A slight pressure difference will then be set up at the orifice, between the receiver and atmosphere, which remains constant when a steady state of flow and equilibrium is reached.   This pressure difference furnishes the means of determining the volume of air delivered per minute by the compressor.

The square-edged circular orifice in a thin plate, mounted between flanges, as shown in Fig. 56, forms one of the best secondary standards of flow measurement.   The orifice is bevelled off on the downstream side at an angle of 45°.   The orifice plate must be thick enough to resist distortion under the difference of pressure.

The discharge coefficients, Table I, apply to orifices with the pressure holes in the plane of the orifice, Fig. 56, and their position in similar orifices for different sizes of pipes should be geometrically similar in order to have the coefficients of discharge identical for the same ratio d_2/d_1.   The advantage of the square-edged orifice is that it is cheaper to make and more easily reproduced with accuracy than a nozzle ; so that if the coefficient of discharge for a particular

ratio of d_2/d_1, and for particular positions of the pressure holes in relation to it, has once been accurately determined, this same coefficient of discharge can be used at corresponding velocities for any other size of orifice in which all these things are geometrically similar.

It has been found by Mr. John L. Hodgson that it is possible, in the case of any one fluid, such as air, to determine experimentally the values of the discharge coefficient, C or Ωa, for various values of the expansion ratio p_2/p_1 for the pressures across the orifice
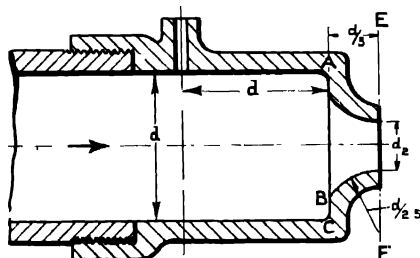
FIG. 57. SHAPED NOZZLE

This discharge coefficient allows for the effects of friction through the orifice, contraction of the jet at the orifice, and the effects of compressibility of the fluid.

The values given in Table I are based on the data obtained by using a square-edged circular orifice—

TABLE I

VALUES OF DISCHARGE COEFFICIENT, $C = \Omega a$

$\dfrac{d_2}{d_1}$	p_2/p_1								
	1·0	·97	·95	·90	·8	·7	·6	·5	·4
·8	·600	·599	·597	590	·565	·540	·508	·475	·443
·6	·630	·627	·622	615	·595	·570	·542	·520	·491
·4	·620	·620	·618	·615	·600	·580	·554	·530	·500
·2	·620	·620	·618	·615	·600	·580	·554	·530	·500
·1	·620	·620	·618	·615	·600	·580	·554	·530	·500

This orifice can be used for measuring air velocities for any value of p_2/p_1 between unity and zero, provided that the value of the discharge coefficient, Ωa, is known for a geometrically similar orifice at the value of p_2/p_1 worked at. It is, however, *preferable and usual* to work at values of p_2/p_1 which lie between unity and 0·97, as then practically no allowance has to be made for the compressibility of the air. Moreover, for an orifice having d_2/d_1 less than 0·6, the coefficient of discharge Ωa is practically constant over a wide range of flow, both for gas and liquids.

The shaped nozzle, Fig. 57, may be fixed at the end of the pipe line, as in Fig. 58. The special point to be noted in the design, Fig. 57, is that the centre of the radius of curvature of the nozzle must always be on the line EF, which passes through the edge of the nozzle. When d_2 is so great that the point B, where the curve of the nozzle intersects the line AC, would be farther from the centre line of the nozzle than the point C, the curve must be struck so as to pass through the point C.

Careful tests show that the discharge coefficient of a nozzle, shaped as in Fig. 57, is about 0·985, and is quite as good as more elaborately curved standard nozzles. But, owing to the difficulty
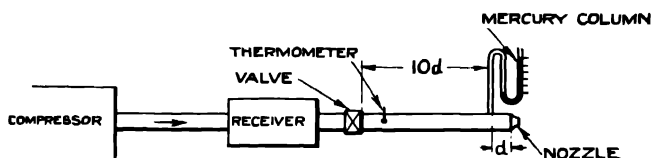
FIG. 58. MEASUREMENT OF COMPRESSED AIR BY NOZZLE

of machining the walls of the nozzle to an exact radius and of avoiding small abrupt changes of curvature in them, the coefficient may be in error by 3 or 4 per cent for any particular nozzle.

TABLE II

VALUES OF DISCHARGE COEFFICIENT, " c " $= \Omega\beta$

For shaped Nozzles, when p_2/p_1 is less than 0·55

d_2/d_1	0·8	·6	·4	·2	·1
" c " $= \Omega\beta$	0·433	·455	·475	·480	·480

The values of $\Omega\beta$ given in Table II apply to nozzles either fixed, as in Fig. 58, or between flanges of the pipe line, provided that d_2 is not greater than about $\frac{1}{4}$ of the diameter of the pipe. In the same way, the values of Ωa, given in Table I, would apply to square-edged orifices discharging into free air, provided d_2 was not greater than about $\frac{1}{4}d_1$.

When measuring air discharges with an orifice or a nozzle, it is necessary to ensure that any water that gets into or condenses in the pressure pipes, will not vitiate the readings. With this end in view, the pressure pipes should be taken out at the top or side of the main, rather than at the bottom. They should be of sufficient diameter to prevent water hanging in them at corners and bends, and they should be laid at a slope to drain, either back into the main or into drainage sumps provided with blow-off cocks placed at their lowest points. Further, there should be no U bends in which water can collect.

The difference of pressure can be measured either by means of a **U** gauge, as in Fig. 55a, or a single tube manometer, or a curved tube manometer, Fig. 59, as shown in Fig. 55. The advantage of the latter is that it gives an equally spaced flow scale down to small fractions of full flow. A screw operates a displacer, which enables the zero to be set when the manometer is equalized without changing the liquid in the reservoir: Drainage sumps fitted with cocks remove any moisture present in the air or gas.

The air from the compressor must be expanded down to nearly atmospheric pressure. This, Fig. 55, is the most accurate method of measuring compressor discharges, for quite another reason than that it allows the most sensitive manometers to be used. The discharge varies as the square root of the difference of pressure across the orifice, so that if the mean of this difference is observed, and the *flow is pulsating*, the discharge calculated from the observations would be considerably in error, owing to the fact that the flow depends upon the mean of the square roots of the differences of pressure, and not on the mean of the differences of pressure.

A further source of error is that there may be *pulsations of both pressure and velocity*, and the pulsations of pressure that travel along the pressure pipes may arrive at the manometer at different times, and so cause spurious readings. It is, therefore, imperative to get rid of such pulsations of flow when using an orifice for testing purposes. This can be effected by discharging the compressor into a receiver, and then expanding the air through a control valve nearly to atmospheric pressure. This is the best way to damp out pulsations before the air reaches the measuring orifice.
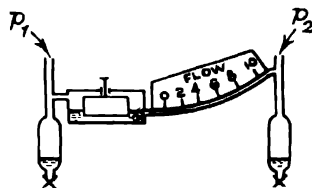
Fig. 59. Curved Tube Manometer

For very accurate work a cellular grid, or a plate drilled with a number of suitably placed holes, Fig. 55, should be placed upstream 10 diameters of the delivery pipe from the orifice, so as to guide the flow in parallel streams of air, diffuse the flow uniformly over the section, and to take out the spiral motion or swirl, which would lower the coefficient of discharge.

If a turbo-compressor is being tested which discharges air without pulsations, obviously a receiver is not necessary.

A point to be noted when using **U** tubes for measuring pressure, or difference of pressure, is that the bore of the tube must be sufficiently large to eliminate surface tension errors. With a manometer of about $\frac{1}{8}$ in. bore (which is frequently used for mercury), the error due to surface tension may easily be 0·2 or 0·3 of an inch of mercury. In the same way, the surface tension error will be 0·2 or 0·3 of an inch of water gauge in a tube of about 0·3 in. internal

diameter in which water is used. The gauge tubes for water or mercury should never be less than $\frac{1}{2}$ or $\frac{3}{4}$ in. internal diameter, if accuracy is desired. Royal Daylight paraffin oil, of specific gravity about 0·8, is one of the best manometer liquids, as it does not evaporate quickly, and the surface tension error in a tube of 0·3 in. internal diameter is inappreciable. The oil can be stained by oil crimson or other suitable dyes, so as to make it easily visible.

In a steady flow through a smooth converging nozzle, like a Venturi tube, running full in a horizontal pipe line so that the streams of fluid are parallel at two cross-sections of areas A_1 and A_2 sq. ft., normal to the stream lines, the same volume of fluid passes each section per second, and is equal to velocity $\times$ sectional area, or $v_1 . A_1 = v_2 . A_2$.

If the flow is adiabatic, neglecting friction, the total energy remains the same at the constricted section or throat, and the loss of potential energy is equal to the gain of kinetic energy, per pound of fluid passing ; also, the velocity at the throat $v_2 = \sqrt{2g\bar{h}}$, h being the head of fluid in feet corresponding to the pressure drop across the throat of the nozzle.

When the fluid is a gas there will be a further increase of velocity and reduction in pressure, owing to the increase of volume when the pressure drops. Hence certain conditions are arranged ; $p_1 - p_2$ must be kept very small, about 12 in. of water gauge when discharging to the atmosphere. Also, the difference of pressures are taken close to each side of a thin plate or diaphragm, as in Figs. 55, 55a, and 56 ; and the values of p_2/p_1 are to be kept between 0·97 and 1, so that the specific weight of air may be practically the same on the up-stream and down-stream sides of the plate.

Then the coefficient of discharge, Ωa or " C," for a square-edged orifice is practically the same for water and gas flows, and the discharge is found by experiment to be proportional to the square root of the pressure difference across the orifice.

Let the observation of the pressure drop, H in., be taken on a water gauge manometer, h being the corresponding head of gas in feet, and the pressure p_1, up-stream, or Hg in. on a mercury column (not shown).

Now $h = (p_1 - p_2)v$, where v is the specific volume in cubic feet per pound of air up-steam of the orifice, and p_1 and p_2 lb. per sq. ft. (abs.). For 1 lb. of gas, $p_1v = RT$, or $v = RT/p_1$ where $R = 53·2$, and T the absolute temperature up-stream.

Substituting these values in the above equation, we have

$$v_2 = \sqrt{2g(p_1 - p_2)\frac{RT}{p_1}} \text{ ft. per sec.}$$

Hence discharge volume $v_o = $ " C " $\times v_2 \times A_2$ cub. ft. per sec., where $A_2 = $ area of orifice or nozzle throat in square feet, and " C " $=$ discharge coefficient.

In order to reduce the discharge volume to that under suction conditions or " free air," of volume v_s, at p_s and T_s (abs.),

$$\frac{p_s v_s}{T_s} = \frac{p_1 v_o}{T},$$

and $\qquad v_s = v_o \cdot \frac{p_1 T_s}{p_s T} = \text{`` } C \text{ ''} A_2 \times \frac{p_1 T_s}{p_s T} \sqrt{2g(p_1 - p_2)\frac{RT}{p_1}}$

$$v_s = C A_2 \frac{T_s}{p_s} \sqrt{2g(p_1 - p_2)\frac{Rp_1}{T}} \text{ cub. ft}$$

When the pressure drop $(p_1 - p_2)$ is observed as H in. " head " of water gauge,

$\qquad p_1 - p_2 = H \times 0.036 \times 144 = 5.18\, H$ lb. per sq. ft.

The pressure up-stream, p_1, is observed (Hg) in. of mercury column, and so is the barometer.

$\qquad \therefore p_1 = (Hg + \text{Barom.}) \times 0.49 \times 144$
$\qquad\qquad = 70.5\,(Hg + \text{Barom.})$. lb. per sq. ft.

also $\qquad p_s = 70.5 \times$ Barometer in inches,

since 0.036 and 0.49 are the pressures in pounds per square inch. due to 1 in. of water and mercury columns, respectively.

Substitute these values in terms of the observed readings and multiply by 60, gives the volume of air discharged under free air (suction) conditions in *cubic feet per minute*—

$$\bullet \qquad 948 \text{ `` } C \text{ ''} A_2 \cdot \left(\frac{t_s + 460}{\text{Barom.}}\right) \sqrt{H\left(\frac{Hg + \text{Barom.}}{t + 460}\right)} \qquad . \quad (14)$$

In case a square-edged circular orifice is used, this result is multiplied by the discharge coefficient Table I appropriate to the value of the ratio d_2/d_1 of the orifice and pipe line worked at.

General Discharge Formula. * Let A_1 and A_2 be the cross-sectional areas in square feet at two sections in a converging nozzle at which the stream lines are parallel, and v_1, p_1, W_1 ; v_2, p_2, W_2 the velocity, pressure, and density respectively. When the flow is steady, the same weight of fluid passes each cross-section per second.

In adiabatic expansion there is no transfer of energy to outside bodies, and the loss of potential energy per pound of fluid between the sections A_1 and A_2 is

$$\frac{p_1}{W_1} \cdot \frac{\gamma}{\gamma - 1} \left\{ 1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \right.$$

while the gain of kinetic energy $= \dfrac{v_2{}^2 - v_1{}^2}{2g}$.

Equating, $v_2{}^2 - v_1{}^2 = 2g \cdot \dfrac{p_1}{W_1}, \dfrac{\gamma}{\gamma - 1}\left\{1 - \left(\dfrac{p_2}{p_1}\right)^{\gamma^{-1}{}_\gamma}\right\}$. (15)

also $\dfrac{W_2}{W_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$; and the discharge $Q = A_1 v_1 W_1$

$= A_2 v_2 W_2$ lb per sec.

Let $\quad n = \dfrac{\text{Area of pipe up-stream}}{\text{Area of orifice or nozzle throat}} = \dfrac{A_1}{A_2} = \dfrac{a_1 \text{ sq. in.}}{a_2 \text{ sq. in.}}$

and $\quad N = \dfrac{a_1}{\left\{\left(\dfrac{a_1}{a_2}\right)^2 - 1\right\}^{\frac{1}{2}}} = \dfrac{n a_2}{(n^2 - 1)^{\frac{1}{2}}}$

Now $\quad v_1 = v_2 \cdot \dfrac{A_2}{A_1} \cdot \dfrac{W_2}{W_1} = v_2 \cdot \dfrac{1}{n}\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$ . . . (16)

Substitute (16) in (15) gives

$$v_2{}^2\left\{1 - \dfrac{1}{n^2}\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{\gamma}}\right\} = 2g \cdot \dfrac{p_1}{W_1} \cdot \dfrac{\gamma}{\gamma - 1}\left\{1 - \left(\dfrac{p_2}{p_1}\right)^{\gamma^{-1}{}_\gamma}\right\} \quad (17)$$

and $\quad Q = A_2 v_2 W_2 = A_2 v_2 W_1 \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{\gamma}},$

hence from (17) we have

$$Q = \Omega A_2 \left[\dfrac{2g \cdot p_1 W_1 \cdot \dfrac{\gamma}{\gamma - 1} \cdot \left\{1 - \left(\dfrac{p_2}{p_1}\right)^{\gamma^{-1}{}_\gamma}\right\}\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{\gamma}}}{1 - \dfrac{1}{n^2}\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{\gamma}}}\right]^{\frac{1}{2}} \quad . \quad (18)$$

where Ω is the discharge coefficient found by experiment.

From this general discharge equation (18), is derived

$$Q = \Omega A_2 \left\{2g \cdot \dfrac{n^2}{n^2 - 1} \cdot p_1 W_1\right\}^{\frac{1}{2}} (\beta) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where $\beta \quad = \left[\dfrac{\dfrac{\gamma}{\gamma - 1}\left\{1 - R^{\gamma^{-1}{}_\gamma}\right\} R^{\frac{2}{\gamma}}}{\dfrac{n^2 - R^{\frac{2}{\gamma}}}{n^2 - 1}}\right]^{\frac{1}{2}}$

Here $R = \dfrac{p_2}{p_1}$, and Q may be calculated in terms of p_1 and the ratio p_2/p_1 which, in discharge measurement, lies between 0·98 and 1 ;

but the pressure drop, $(p_1 - p_2)$, can be more easily measured than the pressure ratio p_2/p_1, and equation (19) can be expressed in terms of $p_1 - p_2$, thus—

$$Q = \Omega A_2 \left\{ 2g \cdot \frac{n^2}{n^2 - 1} (p_1 - p_2) W_1 \right\}^{\frac{1}{2}} (a), \qquad . \qquad . \qquad . \qquad (20)$$

where a

$$= \left[\frac{\left(\frac{1}{1 - R} \right) \left(\frac{\gamma}{\gamma - 1} \right) \left(1 - R^{\frac{\gamma - 1}{\gamma}} \right) R^{\frac{2}{\gamma}}}{\frac{n^2 - R^{\frac{2}{\gamma}}}{n^2 - 1}} \right]$$

For maximum adiabatic flow, it is obvious from (18), or β in (19) to be a maximum, we must have

$$\frac{\left(1 - R^{\frac{\gamma - 1}{\gamma}} \right) R^{\frac{2}{\gamma}}}{n^2 - R^{\frac{2}{\gamma}}} \text{ a maximum,}$$

that is, when

$$\frac{\gamma + 1}{R^{\frac{2}{\gamma}}} - \frac{2}{R^{\frac{\gamma + 1}{\gamma}}} - \frac{\gamma - 1}{n^2} = 0$$

From which, when $n = \infty$,

$$\frac{\gamma + 1}{R^{\frac{2}{\gamma}}} = \frac{2}{R^{\frac{\gamma + 1}{\gamma}}}; \text{ and } \frac{2}{\gamma + 1} = R^{\frac{\gamma - 1}{\gamma}}$$

hence

$$R = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$$

In the case of air, $\gamma = 1.408$, so that if there is a maximum flow, at this critical value, $\dfrac{p_2}{p_1} = .527$, the velocity of the particles of air in the nozzle throat is theoretically equal to the velocity of sound or wave propagation at the same pressure and temperature, and the pressure in the jet, p_2, never gets to be less than the pressure corresponding to maximum discharge, however low the external pressure may be, since this is unable to transmit itself back along the issuing jet and affect the discharge.

Again, based on (20), we have the simple discharge equation—

$$Q = 99.74(\Omega a) N \sqrt{hw} \text{ cub. ft. of " free air " per min.} \quad (21)$$

and

$$w = \frac{2.7p}{T} \text{ for dry air} \quad (22)$$

also

$$w = \frac{2.7}{T} (p - 0.378p_v) \text{ for moist air (see Table III)} \quad (23)$$

where $h =$ " head " across orifice in inches of water

$p =$ pressure in pounds per square inch (absolute)

$p_v =$ vapour pressure in pounds per square inch (as determined by the dew point)

$T =$ temperature Fahr. (absolute)

$w =$ weight in pounds per cubic foot of free air, at 14·7 lb. per sq. in. and 60° F.

Further, for calculation of the discharge by the shaped nozzle,

$$Q = 320(\Omega\beta)Nw\sqrt{T}, \text{ or, substituting (22),}$$

$$Q = 864(\Omega\beta)\frac{Np_1}{\sqrt{T}} \text{ cub. ft. of free air per minute} \quad . \quad (24)$$

TABLE III

VALUES OF $0·378p_v$ AT VARIOUS TEMPERATURES

Temperature of Dew Point, Fahr.	40°	60°	70°	80°	120°	140°	160°	200°
$0·378 p_v$	0·046	·0969	·137	191	638	1·09	1·79	4·355

Flow Conditions. The flow of a fluid through an orifice depends on its viscosity, μ, density, W, and compressibility or adiabatic elasticity. If μ is the coefficient of viscosity in C.G.S. units, the kinematic viscosity is $\frac{\mu}{W}$.

The values of the viscosity of dry air at various temperatures in Table IV are multiplied by 10^6 to reduce the number of figures. Thus $\mu = 0·00001172 = 11·7 \times 10^{-6}$.

TABLE IV

VALUES OF VISCOSITY, μ, FOR DRY AIR

Temperature, Fahr. .	40°	60°	70°	80°	120°	140°	240°
Viscosity, $\mu \times 10^6$ British Engineers' Units	11·72	12·09	12 27	12·46	13 19	13·56	15·36

Suppose that gaseous or liquid flow through an orifice depends only upon the viscosity μ, the density ρ, the average velocity V at the orifice, and the diameter d_2 of the orifice, then the flow conditions are similar for the same values of $Vd_2\rho/\mu$ or Vd_2W/μ. This criterion of flow is known as the " Reynolds' criterion " for viscous flow.

If $4G/\pi d^2$ is substituted for $Vd_2\rho$ the flow criterion becomes $G/d_2\mu$, where G is the discharge in grammes per second. Mr. Hodgson[*] has shown the relation between Ωa and $G/d_2\mu$ by plotting Ωa against $\sqrt{G/d_2\mu}$, for values of the latter from 0 to 300 and 600, also with values of the ratio d_2/d_1 from 0·421 to 0·843, and at pressures of 1 to 4 atmospheres, giving the deviations of the " air curve " and " liquid curve " from the limiting value $\Omega a = 0\cdot600$. He has shown by actual measurement that when the value of $\sqrt{G/d_2\mu}$ is greater than 200 C.G.S. units the flow is turbulent, viscosity and compressibility effects are negligibly small, and the discharge through an orifice is proportional to the square root of the difference of pressure across it. The values of $\sqrt{G/d_2\mu}$ usually employed in air, steam, and water metering lie between 200 and 3,000 C.G.S. units, well above the viscous stage, while the ratio p_2/p_1 is between 1·0 and 0·98.

Usually, in the case of air, the kinematic viscosity, μ/W, is so small, and the velocity so high, that corresponding velocities for similar orifices occur at the same value of the expansion ratio p_2/p_1, rather than at the same values of Reynolds' criterion, $Q/d_2\mu$; and, provided that the value of this criterion is greater than 200,000 British Engineers' units, or $\sqrt{G/d_2\mu}$ is greater than 200 C.G.S. units, then the viscosity effects are negligible. Whereas, when orifices or nozzles are used for the measurement of such relatively viscuous fluids as water, the corresponding velocities would occur at the identical value of Reynolds' criterion.

In the measurement of air flow under ordinary conditions, through an orifice or nozzle of smooth surface, the variations of the discharge coefficient are unaffected by both elasticity and viscosity.

Example 9. Air is measured at 30 lb. per sq. in. (abs.) and 100° F., and the dew point is 60° F. Find the weight of the air.

Given $t_d = 60°$ F. and, from Table III, $0\cdot378 p_v = 0\cdot0969$, and by (23),

$$\therefore w = \frac{2\cdot7}{560}(30 - 0\cdot0969) = \frac{2\cdot7 \times 29\cdot903}{560}$$

$$= 0\cdot1442 \text{ lb. per cub. ft.} \qquad \textit{Answer.}$$

Example 10. In a test of an air compressor using a square edged circular orifice, as Fig. 55, the following results were obtained—

$d_1 = 6$ in. ; $d_2 = 1\cdot3$ in. ; $t = 116°$ F. $= 576°$ F. (abs.)

$p_1 = 29\cdot2$ lb. per sq. in. (abs.)

$h = 14\cdot5$ lb. per sq. in., and $14\cdot5 \times 27\cdot747 = 402\cdot3$ in. of water column.

[*] *Proc. Inst. Mech. E.*, No. 4 (1925), p. 885

Calculate the air discharged, Q, using equation (21).

Given $N = -\dfrac{a_1}{\left(\dfrac{a_1}{a_2}\right)^2 - 1} = \dfrac{28\cdot27}{\sqrt{(21\cdot3)^2 - 1}} = 1\cdot33.$

By equation (22),

$$\dot{w} = \frac{2\cdot7 \times 29\cdot2}{576} = 0\cdot1369 \text{ lb. per cub. ft.}$$

$d_2/d_1 = 0\cdot2167$, and $p_2/p_1 = 0\cdot4966$,

whence, from Table I (p. 149), Ωa or " C " $= 0\cdot53$

Substituting these values, we get

$$Q = 99\cdot74 \times 0\cdot53 \times 1\cdot33 \sqrt{402\cdot3} \times \cdot1369$$
$$= 521\cdot75 \text{ cub. ft. per min.} \qquad\qquad Answer.$$

Again, from Table IV, we have

$$\frac{Q}{d_2\mu} = \frac{521\cdot75}{1\cdot3 \times 0\cdot0000132} = 30{,}405{,}000$$

which shows that we are well above the viscous limit 200,000 (p. 157).

EXAMPLES V

1. A motor is supplied with air at 4 atmospheres pressure and 60° F. The air is expanded adiabatically in the motor cylinder to atmosphere pressure, and then exhausted at this pressure. (*a*) What is the temperature of the exhaust ? (*b*) Calculate the work done per pound of air, and the mean effective pressure in the motor cylinder. (*c*) How many cubic feet per minute of the compressed air are required to give 1 I.H.P., and what is the corresponding volume of "free air" ? Take the specific heat of air at constant pressure and constant volume, 0·24 and 0·1716, respectively.

2. A single-acting, single-stage air compressor is belt-driven from an electro-motor at 400 r.p.m. The cylinder diameter is 6 in. and stroke 7 in. The air is compressed from 15 to 105 lb. per sq. in. (abs.), and the law of compression $pv^{1\cdot3} = $ constant. Find the horse-power of the motor, if transmission efficiency is 97 per cent and mechanical efficiency of the compressor 90 per cent. Neglect clearance effects. (*Assoc. M.Inst.C.E.*)

3. Air is drawn into a cylinder and compressed adiabatically to a pressure of 75 lb. above its original pressure (15 lb. per sq. in.), and is then expelled at this pressure into a receiver ; its original temperature was 60° F. In the receiver the compressed air cools down to its original temperature, and, in order to maintain a uniform pressure in the receiver, an equal weight of compressed air is constantly drawn off and expanded isothermally in a working cylinder down to 15 lb. pressure. Calculate : (*a*) the work spent per pound of air in the compressor ; (*b*) the work done per pound of air in expanding ; (*c*) the temperature of the air as it enters the receiver.

(*U.L., B.Sc. (Eng.)*)

4. Estimate the brake horse-power required to drive a single-stage air compressor which takes in 400 cub. ft. per min. at 15 lb. per sq. in. (abs.), compresses to 120 lb. per sq. in., and delivers it into a receiver at the higher

pressure. The law of compression curve is $pv^{1.26} =$ constant, and the mechanical efficiency of the compressor 88 per cent. Neglect losses due to clearance, cooling, and leakage.

5. An air compressor draws in 260 cub. ft. of air per min. at atmospheric pressure, compresses it adiabatically in one stage to 10 atmospheres, and delivers it to a receiver at this higher pressure. Calculate (*a*) the air horsepower, (*b*) the brake horse-power at the compressor shaft when the mechanical efficiency of the compressor is 86 per cent. Take $y = 1.4$, and neglect all losses due to clearance, cooling, etc.

6. A single-stage double-acting air compressor of 40 I H.P., at 120 r.p.m., takes in air at 15 lb. per sq. in. and delivers it at 120 lb. per sq. in. (abs.), the index of compression curve is $n = 1.35$. Find the diameter of cylinder and length of stroke at the average piston speed of 600 ft. per min. Neglect clearance.

7. A two-stage compressor, with perfect inter-cooler, takes in air at atmospheric pressure and 17° C., and, after adiabatic compression to 9 atmospheres, delivers the air to a receiver at this pressure. Calculate (*a*) the minimum work done, and (*b*) the heat given to the inter-cooler per pound of air compressed. What would be (*c*) the least work done and heat given to the inter-coolers in a three-stage compressor working under the same conditions ? Take $Cp = 0.24$ for air.

8. A two-stage air compressor for a Diesel engine is to deal with 200 cub. ft. of air per min. under atmospheric conditions at 250 r.p.m., and to deliver it at 1,000 lb. per sq. in. Assuming complete inter-cooling, determine the indicated horse-power and the cylinder diameters and stroke. Allow a piston speed of 500 ft. per min., assume $pv^{1.4} = C$ during compression, and neglect clearance and wire-drawing losses. Take atmospheric conditions as 14.7 lb. per sq. in., and 15° C. (*U.L., B.Sc. (Eng.)*)

9. A single-acting two-stage air compressor compressing air for a Diesel engine has piston diameters 4 in. and 1.5 in., and a stroke in each cylinder of 3 in. Air is drawn into the low pressure cylinder at atmospheric conditions of 14.7 lb. per sq. in. and 15° C., and is cooled in the inter-cooler to 15° C. From the high pressure cylinder it is delivered into a storage bottle having a pressure of 600 lb. per sq. in. Neglecting clearance volumes, determine the pressure in the inter-cooler when the compressor is running steadily and the inter-cooler pressure is steady, and find the horse-power used in compressing air at 350 r.p.m. Take index of compression curves, $n = 1.35$.

(*U.L., B.Sc. (Eng.)*, 1923.)

CHAPTER VI

MECHANICAL REFRIGERATION

Refrigeration is the process of taking heat from a body, or from the contents of a cold insulated chamber, and keeping that body at a low temperature by continuously extracting heat as fast as it leaks in from the surroundings. Mechanical refrigeration requires the expenditure of energy, commonly as work spent in driving the machine which acts like a heat pump, on a reversed heat engine cycle, and transfers the heat to a body at a higher temperature.

The *coefficient of performance* of a refrigerating machine is measured by the ratio

$$\frac{\text{Heat extracted}}{\text{Work spent}},$$

both quantities being reckoned in the same units of heat or work.

It has been shown (p. 97) that the ideal heat engine works on a reversible cycle, and may take in heat Q_2 from a cold body at the low temperature T_2 (abs.), and reject heat Q_1 at the higher temperature T_1 (abs.). Let W be the thermal equivalent of the work required to drive this refrigerating machine. Since the machine works on a reversible cycle, by the conservation of energy,

$$Q_1 = Q_2 + W, \text{ or } W = Q_1 - Q_2.$$

Also, by the second law of thermodynamics, we have

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}, \text{ and } \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Hence the coefficient of performance of this ideal reversible refrigerating machine is

$$\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}, \text{ and } Q_2 = W \times \frac{T_2}{T_1 - T_2}$$

By this relation it is clear that, for a given quantity Q_2 of heat to be taken from a cold body, the expenditure of work, W, is proportional to the difference of temperature through which the heat is raised, or the machine works. In other words, the smaller the range of temperature the less work will be required to cool a body by extracting a given amount of heat ; or, *for a given amount of work W spent, the heat Q_2 extracted will be greater the smaller the range of temperature, $T_1 - T_2$.*

Consider a machine working on the ideal Carnot cycle reversed. Starting at the point D or d (Figs. 60 and 61), the cycle is in a

counter-clockwise direction *abcd*. The engine takes in a quantity of heat Q_2, represented by the area *damn*, during isothermal expansion *da*, at the lower temperature T_2; that is, the gain of entropy from the cold body is $Q_2/T_2 = \phi_2 - \phi_1$, represented by the line *da* or *nm*. During adiabatic compression *ab* there is no transfer of heat, and the temperature of the working fluid is raised from T_2 to T_1. Then, during isothermal compression *bc* at the higher temperature T_1, a larger quantity of heat, Q_1, represented by the area *bcnn*, is rejected to the hot body, so that the entropy given to the warmer body is $\dfrac{Q_1}{T_1} = \phi_2 - \phi_1$, represented by the line *bc*.
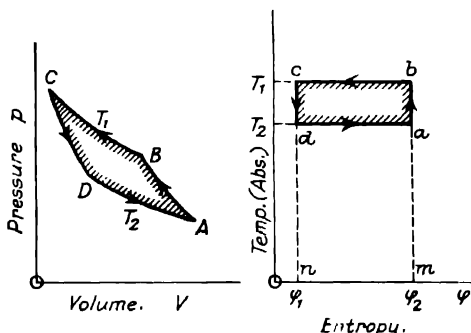
FIG. 60   FIG. 61

CARNOT CYCLE REVERSED

During adiabatic expansion *cd*, the fluid does work and drops in temperature from T_1 to T_2.

The work required to drive this refrigerating machine is represented by the area *ABCD* or *abcd*, equivalent to $Q_1 - Q_2$, which is converted into heat in the machine.

In fact, the reversed perfect engine acts like a heat pump taking heat Q_2 from a cold body at T_2, and delivering more heat Q_1 at a higher temperature T_1, whilst the entropy Q_2/T_2 taken from the cold body is equal in amount to the entropy Q_1/T_1 given to the warmer body.

The *coefficient of performance* is

$$\frac{Q_2}{Q_1 - Q_2} = \frac{\text{Area } damn}{\text{Area } bcnm - \text{area } damn}$$

or

$$\frac{Q_2}{W} = \frac{T_2(\phi_2 - \phi_1)}{T_1(\phi_2 - \phi_1) - T_2(\phi_2 - \phi_1)} = \frac{T_2}{T_1 - T_2}.$$

No refrigerating machine, working between the same limits of temperature, can have a higher coefficient of performance than the reversible one. If a refrigerating machine *B* could be conceived to

have a greater coefficient of performance than the reversible machine A, then the ratio of Q_2/W would be greater in B than in A.   Let the reversible machine (A), acting as a heat engine, be coupled to and drive the more than perfect refrigerating machine (B) that would extract more heat from the cold body than A gives it, while B driven by A would also reject more heat to the hot body than A takes from it.   Thus heat would be continuously taken from the cold body to the hotter body by means of this self-acting machine, which would be contrary to the second law of thermodynamics ; or the axiom that it is impossible for a self-acting machine, unaided by any external agency, to convey heat from a cold body to one at a higher temperature.

Hence all ideal reversible refrigerating machines, working between the same limits of temperature, have the same coefficient of performance.

No refrigerating machine used in practice works on this reversed Carnot cycle.

Highest Efficiency for Refrigeration.   The ideal greatest ratio of heat Q_2, extracted from a cold body at T_2, to the heat Q supplied at the high temperature T, for a given temperature T_1 of the condensing water to which the heat is rejected, is obtained by the combination of a perfect reversible heat engine driving a perfect reversible heat pump.

The perfect heat engine takes in heat Q at T, and rejects heat at T_1, the temperature of the condenser, hence $W = \dfrac{Q}{T}(T - T_1)$.

The perfect heat pump extracts heat Q_2 from the cold body at the low temperature T_2, also delivers heat at T_1, and, since it is reversible, $W = \dfrac{Q_2}{T_2}(T_1 - T_2)$, where W is the heat equivalent of the work given by the heat engine, and used in driving the heat pump.

Hence $\dfrac{Q}{T}(T - T_1) = \dfrac{Q_2}{T_2}(T_1 - T_2)$

from which $\dfrac{Q_2}{Q} = \dfrac{T_2(T - T_1)}{T(T_1 - T_2)}.$

No other combination could extract more heat than Q_2 at the temperature T_2, by means of the same amount of high-temperature heat Q If any other machine could extract more heat than Q_2, by the expenditure of the same amount of heat Q at the high temperature, then with the above combination working reversed, and the other refrigerating machine working direct, the result would be a transfer of heat from the cold body at T_2 to a body at a higher temperature T_1 without the expenditure of energy, by a self-acting agency, which is impossible by the second law of thermodynamics.

Cold Air Machine. Joule's air engine reversed was developed as a refrigerating machine by Bell, Coleman, and Lightfoot, and was one of the earliest used for mechanical refrigeration on board ship and cold storage.

The diagram of the essential parts of the plant, Fig. 62, and the ideal indicator diagram, Fig. 63, may serve to explain the action of this air machine.* Air from the cold chamber R at 18° F. or 32° F. is compressed in a cylinder A, with water-jacket, to about 65 lb. per sq. in. (abs.), and 270° F. or 290° F. The air is passed at this constant pressure through tubes in a cooler C, and heat is extracted by circulating water. This cold air at high pressure is further cooled by expansion in cylinder B, and delivered at about
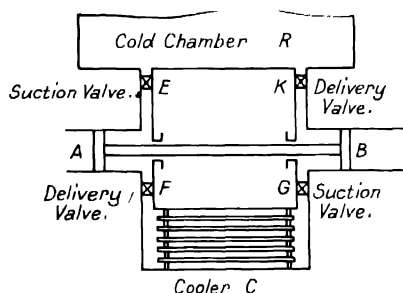
FIG. 62. COLD AIR MACHINE
(Diagram of Essential Parts)
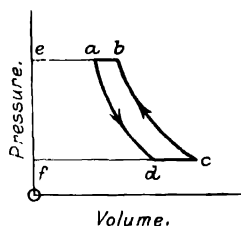
FIG. 63. CYCLE OF
COLD AIR MACHINE

– 80° F. and initial pressure to the cold chamber. During expansion the air does work which helps to drive the machine.

In this type of machine the air coming from the cold chamber carries moisture which was deposited as snow in the expansion cylinder and tended to choke up the valves and passages. This difficulty was met in two ways—

1. Lightfoot carried out the expansion in two stages. In the first the air was expanded to about 35° F., when the greater part of the water vapour was deposited as liquid water and drained away, and the expansion of the dried air was completed in the second cylinder.

2. The other device, in Coleman's machine, is to pass the moist air from the cooler C, before expansion, through "drying pipes," kept cold by the circulation of air from the cold chamber, which reduces the temperature to near the freezing point, and any moisture it may contain is condensed.

The ideal cycle is that of Joule's air engine reversed (p. 111), as in Fig. 63. In the compression cylinder, A, Fig. 62, during the suction

* For the construction and performance of these machines, see Lightfoot. *Proc. Inst. Mech. Eng.* (1881), p. 105, and 1886, p. 201 ; also Coleman, *Min. Proc. Inst. C.E.*, Vol. LXVIII (1882), p. 146.

stroke fc, air, from the cold chamber R, is drawn into the cylinder A, and compressed adiabatically cb. When the valve F opens, the pump delivers the air into the cooler C, completing its stroke from b to e, and the temperature falls, whilst the air gives up heat at constant pressure to the circulating water. Then an equal quantity of air, from C, is taken into the expansion cylinder B at temperature T_a and expands adiabatically, doing work on the piston shown by area under ea and ad. During the return stroke df, in B, the chilled air is discharged into the cold chamber at the initial pressure.

During the cycle, the work done *on* the air in the compressor cylinder A is represented by the area $fcbe$, and the work done *by* the air in the expansion cylinder B is given by the area $eadf$, hence the net work in driving the machine is given by the area $abcd$. The net amount of heat extracted from the cold chamber per pound of air is

$$Q_2 = C_p(T_c - T_d) \; ;$$

and the heat rejected to the cooling water per pound of air is

$$Q_1 = C_p(T_b - T_a),$$

where T_a, T_b, T_c, and T_d denote the absolute temperature of the working air at the points a, b, c, d of the diagram, Fig. 63.

The net amount of work spent in driving the machine is $Q_1 - Q_2$; and the coefficient of performance is $\dfrac{Q_2}{Q_1 - Q_2}$.

Also, since the ratio of expansion in B is the same as the ratio of compression in A, and both are adiabatic,

$$\frac{T_a}{T_d} = \frac{T_b}{T_c}, \quad \frac{T_b - T_a}{T_c - T_d}.$$

$\therefore$ coefficient of performance $= \dfrac{Q_2}{Q_1 - Q_2}$

$$= \frac{C_p(T_c - T_d)}{C_p(T_b - T_a) - C_p(T_c - T_d)} = \frac{T_c - T_d}{T_b - T_a - (T_c - T_d)}$$

$$= \frac{1}{\dfrac{T_b}{T_c} - 1} = \frac{T_c}{T_b - T_c}.$$

This coefficient of performance is less than that of a reversed Carnot cycle between the limits T_a and T_c, namely, $\dfrac{T_c}{T_a - T_c}$, since T_b is greater than T_a. All the heat is not taken in at T_c and rejected at T_a; besides, the transfer of heat from the compressed air at T_b to the circulating water at T_a is not reversible.

In practice, the compression of the air in A is not adiabatic, because the water-jacket on the cylinder reduces the temperature.

so that the slope of the compression curve is not so steep as the adiabatic, and the work spent in compression is reduced.

The actual coefficient of performance obtained with this machine varies from 0·5 to 0·7, and is very much lower than that of a vapour compression machine owing to (1) the necessity of working with a wide range of temperature, since air is a poor conductor and absorber of heat ; (2) air has a small capacity for heat and all machines using air as the working substance must be bulky, and consequently the loss of heat by friction is large. Not only is power lost in friction, but the working air takes up most of the heat so generated, and has less capacity left for taking up heat in useful refrigeration.

Example 1. In a Bell-Coleman refrigerating plant air is drawn into the cylinder of the compressor at atmospheric pressure of 15 lb. per sq. in. and temperature − 5° C., and it is compressed adiabatically to 75 lb. per sq. in., at which pressure it is cooled to 15° C. It is then expanded in an expansion cylinder to atmospheric pressure and discharged into the refrigerating chamber. If the law for expansion is $pv^{1\cdot2}$ = a constant, find the work done on the air per pound, and the coefficient of performance of the refrigerating plant. Specific heat of air at constant pressure is 0·238. (*U.L., B.Sc. (Eng.), 1924.*)

On the indicator diagram, Fig. 63 (p. 163), we have

$$T_c = -5° + 273 = 268° \text{ C. (abs.)}$$

$$\frac{T_b}{T_c} = \left(\frac{75}{15}\right)^{\frac{1\cdot4-1}{1\cdot4}} = 5^{\frac{2}{7}} \therefore T_b = 268 \times 5^{\frac{2}{7}} = 424\cdot45° \text{ C. (abs.)}$$

and $T_a = 15° + 273 = 288° \text{ C. (abs.)}$

Also $\frac{T_a}{T_d} = \left(\frac{75}{15}\right)^{\frac{1\cdot2-1}{1\cdot2}} = 5^{\frac{1}{6}} = 1\cdot3077$

$$\therefore T_d = \frac{T_a}{1\cdot3077} = \frac{288}{1\cdot3077} = 220\cdot24° \text{ C. (abs.)}$$

Now the heat rejected per pound of air at the higher constant pressure is

$$Q_1 = C_p(T_b - T_a) = 0\cdot238(424\cdot45 - 288) = 32\cdot475 \text{ C.H.U.}$$

and the heat extracted per pound of air from the cold chamber

$$Q_2 = \cdot238(T_c - T_d) = \cdot238(268 - 220\cdot24) = 11\cdot367 \text{ C.H.U.}$$

$\therefore$ Work done per pound of air

$$= Q_1 - Q_2 = 32\cdot475 - 11\cdot367 = 21\cdot108 \text{ C.H.U.}$$

$$\text{or } 21\cdot108 \times 1400 = \underline{29,550 \text{ ft.-lb.}} \qquad Answer.$$

The coefficient of performance is

$$\frac{Q_2}{Q_1 - Q_2} = \frac{11\cdot367}{21\cdot108} = \underline{0\cdot54} \text{ (nearly).} \qquad Answer.$$

Reversed Heat Engine as a Warming Machine.* LORD KELVIN, in 1852, made the suggestion that the reversed heat engine cycle might be used to keep a room warm. A machine of the Bell-Coleman type might take in air from the atmosphere at a low temperature, expand it to a lower temperature and pressure, and allow the temperature to rise again by conduction from the external air, after which it would be compressed to atmospheric pressure, and its temperature thereby raised above that of its surroundings. The heated air might then be discharged into the room to be warmed. In this way a reversed heat engine might take heat Q_2 from the atmosphere at a low temperature T_2, and by means of expenditure of work W for expansion and compression, deliver to the room heat Q_1 at a slightly higher temperature T_1. If the machine were reversible,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}, \text{ and } \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

$$\therefore \quad Q_1 = W \frac{T_1}{T_1 - T_2} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad .$$

For example, an oil engine using Russolene of calorific value 20,000 B.Th.U. per lb., consumes 0·5 lb. per brake horse-power hour. Suppose this engine drives a reversed heat engine which takes in air at 50° F. and delivers it at 60° F., how much heat is given to the air per brake horse-power hour if the reversed heat engine works at 80 per cent of the ideal performance ?

Here $W = 1$ B.H.P. hour $= 2545$ B.Th.U., and $T_1 = 60° + 460°$

we have $Q_1 = 2545 \times \frac{520}{60 - 50} \times \frac{80}{100} = 105,872$ B.Th.U.

If the 0·5 lb. of oil were burned directly to warm the air, only 10,000 B.Th.U. would be available, but at a high temperature, which is not taken advantage of in the ordinary methods of heating Whereas by the large temperature drop in the oil engine, combined with the small range of temperature in the reversed heat engine, more than ten times as much heat is given to the air by means of this *Warming Machine.* The oil engine has a brake thermal efficiency of $\frac{2545}{0·5 \times 20,000} = 0·2545$, or 25·45 per cent. Here, heat energy is not created, but the work equivalent to 2545 B.Th.U. from the oil engine is changed into heat, and the extra heat, 105,872 – 2545 B.Th.U., or 103,327 B.Th.U., is raised through a small range of temperature. If this range were increased, say from 40° to 60° F., then the heat given to the air would be 52,936 B.Th.U.

A Vapour Compression Refrigerating Machine consists of four essential parts, shown in the diagram, Fig. 64.

1. A *compressor*, into which vapour is drawn through the suction valve A, from the evaporator at a low temperature T_2, compressed
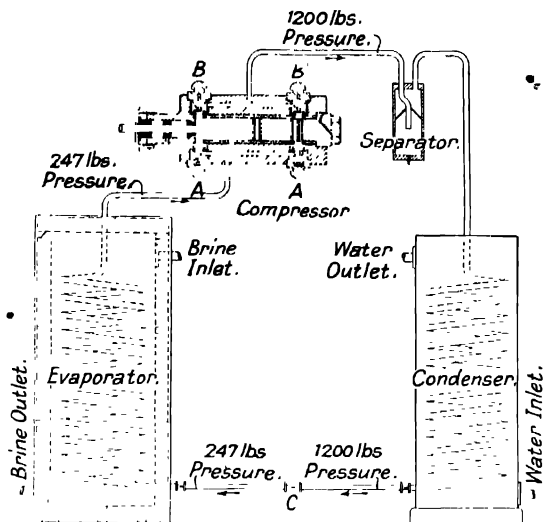
FIG. 64. CARBONIC ACID REFRIGERATING MACHINE (*Haslam*)

to the higher pressure and temperature T_1, and discharged through the delivery valve B, as shown by the indicator diagram, Fig. 65.

A *separator* collects oil, etc., in the vapour from the compressor.

2. A *condenser*, or cooler, consisting of coils of pipe in which the
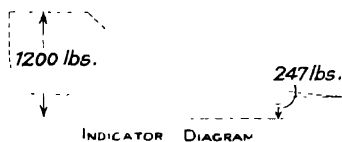
INDICATOR DIAGRAM
FIG. 65

compressed vapour is cooled and condensed at T_1, and gives up its heat to circulating water.

3. The expansion cylinder, replaced in practice by an adjustable throttle valve C, called a *regulator* or *expansion valve*, through which the liquid expands from T_1 to T_2 at the lower pressure.

4. An *evaporator*, or *refrigerator*, consisting of pipes in which the liquid evaporates at the lower temperature and takes up heat from cold brine, which is circulated round the pipes and conveys heat to the working fluid from a cold chamber or from water to be frozen.

In practice, the expansion cylinder is replaced by the expansion valve *C*, which simplifies the machine but reduces the refrigerating effect in the evaporator, because a portion of the liquid vaporizes by free expansion in throttling, the heat required being taken from the fluid itself ; not only is there less liquid left to evaporate usefully, and absorb heat from the cold body, but the work done in expansion is lost. On the other hand, very little work could be recovered by adiabatic expansion of the *liquid*, and this would be reduced by mechanical loss in the expansion cylinder, besides the necessity of adjustment of the expansion for, varying temperature conditions in actual practice.

The loss is negligibly small in the ammonia machine, which is mostly used on land with cold water circulation, but this loss is
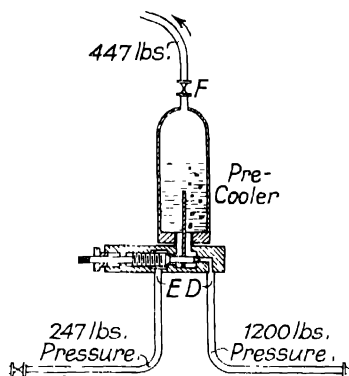
FIG. 66   PRE-COOLER FOR LIQUID CO_2 (*Haslam*)

much greater in the CO_2 machine, as the available circulating water for cooling in the condenser rises in temperature near the critical point.

Messrs. Haslam have found that, on board ship, under tropical conditions, with brine below 0° F. and circulating sea water at 92° F. for cooling purposes, about half of the liquid CO_2 evaporates as it passes through the expansion valve from the high to low pressure, and the refrigerating effect is reduced to about 50 per cent of its value at 15° C. In order to obviate the loss of output, a **pre-cooler**, Fig. 66, is used between the condenser and evaporator. The liquid from the condenser is passed through valve *D* into the pre-cooler at a lower pressure. Hence a portion of the liquid evaporates, cooling the rest of the liquid which collects at the bottom of the receiver, and passes on through valve *E* to the evaporator. The vapour collects at the top of the pre-cooler and passes through pipe *F* direct to the compressor cylinder, where the piston uncovers ports at the end of the suction stroke and admits the high pressure vapour.

This *super-charge* greatly increases the pressure of the charge drawn from the evaporator and already in the cylinder, before the compression stroke begins. The result is increased refrigerating effect and efficiency of the machine. The *pv* diagram, Fig. 67, compared with that of Fig. 65, indicates the increase of work
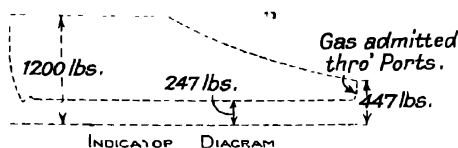
INDICATOR DIAGRAM

FIG. 67. INDICATOR DIAGRAM WITH PRE-COOLER

required to drive the compressor. The results of careful tests show a marked increase of refrigerating effect per unit of work done. There is a larger proportion of liquid in the fluid entering the evaporating coils from the pre-cooler, and the coil surface is more efficient in absorbing heat from the brine.

The actual cycle of the refrigerating machine is like a reversed Rankine cycle, with one important difference, namely, that of throttling through the expansion valve, which is non-reversible.

The liquid is completely evaporated in the refrigerator at temperature T_2. In the temperature–entropy diagram, Fig. 68, P is
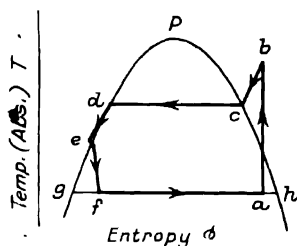
FIG. 68. CO_2 COMPRESSION CYCLE
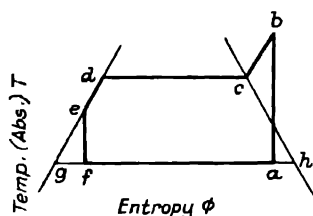
FIG. 69. AMMONIA COMPRESSION CYCLE

the critical point of carbonic acid, and *Pch* the dry saturation line. Suppose adiabatic compression *ab* begins at *a*, when the dryness fraction is *ga/gh*, and the wetness of the stuff is measured by the ratio *ah/gh*. The vapour is compressed to *b*, slightly above the saturation pressure. The dry vapour is thus superheated to a temperature higher than that of the condenser. Then follows cooling, at this *higher constant pressure*, in three stages : *bc*, the superheated vapour is cooled to the temperature of dry saturated vapour at *c* when condensation begins ; during *cd* the vapour is completely condensed at constant temperature ; and in the third or under-cooling stage *de*, the condensed liquid is cooled as low as possible. It is to be noticed that in Fig. 69, *de*, the under-cooling

of the condensed liquid ammonia is practically along the wet boundary line, whereas with CO_2 the cooling curve *de* is quite distinct from the boundary line *dg*.

Next, the line *ef* shows unresisted expansion through the regulating valve ; that is, throttling at constant total heat (p. 43) to the *lower constant pressure* and temperature of the evaporator.

The fraction converted into vapour by passing through the expansion valve is *gf/gh*. During *fa*, we have evaporation of the liquid in the refrigerator coils when heat is extracted from the brine or other body to be cooled.

The *refrigerating effect*, Q_2, that is, the amount of *heat extracted* from the cold body is represented by the area *famr*, under *fa*, down to absolute zero, Fig. 70.
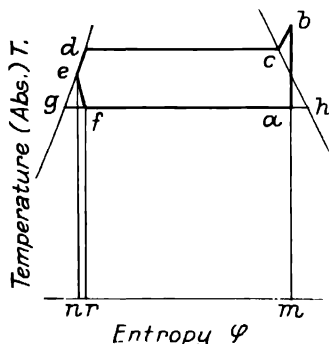
The total amount of *heat rejected* at the higher constant pressure during condensation and cooling, Q_1, is given by the area *mbcden*, under *bc*, *cd*, and *de*. The indicated *work done* in driving the compressor, drawing in the charge at the lower pressure of evaporation, compressing it to the higher pressure, and discharging it to the condenser at this higher pressure, when expressed in thermal units, is the difference $Q_1 - Q_2$.

FIG. 70. AMMONIA COMPRESSION CYCLE

Any interchange of heat with the cylinder walls is neglected, as the compression is supposed to be adiabatic ; but, owing to the non-reversible throttling *ef*, the work spent is greater than the area *abcdefa* by the area *nefr*. The use of the regulating or expansion valve always involves loss of heat, by the liquid refrigerant, equal to its specific heat multiplied by the difference in temperature of the condenser and evaporator, and represented by the area under *ef*, Fig. 71. The smaller the ratio of the specific heat of the liquid to the latent heat of the vapour, the less will be the loss $\frac{ef}{eg}$, in proportion to the refrigerating effect.

Such loss, due to the heat carried over by the liquid in passing through the expansion valve, is practically negligible for NH_3 and SO_2, but it cannot be neglected in the case of CO_2, which has the ratio of the specific heat of its liquid to the latent heat of the vapour much larger than that of the other refrigerants.

Let v_1 = the volume of liquid passing through the throttle valve to the refrigerator, then the heat equivalent of the work which the liquid does in forcing a part through the throttle valve is $v_1(p_1 - p_2)/J$, where p_1 is the higher and p_2 the lower pressure.

Now consider the performance of the vapour compression refrigerating machine with expansion valve, in the three cases in practice represented by the temperature–entropy diagram, Fig. 71, when the vapour, at the end of adiabatic compression is (1) just dry and saturated at the state point b ; (2) at h the vapour is still wet ; and (3) the dry saturated vapour after complete evaporation at g is superheated by compression gv to the temperature T_3.

(1) Assume neither gain nor loss of heat in passing through the expansion valve. The heat in 1 lb. of the working fluid will be the same before and after throttling. The heat before expansion is represented by the area $cepn$, and that after expansion by the area $efrp$. Hence these areas are assumed to be equal, and the effective refrigeration per pound during evaporation fa is the heat extracted, as represented by the area $famr$. Take s the specific heat of the liquid.

Now the area $cepn$ = area $efrp$ = $s(T_1 - T_2)$, and

$$ef = s \cdot \left(\frac{T_1 - T_2}{T_2} \right)$$

Also

$$ea = ed \qquad da = s \log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{}$$

$$\therefore fa = ea - ef = s \cdot \log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} - s \cdot \left(\frac{T_1 - T_2}{T_2} \right)$$

and heat extracted

$$= T_2 \times fa = s \cdot T_2 \cdot \log_\varepsilon \frac{T_1}{T_2} + T_2 \times \frac{L_1}{T_1} - s(T_1 - T_2) \quad (1)$$

Work done = area $abcea$ = area $pecbm$ - area $eamp$

$$= \text{area } pecn + \text{area } cbmn - \text{area } ednp - \text{area } damn$$

$$= s(T_1 - T_2) + T_1 \times \frac{L_1}{T_1} - T_2 \cdot s \log_\varepsilon \frac{T_1}{T_2} - T_2 \times \frac{L_1}{T_1}$$

$$= (T_1 - T_2) \left(s + \frac{L_1}{T_1} \right) - T_2 \cdot s \log_\varepsilon \frac{T_1}{T_2} \quad (2)$$
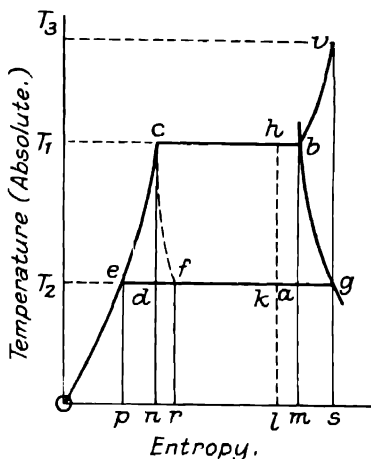
$$\text{Coefficient of performance} = \frac{\text{Heat extracted}}{\text{Work done}}$$

$$= \frac{T_2\left(s \cdot \log \dfrac{T_1}{T_2} + \dfrac{L_1}{T_1}\right) - s(T_1 - T_2)}{(T_1 - T_2)\left(s + \dfrac{L_1}{T_1}\right) - T_2 \cdot s \log_\varepsilon \dfrac{T_1}{T_2}} . \qquad . \qquad . \quad (3)$$

(2) When the vapour is wet at the end of compression. Let the compression begin at the state point k when the dryness fraction is ek/eg; and end at h with dryness $ch/cb = x_1$, and the latent heat $= x_1 L_1$ per lb. of stuff.

We have, heat extracted

$$= \text{area } fklr$$

$$= s \cdot T_2 \cdot \log_\varepsilon \frac{T_1}{T_2} + T_2 \cdot \frac{x_1 L_1}{T_1} - s(T_1 - T_2) \ .. \qquad . \quad (4)$$

Also, work done

$$= cchke = (T_1 - T_2)\left(s + \frac{x_1 L_1}{T_1}\right) - T_2 \cdot s \log_\varepsilon\left(\frac{T_1}{T_2}\right) \quad (5)$$

$\therefore$ coefficient of performance $= \dfrac{\text{Expression (4)}}{\text{Expression (5)}}$

3. When the compressor draws in dry saturated vapour from the refrigerator. The " dry " compression begins at the state point g on the saturation line, Fig. 71, and ends at v, temperature T_3, on the superheat curve of constant pressure drawn from b at saturation temperature T_1.

Heat extracted $=$ area $fgsr$, here $eg = \dfrac{L_2}{T_2}$, and $ef = s \dfrac{(T_1 - T_2)}{T_2}$;

Now $\quad fg = eg - ef = \dfrac{L_2}{T_2} - s \dfrac{(T_1 - T_2)}{T_2}$,

and heat extracted

$$= fg \cdot T_2 = L_2 - s(T_1 - T_2) \qquad . \qquad . \qquad . \qquad . \quad (6)$$

Work done $=$ area $ccbvge =$ area $ecd +$ area $dcbvg$

$$= s(T_1 - T_2) - T_2 \cdot s \log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1}(T_1 - T_2)$$

$$+ C_p(T_3 - T_1) - T_2 \cdot C_p \log_\varepsilon \frac{T_3}{T_1}$$

$$- (T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) + C_p(T_3 - T_1)$$

$$- T_2\left(s \log_\varepsilon \frac{T_1}{T_2} + C_p \cdot \log_\varepsilon \frac{T_3}{T_1}\right) \qquad . \qquad . \qquad . \quad (7)$$

Coefficient of performance $= \dfrac{\text{Expression (6)}}{\text{Expression (7)}}$

To calculate T_3, in superheating from b to v, at constant pressure, the gain of entropy,

$$ag = C_p \log_\epsilon \frac{T_3}{T_1}.$$

But $\qquad ag = eg - ea = \dfrac{L_2}{T_2} - s \cdot \log_\epsilon \dfrac{T_1}{T_2} - \dfrac{L_1}{T_1}.$

Equate these values of ag, we have

$$C_p \cdot \log_\epsilon \frac{T_3}{T_1} - \frac{L_2}{T_2} - s \cdot \log_\epsilon \frac{T_1}{T_2} - \frac{L_1}{T_1}. \tag{8}$$

from which T_3 may be calculated.

Example 2. In an ammonia refrigerating machine the temperature in the refrigerator is 15° F., and after compression 90° F. In the cooler the vapour is condensed at 90° F., and then passes through an expansion valve. Calculate the coefficient of performance when the vapour at the end of adiabatic compression is (*a*) just dry and saturated ; (*b*) 85 per cent dry ; (*c*) just dry and saturated when compression begins. Take the specific heat of liquid ammonia as 1·1, and of the vapour Cp 0·508, and the latent heat of evaporation as $566 - 0.8t°$ F.

Given $L_{90°} = 566 - 0.8 \times 90 = 494$ B.Th.U. per lb.,

$\qquad T_1 = 90° + 460° = 550°$ F. (abs.), and $T_2 = 475°$ F. (abs.)

(*a*) By equation (1) (p. 171), heat extracted

$$= s \cdot T_2 \cdot \log_\epsilon \frac{T_1}{T_2} + T_2 \times \frac{L_1}{T_1} - s(T_1 - T_2)$$

$$= 1.1 \times 475 \times 2.303 \log_{10} \frac{550}{475} + \frac{475 \times 494}{550} - 1.1(90 - 15)$$

$$= 76.65 + 426.6 - 82.5 = 420.75 \text{ B.Th.U.}$$

By equation (2), work done $= (T_1 - T_2)\left(s + \dfrac{L_1}{T_1}\right) - T_2 \cdot s \log_\epsilon \dfrac{T_1}{T_2}$

$$= 75(1.1 + 0.898) - 1.1 \times 475 \times 2.303 \log_{10} \frac{550}{475}$$

$$= 75 \times 1.998 - 76.65 = 73.2 \text{ B.Th.U.}$$

$\therefore$ coefficient of performance $= \dfrac{420.75}{73.2} = \underline{5.74} \qquad Answer.$

(*b*) By equation (4), heat extracted

$$= s \cdot T_2 \cdot \log_\epsilon \frac{T_1}{T_2} + T_2 \cdot \frac{x_1 L_1}{T_1} - s(T_1 - T_2)$$

$$= 76.65 + 0.85 \times 426.6 - 82.5 = 356.76 \text{ B.Th.U.}$$

And by equation (5),

$$\text{Work done} = (T_1 - T_2)\left(s + \frac{x_1 L_1}{T_l}\right) - T_2 \cdot s \log_\varepsilon \frac{T_1}{T_2}$$

$$= 75(1\cdot1 + \cdot85 \times 0\cdot89818) - 76\cdot65 = 63\cdot1 \text{ B.Th.U.}$$

$$\therefore \text{ coefficient of performance} = \frac{356\cdot76}{63\cdot1} = \underline{5\cdot65} \qquad Answer.$$

. (c) Again, when the dry saturated vapour at g is compressed adiabatically to the state point v, and superheated to T_3, to find this temperature, we have $L_{15^\circ} = 566 - 0\cdot8 \times 15 = 554$ B. Th.U. per lb.

and, by equation (8), $C_p \cdot \log_\varepsilon \frac{T_3}{T_1} = \frac{L_2}{T_2} - s \cdot \log_\varepsilon \frac{T_1}{T_2} - \frac{L_1}{T_1}$,

that is, $0\cdot508 \times 2\cdot303 \log \frac{T_3}{550} = \frac{554}{475} - 1\cdot1 \times 2\cdot303 \log \frac{550}{475} - \frac{494}{550}$.

$$1\cdot17 \log \frac{T_3}{550} = 1\cdot1663 - 0\cdot1613 - 0\cdot8982 = 0\cdot1069$$

and $\qquad \log T_3 - \log 550 = \frac{0\cdot1069}{1\cdot17} = 0\cdot09137$

from which $\qquad\qquad T_3 = 678\cdot8^\circ$ F. (abs.).

Then, by equation (6), heat extracted

$$= L_2 - s(T_1 - T_2)$$

$$= 554 - 1\cdot1(90 - 15) = 471\cdot5 \text{ B.Th U. per lb.}$$

and, by equation (7), work done

$$= (T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) + C_p(T_3 - T_1) - T_2\left(s \log_\varepsilon \frac{T_1}{T_2} + C_p \cdot \log_\varepsilon \frac{T_2}{T_1}\right)$$

$$= 75\left(1\cdot1 + \frac{494}{550}\right) + 0\cdot508 \times 128\cdot5 - 475 (\quad . \quad . \quad . \quad)$$

$$\left(1\cdot1 \times 2\cdot303 \log \frac{550}{475} + \cdot508 \times 2\cdot303 \log \frac{678\cdot8}{550}\right)$$

$$= 75 \times 1\cdot99812 + 0\cdot508 \times 128\cdot5 - 475(0\cdot1613 + 0\cdot1069)$$

$$= 149\cdot859 + 65\cdot278 - 127\cdot39 = 87\cdot74 \text{ B.Th.U.}$$

$$\therefore \text{ coefficient of performance} = \frac{471\cdot5}{87\cdot74} = \underline{5\cdot37} \qquad Answer.$$

Example 3. The following are approximate expressions for the entropy of ammonia liquid and dry saturated vapour : liquid, $0\cdot00184\,(t-32)$; vapour, $1\cdot158 - 0\cdot00192\,(t-32)$, t being the temperature on the Fahrenheit scale. Obtain corresponding expressions of the form $a + bt_c$, t_c being the temperature on the Centigrade scale. Draw the θ, ϕ chart between temperatures 14° F. and 77° F. (-10° C. and 25° C.). Find the coefficient of

performance of a refrigerator working on a reversed Rankine cycle between these limits, the vapour being 5 per cent wet at the end of compression. If the actual performance is 0·6 of the amount in the above ideal case, calculate the pounds of ice produced per horse-power from water at the freezing point. Latent heat of ice, 144 B.Th.U. (80 C.H.U.).

<div align="right">(<i>U.L., B.Sc. (Eng.)</i>)</div>

At 77° F. or 25° C. $\begin{cases} \text{liquid} & \phi = 0.00184(77-32) = 0.0828 \\ \text{vapour} & \phi = 1.158 - 0.00192 \times 45 = 1.0716 \end{cases}$

At 14° F. or –10°C. $\begin{cases} \text{liquid} & \phi = 0.00184(14° - 32°) = -0.03312 \\ \text{vapour} & \phi = 1.158 - 0.00192 \times (-18) \\ & \quad = 1.158 + 0.03456 = 1.19256 \end{cases}$
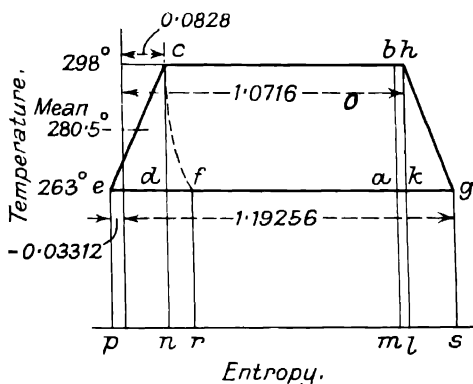
FIG. 72. AMMONIA COMPRESSION CYCLE

On C scale, $liquid$ $-0.03312 = -10b$ $\therefore b = 0.00312$

$+ 0.08280 = 25b$ and $\phi = 0.00312t_c$

$0.04968 = 15b$ *Answer.*

$vapour$ $1.07160 = a + 25b$

$1.19256 = a - 10b$ $. b = -0.003456$

$\overline{-0.12096 \quad 35b}$

$a = 1.19256 - 0.03456 = 1.158$

$\therefore \phi = 1.158 - 0.003456t_c°$ *Answer.*

Plotting the above values on $\theta\phi$ chart, as shown in Fig. 72,

Dryness fraction $bc/ch = 95$ per cent.

⁕ $bc = 0.95(1.0716 - 0.0828) = 0.95 \times 0.9888 = 0.93936$

$cd = 0.0828 + 0.03312 = 0.11592$

Temperature range 35° C, and area $ecd = dfrn$,

$$df = \frac{17.5 \times 0.11592}{263} = 0.00771$$

$$\therefore \quad fa = 0.93936 - 0.00771 = 0.93165$$

Refrigeration per pound $= 0.93165 \times 263 = 245.024$ C.H.U.

Work done $= 17.5 \times 0.11592 + 0.93936 \times 35$

$$= 2.0286 + 32.8776 = 34.906 \text{ C.H.U.}$$

$$\therefore \text{ coefficient of performance} = \frac{245.024}{34.906} = 7.02$$

Actual coefficient $= 7.02 \times 0.6 = 4.212$
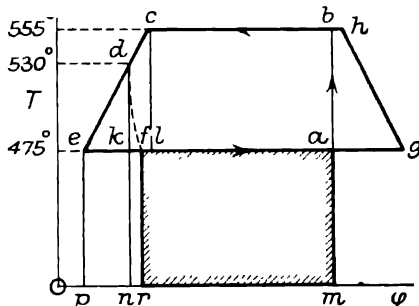
FIG. 73. AMMONIA COMPRESSION CYCLE

Hence, ice produced per horse-power hour

$$\frac{33,000 \times 60}{1400} \times \frac{4.212}{80} = 74.46 \text{ lb.} \qquad \textit{Answer.}$$

Example 4. In an ammonia refrigerating machine, the temperature of evaporation of the liquid is 15° F. The vapour is compressed and eventually delivered to the cooler at a temperature of 95° F., the dryness at the end of compression being 0.95. In the cooler the vapour is condensed at 95° F., and then cooled to 70° F.; after which the liquid is passed through a throttling valve into the evaporating coils. Calculate for this machine the ratio of the heat abstracted to the work done if the specific heat of the liquid is 1.1, and the latent heat of the vapour is given by $566 - 0.8t$, where t is the temperature of the vapour in degrees Fahrenheit. (*U.L., B.Sc. (Eng.).*)

At 95° F., $L = 566 - 0.8 \times 95 = 566 - 76 = 490$ B.Th.U.

Fig. 73 shows the cycle, *abcdfa*, on the temperature entropy diagram or chart.

$bc = 0.95 \times \frac{490}{555} = 0.84$, since the dryness fraction is $\frac{bc}{ch} = 0.95$.

The total heat is constant during throttling, from d to f.

∴ area $pedn =$ area $pefr = ef \times 475 = 1\cdot1(70 - 15) = 60\cdot5$ B.Th.U.

$$ef = \frac{60\cdot5}{475} = 0\cdot1274$$

Also $\quad el = 1\cdot1 \times 2\cdot303 \times \log\dfrac{555}{475} = 0\cdot1712$

∴ $fa = ca - ef = bc + el - ef = 0\cdot84 + 0\cdot1712 - 0\cdot1274 = 0\cdot8838$

∴ Heat extracted per pound of NH_3, represented by the area $famr$, down to absolute zero

$$= 0\cdot884 \times 475 = 419\cdot8 \text{ B.Th.U. per lb. per cycle.}$$

Work done, represented by the area $abcde =$ area $cel +$ area $abcl$

$$= (1\cdot1 \times 80 - 0\cdot1712 \times 475) + 0\cdot84(95° - 15°)$$

$$= 88 - 81\cdot32 + 67\cdot2 = 73\cdot88 \text{ B.Th.U}$$

∴ coefficient of performance $= \dfrac{\text{Heat abstracted}}{\text{Work done}} = \dfrac{419\cdot8}{73\cdot88}$

$$= 5\cdot68. \qquad\qquad Answer.$$

Refrigerating Effect and Work Done in Terms of Total Heat.

During a reversible operation at *constant pressure*, like evaporation or condensation, the heat taken in or given out by a fluid is the change of its total heat (see p. 44). In the evaporating coils, the amount of heat taken in from the cold body by 1 lb. of refrigerant is $H_a - H_f$ (Fig. 70) ; also the heat rejected to the condenser and cooler is $H_b - H_c$. During throttling in passing the expansion valve, the total heat is unchanged, and $H_f = H_c$. Therefore the heat rejected is $H_b - H_f$. The work done in driving the compressor, represented by the area of the pv indicator diagram (Fig. 21, p. 44) or its thermal equivalent, $A \int_a^b v \, . \, dp = H_b - H_a$. Hence the quantities of heat dealt with during the whole refrigerating cycle are

$$\text{Work expended} = \text{Heat rejected} - \text{Heat taken up}$$

$$H_b - H_a = (H_b - H_f) - (H_a - H_f)$$

The coefficient of performance $= \dfrac{H_a - H_f}{H_b - H_a}$.

This represents the performance of the standard refrigerating machine working on the *ideal cycle*, with adiabatic compression between constant pressure limits of suction and discharge, chosen by the Institution of Mechanical Engineers, and clearly explained by Sir Alfred Ewing in the Report of the Refrigeration Research Committee.*

* See *Min. Proc. Inst. Mech. Eng.*, Oct., 1914.

The standard *unit of refrigeration* proposed for rating refrigerating machines is 1 *calorie per second*, the calorie being the amount of heat required to change the temperature of 1 kilogram of water by 1° C. ; 1 calorie is $2.2046 \times \dfrac{9}{5}$, or 3.9683 B.Th.U. ; and 1 calorie per second is equivalent to $3.9683 \times 60 \times 60 \times 24$, or about 342,860 B.Th.U. per day of 24 hours.

The refrigeration may be stated for particular temperature conditions, or for *standard conditions* defined as follows : The temperature limits to be steady ; the temperature of the cooling water to range from 15° C. (59° F.) at inlet to 20° C. (68° F.) at outlet, and the temperature of the brine to range from 0° C. (32° F.) to −5° C. (23° F.). The refrigeration produced under standard conditions is called the *rated capacity* of the machine. *A machine of unit rated capacity produces a refrigeration of* 1 *calorie per second in steady working under standard conditions.* In the case of cooling by direct expansion, without the use of brine, the lower limit of temperature of the vapour in the evaporator is taken as − 10° C. (14° F.), and the temperature of the cooling water the same as those for machines using brine.

" In the *ideal standard cycle* for comparison with actual performance, the most efficient cycle employing adiabatic compression is that in which the pressures of suction and condensation, measured just after and just before the substance passes the expansion valve, are the same as those that occur in the actual process of working, and in which the substance is cooled before passing the expansion valve to a temperature equal to that at which, in the actual process, the cooling water is supplied."

The *efficiency ratio* is the fraction—

$$\frac{\text{Coefficient of actual performance}}{\text{Coefficient of ideal performance}}.$$

The work may be reckoned either as the indicated work done in the compressor or as the total work required to drive it, and, when practicable, both of these should be given in stating the results of a test.

For the calculation of the ideal performance, the Mollier charts of entropy and total heat for CO_2, NH_3, and SO_2, the three substances in practical use, have been drawn on a large scale, and tables giving the pressure temperature and volume of saturated vapour prepared by Professor C. F. Jenkin for the Committee.

Mollier, in his total heat entropy chart, uses oblique co-ordinates in order to open up or spread out the diagram and exhibit the cycle more clearly. When plotting on ordinary squared paper, the co-ordinates are found by drawing lines parallel to the direction of the axes through each state point to cut the scales. The rectangular co-ordinates of the point B, Fig. 74, are $y = 4$, $x = 3.5$, and of

C, 4, 4·5.   By drawing the axis of *x*, inclined at a small angle *α* in Fig. 75, the vertical scale is not changed, but the horizontal scale is
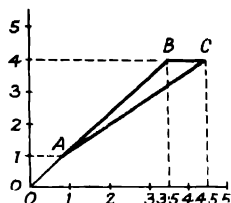
FIG. 74.   RECTANGULAR
CO-ORDINATES
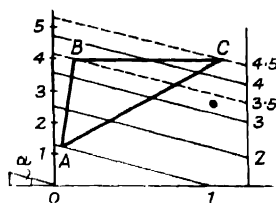
FIG. 75.   OBLIQUE
CO-ORDINATES

enlarged 4 times, the line *BC* is made 4 times its original length, and the triangle *ABC* is enlarged for the same size of paper, since
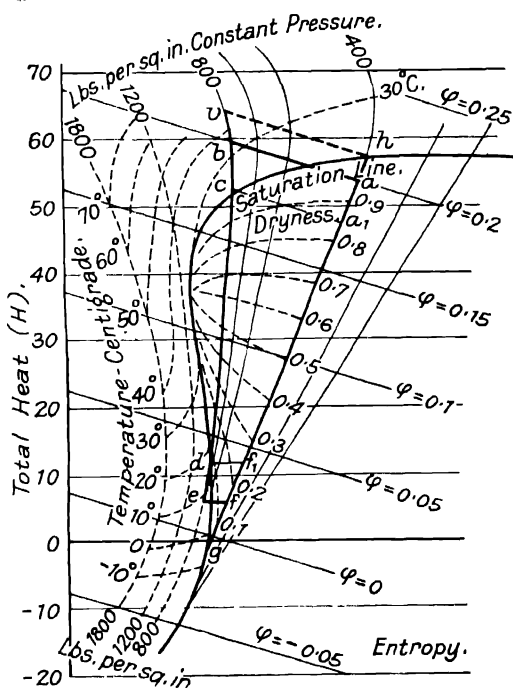
FIG. 76.   *Hφ* CHART FOR CARBON DIOXIDE

the values of *x* can be given as shown on the constant *φ* lines to the right.

Fig. 76 shows, on a small scale with a few lines, a total heat entropy chart for carbonic acid with oblique co-ordinates. Adiabatics (*φ* constant) are drawn parallel to the oblique axis, and lines

representing constant total heat are horizontal. During evaporation or condensation of the CO_2 at *constant pressure* and temperature, the change of total heat, $dH = dQ = T \cdot d\phi$, and $\dfrac{dH}{d\phi} = T$, so that the slope of a constant pressure line measures the temperature. Within the saturation line or boundary curve, representing the mixture of liquid and saturated vapour, the temperature of saturation along any constant pressure line is constant, and therefore any constant pressure line in the wet region is straight, but, upon crossing the dry saturation line into the region of superheat, the temperature rises and the line curves. Each of the fan-shaped lines is thus a line of constant pressure and temperature, and represents evaporation in the refrigerating coils. The effect of drawing the oblique axis is to open out the fan. Again, since the heat taken in by the liquid during evaporation is proportional to the dryness, equal distances along any constant temperature line *gh* correspond to equal increments of total heat, or equal changes of dryness. Any evaporation line like *gh* may be divided into 10 equal parts, representing dryness fractions.

Vapour Compression Cycle on the $H\phi$ Chart. In Fig. 76, which shows the changes of total heat in relation to entropy for carbonic acid, the same letters *a, b, c, d, e, f* represent the state points of the refrigerant throughout the cycle as are given in the $T\phi$ diagram, Fig. 68 (p. 169). The point *a* represents the state of the fluid mixture in the evaporating coils before it enters the compressor.

During evaporation along *fa* the constant pressure is 400 lb. per sq. in., and the temperature about $-8.6°$ C. At *a* the wetness of the vapour is *ah/gh*, and the dryness fraction *ag/gh* is 0.95, when the mixture is drawn into the compressor. The line *ab* parallel to the oblique lines of constant entropy represents adiabatic compression, and is drawn to intersect at *b* the curve giving the higher limit of pressure, namely, that in the condenser, 800 lb. per sq. in. The temperature of the superheated vapour is then $35°$ C.

If evaporation were complete and only dry saturated vapour drawn into the compressor, then " dry " adiabatic compression in the region of superheat would be shown by the broken line *hv*, drawn from *h* on the dry saturation line ; and the temperature at *v*, 45° C.

On the other hand, if the dryness fraction of the mixture were only 0.83, at a_1 when taken into the compressor, such that the line a_1c would not pass the dry saturation line but meet it just at the condenser pressure, then there would be no superheating, and the compression is called entirely " wet " compression, and the temperature of the dry saturated vapour 18.6° C.

The difference between the readings on the total heat scale of the ends *b* and *a* of the compression line, namely $H_b - H_a = 6$, gives the amount of work in heat units necessary to compress 1 lb.

of the refrigerant. For hv, the work done is equivalent to 6·7 C.H.U. ; and for a_1c it is 5·2 C.H.U.

The superheated vapour at b is cooled in the condenser at constant pressure, 800 lb. per sq. in., to dry saturated vapour at c ; followed by condensation cd until all is liquid at d, and temperature 18·6° C., and the liquid is further cooled to 10° C. at the state point e, found by the intersection of the constant pressure line from d with the line of constant temperature appropriate to the cooling water. Now the liquid at 10° C. passes through the expansion valve into the evaporating coils. From e draw a line ef parallel to the horizontal lines of total heat to meet at f, the line representing the temperature – 8·6° C. and pressure 400 lb. per sq. in. in the evaporator. The line ef of constant total heat represents throttling through the expansion valve, and determines the state point f, giving the conditions when evaporation starts in the coils of the refrigerator. The dryness fraction gf/gh indicates the fraction of liquid which has evaporated in the throttling process to reduce the mixture to the temperature and pressure in the evaporating coils.

The line fa shows evaporation at the lower constant temperature and pressure during the effective refrigeration. The difference between the readings on the vertical total heat scale, corresponding to the state points a and f, the ends of the evaporation line, gives the amount of heat which 1 lb. of the carbonic acid in the evaporator takes up from the brine or cold body.

In this case the refrigerating effect, $H_a - H_f = 48$ C.H.U. per lb., and the coefficient of performance is 48/6, or 8. When the evaporation is carried on to completely dry vapour at h, $H_h - H_f = 51$ C.H.U. per lb., and the coefficient of performance is 51/6·7, or 7·6. In the third case, that of " wet " compression, $H_{a1} - H_{f1} = 40·8$ C.H.U per lb., giving a coefficient of performance 40·8/5·2, or 7·85.

Sir Alfred Ewing points out[*] a simple method of determining the dryness before compression which gives the maximum coefficient of performance in the ideal theoretical refrigerating machine.

"The refrigerating effect of any degree of dryness a, Fig. 76, before compression is proportional (on some scale) to the length fa. The work done is proportional (on another scale) to the length ab. Hence the position of b which will give the highest coefficient of performance is that which gives the smallest ratio of ab to fa. This is found by drawing a tangent from f on the evaporation line to the line of constant pressure for the condenser, meeting that line in b. Then the compression line ab is the one which gives the maximum coefficient of performance in the ideal cycle."

In order to determine the standard cycle a point e is taken, which gives the condition of the liquid carbonic acid at 10° C. in this example before it enters the evaporator. Draw the line ef

[*] See Appendix II of the *Report of the Refrigeration Committee of the Institution of Mechanical Engineers*, 1914, p. 681.

parallel to the horizontal lines of total heat to meet the line representing the required temperature and pressure in the evaporator.

Then the tangent from f to the condenser pressure curve determines the point b in the Fig. 76, hence the state point a of dryness at which compression should begin. It is advisable to use the large scale charts, drawn up by Professor C. Frewen Jenkin, from
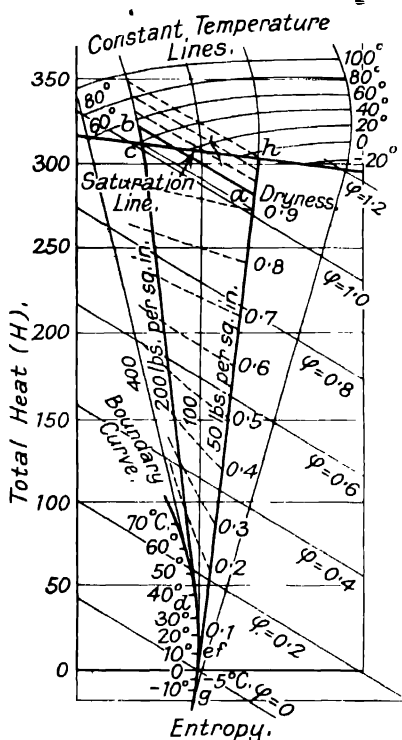
Fig. 77. $H\phi$ Chart for Ammonia

which the figures given in this example were found by measurement.

Again, suppose the condensed liquid at $18.6°$ C., in the state d, instead of being cooled to $10°$ C., as in the previous example, is allowed to pass directly through the expansion valve. The throttling is represented by the constant total heat line df_1, and effective refrigeration starts at f_1, when the dryness fraction is already gf_1/gh. Now, for maximum performance, draw a tangent from f_1 to the condenser pressure curve bc, and the constant ϕ line from the point of contact meets the evaporation line gh, where the dryness is 0.975. The maximum coefficient of performance is found to be 7.25, instead of 8. Obviously, the performance of the machine is improved by cooling the liquid refrigerant after condensation and before it passes the regulating or expansion valve to the lowest temperature to take up heat from the cold body.

Another example is given in Fig. 77, to illustrate the use of the $H\phi$ diagram for ammonia. The temperature of evaporation taken is $-5°$ C. at the corresponding constant pressure 50 lb. per sq. in., and the ammonia is condensed at $35.5°$ C., the pressure being 200 lb. per sq. in., and the liquid cooled to $15°$ C.

Here the cooling curve de of the liquid ammonia practically coincides with the boundary curve, even on the large scale chart, while the points e and f coincide on the diagram, Fig. 77.

When there is uncertainty as to the exact point of contact of the tangent fb to the curve, the point a for the greatest coefficient of performance may be found by taking several values of the dryness

before compression, so that the vapour is dry or slightly superheated at the end of adiabatic compression.

Starting with the vapour of dryness 0·9 when drawn into the compressor, which gives dry saturated vapour when the pressure is 200 lb. per sq. in., it is only necessary to take the two readings of H_a, H_b on the chart for each compression line, and of H_f on the total heat scale. The following results were obtained—

Dryness before Compression	Refrigerating Effect	Work in the Compressor	Coefficient of Performance
	$H_a - H_f$	$H_b - H_a$	
1·0	286·6	45·5	6·30
0·975	277·6	43·7	6·35
0·95	270·2	41·7	6·48
0·925	258·6	39·0	6·63
0·9	255·6	38·8	6·58

Thus, according to the ideal standard cycle for ammonia, on the assumption of *adiabatic* compression, the greatest coefficient of performance is found by having the vapour slightly superheated at the end of compression.

From the $H\phi$ charts it is easy to determine the amount of heat which 1 lb. of ammonia or carbonic acid can extract from the cold body when working under given conditions as to temperature and pressure of evaporation and condensation. It is also necessary to know the volume of 1 lb. of the working fluid in the state of vapour from tables. Then, making due allowances for loss of heat, mechanical and volumetric efficiencies, the power and size of the compressor may be estimated.

In the **actual machine,** instead of the ideal adiabatic compression there is interchange of heat between the vapour and the walls of the compressor cylinder, and this transfer of heat is less with dry compression than with wet, because the liquid is a better heat conductor and has greater specific heat than the dry vapour. In addition to the indicated work in the compressor, there is heat, given to the vapour, equivalent to work spent in friction of the piston and glands. Every part of the plant at a temperature below that of the atmosphere must gain heat by leakage into it from the surroundings.

With saturated vapour in the compressor at the end of the stroke, the loss due to **clearance** may be greater than with superheated vapour. It is all-important to have the clearance between the piston and cylinder cover, reduced to the smallest practicable. Any vapour remaining in the clearance at the end of the discharge stroke is at the higher pressure, and the piston must travel some

13—(5434)

distance in the suction stroke before this vapour expands below the lower pressure to allow the fresh charge to be admitted.

Superheat at the end of compression is also desirable, to avoid liquid being pumped idly round the whole cycle, and to ensure that the clearance space does not contain liquid which would evaporate during the following suction stroke, and seriously diminish the circulation of the vapour and the **volumetric efficiency** of the compressor. Thus by careful adjustment for slightly wet compression, a greater quantity of refrigerant would be circulated and greater refrigerating effect apparently produced than with all dry compression for the same volume swept by the piston. Generally, in practice, the liquid is completely evaporated in order to avoid the absorption of heat by evaporation in the compressor cylinder, which is found to counterbalance the slight theoretical gain by partially wet compression.

FIG. 78. *Hϕ* CHART

When the temperature of water entering the condenser is near the critical point of carbonic acid, superheating is necessary. With cooling water at a lower temperature, and under ordinary working conditions, the record of tests shows the best results when the CO_2 vapour is discharged from the compressor at a temperature about 50° to 70° C.

Trials. In tests by Linde to compare the performance of a carbonic acid machine and an ammonia machine (*a*) when the condensed liquid was cooled to the temperature of the refrigerator before passing through the expansion valve, he found practically the same amount of refrigeration in each machine. (*b*) When the temperature of the condenser was 72° F. and that of the brine 15° F., the co-efficient of performance in the CO_2 machine was 82 per cent of that in the ammonia machine. (*c*) When the condenser was raised to 95° F. and the two machines working between the same limits of temperature, the co-efficient of performance of the CO_2 machine was 50 per cent of that in the ammonia machine.

Professor Schrotter made careful trials of a Linde ammonia compression machine. The cooling water was supplied at 10° C. and raised to 20° C. in the condenser. In one case the mean temperature of the brine was – 3·5° C., the indicated horse-power of the compressor 14·5, and the actual co-efficient of performance 7·21 ; under the same conditions the ideal performance of the cycle with an expansion valve would be 10·2.

Example 5.   An ammonia vapour compression refrigeration plant is to effect a refrigeration of 20 lb. calories per second, and the working limits are as shown in the heat entropy chart, Fig. 78. The net heat from the cold

chamber is only 85 per cent of the possible amount shown on the chart, and the mechanical efficiency of the compressor is 65 per cent. Determine the horse-power required to drive the compressor and the amount of fluid circulation required for the stated performance.

If the specific volume of saturated ammonia vapour at 40 lb. per sq. in. is 7 cub. ft. per lb., determine a suitable size of compressor at 80 r.p.m., single-acting, with a piston speed not exceeding 150 ft. per min.

$$(U.L., B.Sc. (Eng.), 1921.)$$

By the chart, at the state points a, b, c, e, f of the cycle, we have
Heat extracted $= H_a - H_f = 274 - 22$

$$= 252 \text{ calories per lb. of NH}_3 \text{ evaporated.}$$

Work done $\quad = H_b - H_a = 310 - 274 = 36$ calories.

Net refrigeration per lb. $NH_3 = 252 \times 0.85$

$$= 214.2 \text{ C.H.U. per cycle or rev.}$$

Let $\qquad V =$ specific volume of gas leaving evaporator

$\qquad s =$ specific volume of liquid

$\qquad x =$ dryness fraction of gas leaving evaporator

Then effective volume per lb. of NH_3

$$= Vx + s(1 - x)^* = 7 \times 0.91 = 6.37 \text{ cub. ft.}$$

$\therefore$ refrigeration per cubic foot of compressor cylinder volume

$$= \frac{H_a - H_f}{Vx} = \frac{214.2}{6.37} = 33.62 \text{ calories.}$$

The refrigeration required is 20 calories per sec., which would be given by

$$\bullet \quad \frac{20}{33.62} \times 60 \text{ cub. ft. of NH}_3 \text{ per min.} = 35.7 \text{ cub. ft. per min.}$$

and there are 80 rev. or cycles per min.

Hence each charge per cycle or revolution $= \dfrac{35.7}{80}$ cub. ft. of NH_3

Now, work done per lb. NH_3 per cycle $= 36$ C.H.U. $= 36 \times 1400$ ft.-lb

and the work done per cubic foot $= \dfrac{36 \times 1400}{6.37}$ ft.-lb.

$\therefore$ Horse-power required at 80 cycles per min. is

$$\frac{36 \times 1400 \times 35.7}{6.37 \times 33,000} \times \frac{100}{65} = 13.16 \text{ H.P.}$$

Again, at 80 rev. or 160 piston strokes per min., and piston speed 150 ft. per min., the length of one stroke is $\dfrac{15}{16}$ ft. But the *volume*

---

* The term $s(1 - x) = 0.02563(1 - 0.91) = 0.0023$ for NH_3 is negligible for practical purposes.

swept by the piston during one suction and one compression stroke must be 2×35.7, or 71.4 cub. ft. per min.

Let $A =$ sectional area in square feet of compressor cylinder,

then $A \times \dfrac{15}{16} \times 160 = 71.4$, and $A = 0.476$ sq. ft.

∴ Diameter of cylinder in inches $= \dfrac{0.476 \times 144}{0.7854} = 9.342$ in.,

and piston stroke $= \dfrac{15}{16} \times 12 = 11\frac{1}{4}$ in.
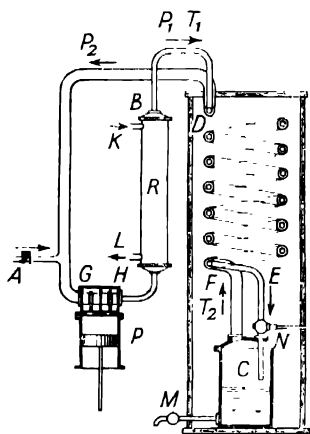
FIG. 79. LINDE REGENERA-
TIVE MACHINE

Regenerative Cooling. The Linde regenerative process of refrigeration, applied to attain the lowest temperatures for the liquefaction of air and gases, is based on the Joule-Thomson cooling effect.

In the diagram Fig. 79, the compression pump, P, draws in air, or other gas, by the suction valve G, and delivers it under high pressure, p_1, of 100 to 200 atmospheres, by the valve H to the cooler R, through which cold water circulates by the inlet K and the outlet L. The compressed air at high pressure, p_1, and temperature T_1, then passes through the *counter current interchanger* by the inner spiral pipe DE, to the throttle valve or orifice N, into the vessel C. During throttling the drop of temperature to T^1 occurs with the pressure drop from p_1 to p_2.

Then the air returns through the outer pipe FD, taking up heat from the compressed air flowing in the opposite direction through the inner coil DE, and is raised very nearly to the initial temperature T_1 at D. The same air is again drawn into the compression pump P, and goes through the cycle of compression, cooling throttling, and taking up heat during its return.

The apparently slight cooling effect or drop of temperature by throttling, which in the case of air is only $\frac{1}{4}°$ C. for every atmosphere fall in pressure, when repeated continuously under the difference of pressure, $p_1 - p_2$, enables the air after throttling to take up heat in the interchanger from the other portion of compressed air on its way through DE to the throttle valve. By repeating the cycle with continuous abstraction of the same quantity of heat from the interchanger coils, and air in them, any gas is gradually reduced below its critical temperature and to T_2, the boiling point at the

lower pressure p_2, which produces liquefaction. Thus the steady state of thermal equilibrium is established at the lowest temperature in the vessel C.

The ideal conditions are : No heat leakage from outside to the interchanger, and complete transfer of heat in the interchanger at every point along the inner coil at the high pressure, p_1, to the outer coil. Then there will be a gradual fall of temperature from T_1 at D, to E, where the drop of temperature by throttling eventually reduces it to T_2 at F in the low pressure coil FD, in which the returning gas at pressure p_2 receives heat from the high pressure coil and gas, until at D its temperature is raised to T_1.

There are two losses : (1) That due to leakage of heat from the outside by conduction and radiation, which thorough insulation may reduce, but never quite prevent. (2) Imperfect interchange of heat, which is overcome by the counter current in very long pipes of copper or other good conductor of heat, one inside the other, formed into a spiral coil or worm, giving close thermal connection.

The Joule-Thomson cooling effect, $T_1 - T^1$, for a given pressure drop may be measured by the quantity of heat, Q per lb. of gas, necessary to raise the gas after throttling to the temperature T_1, at which it enters the interchanger. That is, $Q = Cp(T_1 - T^1)$, where C_p is the mean specific heat of the gas between these temperatures and at the lower pressure p_2.

Now, the total heat, H, is a function of the pressure, and for a given temperature the total heat of a gas is greater at the lower pressure. Each pound of gas entering the interchanger at p_1, T'_1, has internal energy E_1, and the work done upon it is p_1v_1 If each pound of returning gas leaves the interchanger at the same point D, under pressure p_2, and at the same temperature T_1, having taken up a quantity of heat, Q, in the interchanger, it has internal energy E_2, and does work against external pressure equal to p_2v_2. Therefore, the difference in the total heat is the amount of heat taken from the apparatus per pound of gas that flows through in one cycle—

$$H_2 - H_1 = E_2 + A p_2 v_2 - (E_1 + A \cdot p_1 v_1)$$

This amount of heat Q is that necessary to raise the gas on leaving to the temperature T_1, hence $Q = H_2 - H_1$.

The value of Q depends only on the pressures p_1 and p_2, and the temperature T_1, and may be determined experimentally by observing the Joule-Thomson drop of temperature, $T_1 - T^1$, during throttling from p_1 to p_2, without any interchange of heat

When the gas begins to liquefy, the part that is liquid collects in the vessel C, and is drawn off by the stop-cock M. More air is pumped in through the stop valve A by an auxiliary low pressure pump, to replace the fraction that is liquefied.

The effective refrigeration is greatly increased with a lower initial temperature, T_1, hence the compressed air or gas is pre-cooled by

separate refrigeration before being supplied through the stop valve *A*. This pre-cooling is essential for hydrogen, in which throttling at the ordinary temperature has a heating effect that becomes a cooling effect at a low initial temperature.

Linde also showed that, while the cooling effect in throttling is increased by a larger pressure drop or difference of pressures $p_1 - p_2$, the work done in compression depends on the ratio of the pressures $r = \dfrac{p_1}{p_2}$, which should be kept small to obtain greater practical efficiency. In the isothermal compression of a perfect gas the work spent is $RT_1 \log_\varepsilon r$, and in adiabatic compression from constant initial temperature, $\left(\dfrac{p_1}{p_2}\right)^{\gamma' - 1} \gamma'$ is the variable factor. Hence, by working at a comparatively high back pressure p_2, the work spent is greatly reduced. If p_1 is 100 atmospheres and p_2 is one atmosphere, the cooling effect is proportional to 99, and the work of compression is nearly proportional to log 100. Whereas, when p_2 is 50 atmospheres, the cooling effect is proportional to 50 and the work of the compression pump to log 2. For adiabatic compression the work is still more reduced. Thus, working with p_2 at 50 atmospheres gives higher thermodynamic and practical efficiency. When p_1 is 200 atmospheres, p_2 may be 100 for efficient working.

As liquefaction proceeds, the apparatus remains at a steady temperature condition in thermal equilibrium, not getting any colder, neither gaining nor losing heat on the whole. At this stage the larger fraction of gas not liquefied continues to go through the same cycle, and the gas leaves the interchanger at the same temperature T_1 as the gas that enters at D.

If x is the fraction not liquefied, and $1 - x$ the small fraction drawn off as liquid at T_2; then the total heat H_1 of the gas entering the interchanger is equal to xH_2, that of the gas returning at D_1 together with $(1 - x)H_c$ of the liquid leaving at C.

$$H_1 = xH_2 + (1 - x)H_c$$

Suppose the liquid leaving at C were evaporated at its boiling point T_2, and the vapour heated at the same pressure p_2, from T_2 to T_1, to bring the whole of the fluid leaving the interchanger to the temperature T_1 at D; the heat required would be

$$(1 - x)\left\{L + C_p(T_1 - T_2)\right\} = Q$$

which gives the fraction $(1 - x)$ that is liquefied.

In Norway the Linde plant liquefies air on a large scale for the purpose of supplying nitrogen, which is passed over hot calcium carbide in the manufacture of a nitrogenous fertilizer.

Rectification of Liquid Air. In the Linde process of rectification of liquefied air, the more volatile nitrogen evaporates more readily than oxygen, and the liquid becomes richer in oxygen. Under

atmospheric pressure the boiling point of nitrogen is about – 194°C. or 79° C. (abs.), and that of oxygen – 182° C., or 91° C. (abs.).

In one process the upper part of a tall rectifying column is filled with glass balls, through which liquid air trickles down ; and at the bottom there are condenser pipes, where nearly pure liquid oxygen collects. Highly compressed air, cooled by passing through a counter current interchanger, enters at the bottom, becomes lique- fied in the condenser pipes, and its latent heat serves to evaporate some of the liquid oxygen. A stream of oxygen gas passes up the rectifying column and meets the liquid air trickling down. The oxygen gas condenses and returns with the down-stream, while the latent heat given out by the condensation evaporates the nitrogen, which passes off at the top of the column mixed with about 7 per cent oxygen ; while nearly pure oxygen collects at the bottom. The gas from the top of the column is passed through a counter current interchanger to take heat from the incoming compressed air.

In a modification of this process, a fractional method of lique- faction is adopted to separate a liquid rich in nitrogen into practically pure nitrogen, which passes off as gas at the top of the rectifying column, and pure oxygen at the bottom.

Ammonia Absorption Machine. Ammonia vapour (NH_3) is very soluble in cold water, and heat is given out when the NH_3 is absorbed by water. The ammonia vapour may be driven out of the solution by heating it, and a larger quantity of heat is needed to evaporate the liquid when in solution with water than to evaporate the same weight of pure liquid ammonia, at the same pressure and temperature.

The difference is the heat energy required to overcome the affinity of the substances and to separate the NH_3 from the water solution. It takes about 450 to 470 C.H.U. to separate 1 lb. of NH_3 from the solution in water and change its state from that of liquid to vapour, whereas the latent heat of evaporation from pure liquid ammonia is only 300 to 320 C.H.U. per lb. (See Table 11, p. 192.) When the NH_3 vapour, driven out of its water solution by steam heating in the generator, Fig. 80, is cooled by water circulation in a *condenser*, it is readily liquefied by its own pressure above 150 lb. per sq. in. (abs.) at the ordinary temperature. In the process of refrigeration, the liquid NH_3, at this pressure, is allowed to pass through a *regulating* or *expansion valve* into the evaporating coils in the *refrigerator*, where it extracts its latent heat of evapora- tion from the brine or other body to be cooled.

The vapour pressure is reduced, say to about 30 lb. per sq. in., and evaporation encouraged by *absorption* of the vapour in the solution in the *absorber* at 25 lb. per sq. in., which acts like the suction stroke in the vapour compression machine. The water solution, rich in NH_3, is pumped from the *absorber* back through

the *interchanger*, where it is heated, to the *analyser*, where it drips down over trays to the generator.

While the NH_3 is driven out of the solution in the generator, *heated by steam coils*, the liquid becomes denser and falls to the bottom. This weak solution from the bottom of the generator at the higher pressure and temperature is allowed to pass the *check valve* and through coils in the interchanger or economizer, where it gives up heat to the rich solution pumped from the absorber, and, thus *cooled*, passes again to the absorber, which requires to be cooled because of the amount of heat given out by the NH_3 in dissolving.
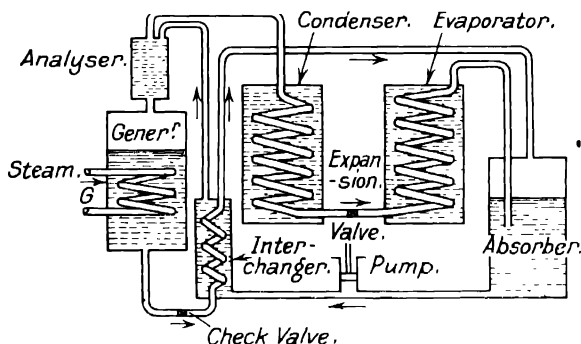
FIG. 80. AMMONIA ABSORPTION REFRIGERATING MACHINE

The wet ammonia vapour leaving the generator passes up through the *analyser*, or separator, containing a series of trays which separate more water from the NH_3. The vapour is further cooled and dried in a *rectifier*, not shown in the diagram, Fig. 80, on the top of the condenser, and the water condensed is returned to the upper part of the analyser.

In this absorption process of refrigeration the efficiency is measured by the ratio Q_2/Q, where Q_2 is the refrigeration effect, that is, the amount of heat extracted from the brine in the evaporator, and Q is the heat supplied to the refrigerating machine at high temperature in the steam, not only in the generator coils, but also that required to drive the pump. It is obvious this efficiency must be less than unity, on account of the additional heat needed to drive the ammonia from the solution in water, above the latent heat of evaporation which gives the effective refrigeration. In practice, taking into account all the losses, the refrigeration effect is about a quarter of the heat supplied in the steam. For instance, tests show that an ammonia absorption machine is capable of making 15 tons of ice per ton of coal. The calorific value of ordinary coal is about 12,000 B Th.U. per lb., and the heat taken from 1 lb.

of water in cooling and freezing, is about 200 B.Th.U. of effective refrigeration. Then the heat ratio, $\dfrac{Q_2}{Q} = \dfrac{15 \times 200}{12,000}$, or 25 per cent.

In the *absorption refrigerating machine* invented by Mr. W. W. Seay, anhydrous NH_3 is absorbed by anhydrous salt like sulphocyanide of ammonium, for which it has great affinity. The proportion in the mixture is 1 lb. of NH_2 to 3 lb. of the dry salt, which unite to form a liquid solution, and the NH_3 vapour can be driven off by heating, leaving the salt dry again. There is no water in this working substance, only in the absorber cold water circulates through a coil of pipe to take up the heat given out during absorption of the ammonia vapour by the salt. This heat is much less than the latent heat of the vapour, for part of it is used to liquefy the salt. In the same way, in the generator, steam or hot water through a coil of pipe supplies the heat required to drive off the NH_3 vapour. The amount of heat required is also less than the latent heat of the vapour, for part of the heat required is given out in the solidification of the salt. The NH_3 vapour from the generator is cooled by circulating water and liquefied in the condenser, then it passes through an expansion valve to the evaporator, where it extracts heat from the brine in the usual way.

The performance of the Seay machine is stated to be more efficient than in the method by absorption of ammonia in water. However, the process is intermittent and is not likely to show greater economy than the vapour compression machine in practical working.*

Properties of the Chief Refrigerants. Tables† I and II show that the pressure of CO_2 is higher and the volume of the vapour smaller than those of ammonia. Consequently the CO_2 machines are the smaller, but care has to be taken to prevent leakage past the piston, piston-rod, and glands.

CO_2 is colourless and practically odourless and, being heavier than air, displaces the air in a room or confined space and causes suffocation.

Ammonia is also colourless, has a pungent odour, and is noxious. It attacks brass and copper, so that only iron or steel pipes can be used.

The Board of Trade regulations only allow ammonia machines on board ship when enclosed in a separate room, usually on deck ; while small CO_2 machines may be placed in the engine-room.

---

* For further particulars on the theory of refrigeration, see *The Mechanical Production of Cold*, by Sir Alfred Ewing (Pub., Cambridge University Press) ; and on " Vapour Compression Refrigerating Machines," *Proc. Inst. Mech. E.*, Nov., 1912, and Oct., 1914.

† Table I is abridged from results by Professor H. L. Callendar ; and most of the data in Table II are from Dr. Mollier, converted into British Engineers' units.

Joly measured the mean specific heat of CO_2 vapour at constant volume and high density over the range $10°$ C. to $100°$ C., and found $C_v = 0.2108$. The values of the specific heat of CO_2 vapour, at different pressures and temperatures, observed by Jenkin and Pye for their $H\phi$ chart are as follows—

Pressure : lb. per sq. in.	150	200	300	400	500	700
Temperature °C.						
0°	0·23	0·25	0·28	0·33	0·40	—
15°	·23	·23	·26	·29	·35	0·55
30°	·23	·23	·25	·26	·30	·38

The mean volume of liquid ammonia is 0·0256 cub. ft. per lb., and its specific heat about 0·9 to 1·02. The specific heat of the vapour at constant pressure is 0·508.

TABLE I
PROPERTIES OF CARBONIC ACID (CO_2)

Temp. $t°$ C.	Pressure in lb. per sq. in. p	Volume of liquid in cub. ft. per lb.	Volume of satu'd vapour in cub. ft per lb.	Heat of liquid in lb. calories C.H.U. per lb. h	Latent Heat of satu'd vapour C.H.U. per lb. L	Total Heat C.H.U. per lb. H	Entropy (ϕ) Liquid	Entropy (ϕ) Vapour	Specific Heat at Constant Pressure Vapour	Specific Heat at Constant Pressure Liquid
-15°	331	0·0159	0·258	-8·7	63·5	54·8	0·032	0·214		
-10°	383	0·0163	0·220	-5·9	60·6	54·7	0·021	0·209	0·33	0·58
-5°	441	·0168	0·188	-3·0	57·5	54·5	0·011	0·204		
0°	505	·0173	0·161	0	54·1	54·1	0·011	0·198	0·40	0·63
5°	576	·0180	0·137	3·1	50·4	53·5	0·011	0·192		
10°	653	·0187	0·117	6·5	46·2	52·7	0·022	0·185	0·51	0·74
15°	737	·0196	0·099	10·0	41·4	51·4	0·034	0·178		
20°	829	·0208	0·083	13·9	35·7	49·6	0·047	0·169	0·92	0·99
30°	1037	·0264	0·048	24·5	15·4	39·9	0·081	0·132	0·83	
{ Critical Point 31·5°	1071	0·0344		31·4	0	31·4	0·103	0·103	Inf.	

TABLE II
PROPERTIES OF AMMONIA (NH_3)

Temp. $t°$ C.	Pressure in lb. per sq. in. p	Volume of saturated vapour in cub. ft. per lb.	Heat of liquid in lb. calories C.H.U. per lb. h	Latent Heat of saturated vapour in C.H.U. per lb. L	Total Heat, lb calories C.H.U. per lb. H	Entropy (ϕ) Liquid	Entropy (ϕ) Vapour
-10°	41·5	6·8	-8	322	314	-0·033	1·193
-5°	51	5·6	-4·5	319·5	315	-0·017	1·175
0°	61	4·7	0	316	316	0	1·158
5°	74·5	4·0	4·5	312·5	317	0·017	1·141
10°	89	3·3	9·2	308·6	317·8	0·033	1·123
15°	106	2·7	14	304	318	0·050	1·107
20°	125	2·3	18·6	300	318·6	0·066	1·089
30°	170	1·7	28·2	290	318·2	0·098	1·055

Ethyl Chloride. A refrigerant in common use consists of a mixture of ethyl chloride* with a small proportion of methyl chloride which raises the vapour pressure.

Ethyl chloride (C_2H_5Cl) is a colourless liquid ; its boiling point at atmospheric pressure is $12.5°$ C. It is very inflammable, a mixture with air can be exploded by a spark when the proportions by volume of the ethyl chloride vapour are not less than 4·9 per cent nor more than 13·5 per cent. For petrol vapour the corresponding limits of inflammability are 1·5 per cent and 6·4 per cent.

Ethyl chloride does not attack iron or other metals. It attacks india-rubber, pure or vulcanized, makes wood swell, but lignum vitae can be used for valves as a non-conductor of heat. Liquid ethyl chloride dissolves a small quantity of water ; if exposed to the air in a shallow glass, it evaporates, rapidly falls in temperature, and ice crystals form from the moisture in the atmosphere. Similar ice crystals quickly choke the expansion valve in a refrigerating machine if there is any water dissolved in the ethyl chloride.

The latent heat of methyl chloride is 102·45 C.H.U. per lb. at its boiling point- -23·8° C., and atmospheric pressure. The mean specific heat of its vapour from 15° C. to 100° C. is 0·222, and that of ethyl chloride vapour is 0·245 for the same range and at 1 atmospheric pressure. The specific heat of the superheated vapour of ethyl chloride increases to 0·273 at 110° C. and 30 lb. per sq. in. The starting-point for the charts is 0° C., where $\phi = 0$, and the total heat $H = Apv = 0.02$.

TABLE III
PROPERTIES OF ETHYL CHLORIDE (C_2H_5Cl)

Temp $t°$ C.	Pressure lb. per sq in p	Specific Volume saturated vapour, cub ft per lb.	Heat of Liquid, C.H U. per lb. h	Specific Heat Liquid	Latent Heat of saturated vapour, C H U. per lb. L	Total Heat, C H U. per lb. H	Entropy Liquid	Vapour
35°	32 03	2 80	13 68	0 408	86 73	100 41	·0472	·3285
30°	27 33	3 01	11·66	0 404	87 85	99 51	·0404	·3301
25°	23 07	3 52	9 69	0 399	88·92	--	·0340	·3319
20°	19·38	4 08	7 73	0 394	89 93	97·66	·0272	·3340
15°	16 14	4 96	5 78	0 390	90·98	--	·0206	·3361
10°	13·35	6 01	3 84	0 385	91 97	95 76	·0137	·3383
5°	10 97	7 37	1 95	0·380	92 96	94·90	·0070	·3410
0°	8·92	8·89	0 02	0 376	93 86	93 88	·0000	·3436
− 5°	7·18	10 89	− 1·82	0·371	94·72	- -	−·0008	·3463
−10°	5 73	13·20	3·67	0·367	95 54	91·87	−·0138	·3494
15°	4 52	16 18	5·51	0 362	96·41	--	−·0207	·3527

* Report No. 14 on the "Thermal Properties of Ethyl Chloride," by Professor C. F. Jenkin and Mr. D. N. Shorthose, for the Engineering Committee of the Food Investigation Board, Department of Scientific and Industrial Research ; also Report No. 19 on the "Thermal Properties of Methyl Chloride," by Mr. D. N. Shorthose, M.A.

Calcium Chloride Brine, which is cooled in the evaporator and conveys the heat from the cold body to the evaporating coils, is of mean density about 1·2 at 60° F. (15·5° C.), and of specific heat—

Temperature °C.	– 15°	– 10°	5°	0°	+ 5°	+ 10°
Specific Heat .	0·695	0 700	0 705	0·710	0·715	0·720

TABLE IV   FIXED TEMPERATURES

Substance	Critical Pressure Atmospheres	Critical Temperature ° C.	Boiling Point ° C.	Freezing Point ° C.
Water .	200	365°	100	0
Ammonia .	115	130°	– 33°	– 77°
Carbon Dioxide .	75	31°	80°	56°
Oxygen .	51	– 118°	– 182°	
Atmospheric Air .	39	– 140°	194°	
Nitrogen .	35	116°	– 194°	– 214°
Hydrogen .	20	– 241°	– 243°	– 258°

EXAMPLES VI

1. An ammonia compression refrigerating machine has to do an amount of refrigeration equal to the production of 25 tons of ice per 24 hours from and at 32° F. If the temperature limits in the compressor are 75° F. and - 5° F. ; calculate the horse-power of the compressor (a) on the assumption that the cycle is a perfect one, and (b) if the actual performance is 60 per cent of the ideal. The latent heat of ice, 142 B Th.U. per lb. (*U.L., B.Sc. (Eng.)*.)

2. Find the horse-power of a perfect reversed heat engine that will make 900 lb. of ice per hour at 27° F. from water at 60° F. What amount of heat must be extracted per pound of ice Given the specific heat of ice, 0·5 ; and the latent heat 142 B.Th.U. per lb.

3. In a Bell-Coleman refrigerating machine the air is drawn in from the cold chamber at 23° F. and atmospheric pressure of 15 per sq in. (abs), and, after adiabatic compression to 60 lb per sq. in (gauge), the air, at this constant pressure, is cooled to 60° F. Then the air is expanded adiabatically to atmospheric pressure and discharged to the cold chamber What is (a) the net amount of work expended per pound of air ; (b) the heat extracted from the cold chamber ; and (c) the coefficient of performance ? C_p = 0·238 for air.

4. In an open cycle air type of refrigerating machine circulating 1,500 lb. of air per hour, air is drawn from a cold chamber at a temperature of 10° C. (50° F.), and compressed adiabatically to 67 lb. (abs.). It is afterwards cooled at this pressure to 25° C., the temperature of the condenser, and then expanded adiabatically to atmospheric pressure and returned to the cold chamber. Find the number of units of heat (a) extracted per hour from the cold chamber, and (b) the heat rejected. If the indicated horse-power of the compressor is 25·0, find (c) the coefficient of performance of the machine. The specific heat of air may be taken as 0·241, and the law of expansion and compression as $pv^{1\cdot4}$ = constant.   (*U.L., B.Sc. (Eng.)*.)

5. The temperature range in an ammonia plant is from 20° C. to - 10° C. Find the coefficient of performance (a) for the corresponding reversed Carnot cycle, and also (b) for the ammonia plant if, after compression, the ammonia gas is 0·95 dry. (c) What improvement in the coefficient of performance would be obtained if the plant were so managed that the temperature of the gas after compression were 35° C., the pressure range remaining as before ?
The values of the entropy for the liquid and dry vapour at 20° C. are 0·066 and 1 089 ; and at – 10° C. they are – 0·033 and 1·193 respectively ; and the specific heat of the vapour is 0·508.   (*U.L., B.Sc. (Eng.) 1925.*)

CHAPTER VII

COMBUSTION OF FUELS

THE fundamental law of chemical combination is that all compounds are constant in composition ; in other words, the molecule of any substance has always the same atoms similarly combined. Every molecule of water always consists of 1 atom of oxygen combined with 2 atoms of hydrogen, and is therefore represented by the symbol H_2O. These letters tell us the *weight* of the substance. H_2O means 18 parts by weight of the compound water, or steam, containing 2 parts by weight of hydrogen, and 16 parts by weight of oxygen. The *molecular weight* of a compound is the sum of the atomic weights of its constituents.

The following table gives, approximately, in round numbers, the atomic or combining weights, and the molecular weights of the substances—

Symbol of Substance . .	H_2	O_2	N_2	C
Combining Weight . . .	1	16	14	12
Molecular Weight . . .	2	32	28	—

Thus CO_2 means $12 + 2 \times 16$, that is, 44 parts by weight of carbon dioxide. CO represents 28 parts by weight of carbon monoxide ; and CH_4, 16 parts by weight of methane or marsh gas.

All chemical changes take place between fixed definite proportions by weight of substances.

When an electric current is passed through water containing a little sulphuric acid to conduct the current, bubbles of gas are formed at the two platinum plates immersed in the liquid and connected with a battery or other generator of electric current. After making allowance for solubility of the gases and secondary chemical changes, and the gases collected separately, exactly two volumes of hydrogen are given off at the platinum plate by which the electric current leaves the water, for every volume of oxygen collected at the other plate by which the current enters. The *weights* of the hydrogen and oxygen so set free by electrolysis are always in the fixed proportion of 1 to 8, and their sum is equal to the weight of liquid decomposed. The gases hydrogen and oxygen are entirely different in properties from each other and from water. The fact that distilled water is always composed of these two gases combined in this proportion may be verified by mixing two volumes of hydrogen gas with one volume of oxygen above mercury in a

glass tube surrounded by a steam jacket. Passing an electric spark through the mixture causes an explosion ; the gases combine, giving out a great amount of heat, whilst steam or water vapour is formed. This steam, on being reduced to the original pressure and temperature, occupies two volumes, and is exactly the same weight as the hydrogen and oxygen combined. Moreover, the heat evolved in the combination is equivalent to the electric energy spent in the electrolysis of water.

The combination of hydrogen and oxygen to form water is represented by the equation

$$2H_2 + O_2 = 2H_2O, \text{ and heat evolved} \qquad . \qquad . \qquad . \qquad (1)$$
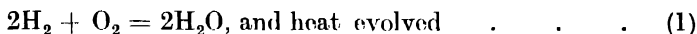
and means that two molecules of hydrogen unite with one molecule of oxygen to form two molecules of water or steam, and heat is generated. Thus 4 parts by weight of hydrogen combine with 32 parts by weight of oxygen and form 36 parts by weight of steam ; or, in the same proportion, 1 *lb. of hydrogen unites with* 8 *lb. of oxygen to form* 9 *lb. of steam.*

Again, the molecular volumes of all perfect gases are equal, and Avogadro's law states that equal volumes of different gases, under the same conditions of temperature and pressure, contain the same number of molecules. Therefore the weight of unit volume, that is the density, is proportional to the molecular weight. Taking hydrogen as unit weight, the law holds that the density of every perfectly gaseous compound is half its molecular weight. The density of hydrogen at $0°$ C. and 14·7 lb. per sq. in. is nearly equal to 0·00559 lb. per cub. ft.

Hence the weight of a gas in pounds per cubic foot, at standard pressure and temperature, is equal to half its molecular weight multiplied by 0·00559. For instance, 1 cub ft. of methane (CH_4), at $0°$ C. and 14·7 lb. per sq in., weighs

$$\frac{12 + 4}{2} \times 0 \cdot 00559 = 0 \cdot 0447 \text{ lb.}$$

Equation (1) agrees with experiment, which also shows that 2 volumes of hydrogen burned with 1 volume of oxygen yields 2 volumes of water vapour at the same temperature and pressure, so that, for complete combustion, 1 *cub. ft. of hydrogen requires* $\frac{1}{2}$ *cubic ft. of oxygen to form* 1 *cub. ft. of water vapour.*

Again, when solid carbon burns to carbon dioxide, the equation $C + O_2 = CO_2$ shows that 12 parts by weight of carbon combine with 32 parts by weight of oxygen to form, by complete combustion, 1 molecule or 44 parts by weight of carbon dioxide. Reducing the combining weights in the same proportion, it follows that 1 lb. of carbon requires $2\frac{2}{3}$ lb. of oxygen to form $3\frac{2}{3}$ lb. of carbon dioxide.

The oxygen required for the combustion of fuel is supplied by the

air. For the purpose of calculation, atmospheric air, when pure and dry, may be taken as a mixture of

	Per Cent by Weight	Per Cent by Volume
Oxygen	23·1	20·9
Nitrogen	76·9	79·1

There is always water vapour in the air, which helps to start the chemical action, and part of the heat of combustion evaporates the moisture.

Experiment shows that an *inert gas* like nitrogen retards combustion, not only by taking up heat, thereby lowering the temperature of combustion and reducing the velocity of the gaseous molecules, but also by preventing contact between the molecules of the combustible gases with oxygen. Hence an excess of air, usually at least 1·5 times the theoretical quantity required, is drawn through the boiler furnace, because the oxygen in the air supply does not get so intimately into contact with the combustible in the lumps of solid fuel, in order to burn the coal perfectly on the fire-grate, as it does with finely divided coal or liquid fuel when sprayed.

One lb. of oxygen is contained in $\dfrac{100}{23 \cdot 1} = 4 \cdot 33$ lb. of air, therefore, to find the theoretical weight of air supply for the complete combustion of 1 lb. of a fuel, the necessary weight of oxygen is multiplied by 4·33. The volume of air may then be obtained, since 1 lb. of dry air occupies 13·09 cub. ft. at 15° C. and 14·7 lb. per sq. in.

Table I shows the theoretical minimum weight of oxygen or air required to burn 1 lb. and 1 cub. ft. of each combustible separately

TABLE I

CALORIFIC VALUES OF SUBSTANCES AND OXYGEN REQUIRED FOR COMPLETE COMBUSTION

Substance	1 lb. Substance requires:		Products formed with Oxygen		Calorific Values C.H U. per lb.		Cub. ft of Oxygen required per 1 cub. ft. burned	Calorific Values of Gas			
								C.H.U. per cub.ft. burn'd		Kil-cal. per cub. metre	
	Oxygen lb.	Air lb.	Water lb.	CO$_2$ lb.	Higher	Lower		Higher	Lower	Higher	Lower
Hydrogen H$_2$	8	34·63	9		34,500	29,000	½	191	162	3,055	2,555
Carbon C	2⅔	11·54		3⅔	8,080	8,015					
Carbon Monoxide CO	⁴⁄₇	2·47		1⁴⁄₇	2,420	2,392	½	190	188	3,040	3,012
Marsh Gas, Methane CH$_4$	4	17·3	2¼	2¾	13,100	11,650	2	591	529	9,470	8,464
Olefiant Gas, Ethylene C$_2$H$_4$	3⁴⁄₇	14·84	1²⁄₇	3⅔	11,900	11,060	3	930	867	14,890	13,880

Pound-calories (C.H.U.) × 1·8 = B.Th.U.
Kilo-calories per cubic metre × 0·1124 = B.Th.U. per cubic foot.

aple 1.   The analysis by weight of petroleum is 86 per cent of carbon and 14 per cent of hydrogen ; calculate the quantity of air required for the complete combustion of 1 lb. of this oil fuel.

Given 1 lb. of the oil consists of 0·86 lb. of carbon, and 0·14 lb of hydrogen, from Table I, the minimum weight of air required is

$$0·86 \times 11·54 + 0·14 \times 34·63 = 9·92 + 4·85 = 14·77 \text{ lb.}$$

If the air is at 15° C. and atmospheric pressure, its volume will be

$$14·77 \times 13·09, \text{ or } 193 \text{ cub. ft.}$$

Quantity of Air Required for the Complete Combustion of Gaseous Fuel.

In the case of gaseous fuel the volume of oxygen required for its complete combustion is calculated from the chemical equations, and the cubic feet of oxygen multiplied by $\dfrac{100}{20·9} = 4·785$ gives the cubic feet of air necessary.

Example 2.   A coal gas has the following percentage analysis by volume : Hydrogen, 46 ; methane, 39·5 ; ethylene, 2·53 ; tetrylene, 1·27 ; carbon monoxide, 7 5 ; nitrogen, 1·2 ; and water vapour, 2.

Calculate (*a*) the minimum amount of air required for the complete combustion of the gas ; (*b*) the contraction that takes place on combustion when the products are reduced to the same pressure and temperature, without condensation of water vapour.

Taking each combustible constituent in 1 cub. ft. of the gas burnt separately—

(1) *Hydrogen.* $2H_2 + O_2 = 2H_2O$
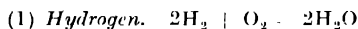
i.e.   2 cub ft. H +   1 cub. ft. O form   2 cub ft Water vapour

∴ 0·46 ,,   ,, + 0 23 ,,   ,,   ,,   0·46   ,,   ,,   ,,

(2) *Methane.* $CH_4 + 2O_2 = CO_2 + 2H_2O$
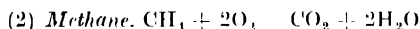
i.e.   1 cub. ft. CH_4 + 2 cub. ft. O form   1 cub ft. CO_2 +   2 cub ft H_2O

∴ 0·395 ,,   ,, + 0 79 ,,   ,,   ,, ·395 ,,   ,,   + 0·79 ,,   ,,

(3) *Ethylene.* $C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$
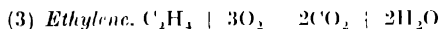
i.e.   1 cub ft. C_2H_4 + 3 cub. ft. O form 2 cub. ft CO_2 +   2   ,,   ,,

∴ 0·0253 ,,   ,, + ·0759 ,,   ,,   ,, ·0506 ,,   ,,   + ·0506 ,,   ,,

(4) *Tetrylene.* $C_4H_8 + 6O_2 = 4CO_2 + 4H_2O$
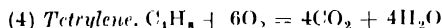
i.e.   1 cub ft. C_4H_8 + 6 cub. ft. O form   4 cub. ft. CO_2 +   4 cub. ft. H_2O

∴ 0·0127 ,,   ,, + ·0762 ,,   ,,   ,, 0 0508,,   ,,   + 0·0508 ,,   ,,

(5) *Carbon Monoxide.* $2CO + O_2 = 2CO_2$
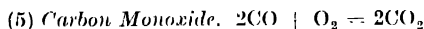
i.e.   2 cub. ft. CO   + 1 cub. ft. O form   2 cub. ft. CO_2

∴ 0·075 ,,   ,, + 0·0375 ,,   ,, ,,   0·075   ,,   ,,

Arranging these figures in a table, we get

Constituents in Coal Gas	Volume of Gas in 1 cub. ft. of Coal Gas	Volume of Oxygen required by each Constituent	Volume of Products of Combustion		N_2
			CO_2		
		Cub. ft.	Cub. ft.	Cub. ft.	Cub. ft.
H_2	0·46	0·23	0·46		
CH_4	·395	·79	·79	0·395	
C_2H_4	·0253	·0759	·0506	·0506	
C_4H_8	·0127	·0762	·0508	·0508	
CO	·075	·0375	·075		
N_2	·012				0·012
H_2O	·020		0·0200		
Total	1·000	1·2096	1·3714	0·5714	0·012	1·9548

(a) Since 1·2096 cub. ft. of oxygen are required for combustion, the air necessary to supply this volume of oxygen is

$$\frac{100}{20\cdot9} \times 1\cdot2096 = 5\cdot787 \text{ cub. ft. air.}$$

(b) This air contains $5\cdot787 - 1\cdot2096 = 4\cdot577$ cub. ft. of nitrogen, which apparently retards combustion and is added to the burnt products, making a total of $1\cdot955 + 4\cdot577 = 6\cdot532$ cub. ft.

A mixture of 1 cub. ft. of coal gas and 5·787 cub. ft. of air becomes after complete combustion 6·532 cub. ft., consisting of water vapour, carbon dioxide, and nitrogen reduced to the same temperature and pressure. Hence the contraction in volume during combustion is

$$6\cdot787 - 6\cdot532 = 0\cdot255 \text{ cub. ft., or}$$

$$\frac{0\cdot255}{6\cdot787} \times 100 = 3\cdot76 \text{ per cent.}$$

In a gas engine, for efficiency in working, there is always an excess of air, in the ratio of about 9 cub. ft. of air to 1 cub. ft. of coal gas, making the charge 10 cub. ft., and this mixture has added to it 2 cub. ft. of residual products from the previous charge in the engine cylinder, hence the total volume would be 12 cub. ft. before combustion, becoming, with the same contraction, 11·745 cub. ft. after combustion, which will now be $\frac{0\cdot255}{12} \times 100$, or 2·125 per cent.

Example 3. The analysis by weight of coal is C, 80 per cent; H_2, 5 per cent; S, 0·5 per cent. Calculate the theoretical quantity of air necessary for the complete combustion of 1 lb. of the coal. If 20 lb. of air are supplied per pound of coal, and the combustion is complete, estimate the analysis of the flue gas by weight.

By Table I, the air required per pound of coal is

$$0.8 \times 11.54 + 0.05 \times 34.63 + 0.005 \times 4.33$$
$$= 9.23 + 1.73 + 0.02 = 10.98 \text{ lb. air.}$$

The oxygen needed for the given weights of C, H, and S in 1 lb. of coal might have been calculated, and the sum divided by 0.231 lb. the oxygen per pound of air.

The total weight of flue gas from 20 lb. of air and the weight of combustibles in 1 lb. of the coal is 20.855 lb.

The weights of the burnt products are—

		Lb.	Per cent
CO_2	$= 0.8 \times \dfrac{44}{12}$	2.933	14.06
H_2O	0.05×9	0.45	2.16
SO_2	$= 0.005 \times 2 =$	0.01	0.05
N_2	$= 20 \times 0.769$	15.38	73.75
O_2 (by difference)		2.082	9.98
Total weight		20.855	100.00

To find the percentages, divide each constituent by the total weight, gives the weight of each in 1 lb. of the flue gas; then multiply by 100, thus

$$CO_2 = \frac{2.933}{20.855} \quad 100 \text{ gives } 14.06 \text{ per cent.}$$

Conditions for Combustion. *Rapid combustion*, or burning, is chemical combination of combustible substances with oxygen, accompanied by the evolution of heat, and generally of light. When the resulting compound is decomposed or split up into the original combustibles, an amount of heat is absorbed, or disappears as sensible heat, equal to that generated in forming the compound. The actual process of combustion is very complicated, and the chemical equations given above only represent the results.

A combustible gas combines completely with oxygen only when (a) the mixture contains the two gases within proper proportions; and (b) the temperature and pressure of the mixture are within fixed limits.

As stated above (p. 196), the electrolytic mixture of hydrogen and oxygen at 18° C. and 14.7 lb. per sq. in. may be ignited by the electric spark, giving an explosion. Sir Humphry Davy showed that if the same mixture be reduced to 0.8 lb. per sq. in. at 18° C., combustion will not take place when the spark is passed; but, keeping the mixture at the same lower pressure, it becomes inflammable when the temperature is raised sufficiently. When the mixture is heated up, slow combustion may take place and the

mixture heats itself, and its temperature will rise very rapidly until flame appears.

When this electrolytic, or true *explosive mixture* of hydrogen and oxygen, is fired by adiabatic compression it ignites at 526° C. with a sharp explosion.

During the initial period of inflammation at atmospheric pressure the flame is propagated through the mixture with a uniform velocity of 20 metres per sec. (65·6 ft. per sec.). The early experiments of Mallard and Le Chatelier on the velocity of flame propagation in mixtures of hydrogen and air, at constant pressure, showed that the spread or travel of flame is most rapid when there is a slight excess of the inflammable gas in the mixture. Thus the mixture 1 volume of hydrogen to 2·5 volumes of air contains the proportion of oxygen for complete combustion, and the velocity of the flame was 11·5 ft. per sec. ; while with excess of hydrogen 1 volume to 1·5 of air, the maximum velocity 14·4 ft. per sec. was attained ; but with greater excess, as in a mixture consisting of equal volumes of hydrogen and air, the velocity of flame propagation was reduced to 12·4 ft. per sec. Dr. Neumann, of Dresden, has shown that mixtures of petrol vapour and air in the proportion of 1 to 12 gave the maximum velocity of flame propagation, while the proportion for complete combustion is about 1 to 15.

The effects of different methods of firing upon the rate of combustion in the case of solids were strikingly shown by Sir Frederick Abel with gun-cotton. A train of gun-cotton, laid loosely on a table open to the air, when ignited by a flame, burns rapidly and harmlessly without explosion. When laid in the same way, the gun-cotton, fired by a percussion detonator to produce a high initial pressure or shock, gives a most violent explosion, due to the instantaneous change of the solid into the gaseous state.

Explosion Wave. Berthelot discovered when detonation is set up in the electrolytic mixture of hydrogen and oxygen in a closed tube, fired by an electric spark and a grain of fulminate, the shock and ignition with combustion are transmitted through the whole gaseous mixture by an *explosion wave* of adiabatic compression at a velocity of 2,820 metres per sec. Each layer is compressed before being fired, and then compresses the next layer. It has been proved, by collecting the residual gas and exploding it, that the combustion at the highest temperature in the wave front is not *wholly* complete. The velocity depends on the nature of the mixture. The rate of explosion is retarded by inert gases like nitrogen, according to their volume and density, which, when present in excess, may stop detonation.

Professor H. B. Dixon confirmed these results[*] for various gaseous mixtures, and showed that in the explosion of a volatile carbon compound with oxygen, the carbon burns first to carbonic oxide,

[*] *Phil. Trans. Roy. Soc.*, Vol. 184 (1893) ; and *Phil. Mag.*, January, 1899.

and afterwards, if oxygen is present in excess, the carbonic oxide first formed burns to carbonic acid.

The explosion wave is not easily produced in the gas engine cylinder. The sudden blow or shock by explosion of petroleum vapour, known as " pinking " in the petrol engine, at high temperature has nearly the the same effect, and is called " detonation." Yet the ordinary propagation of inflammation is quite different from the detonation of gun-cotton, dynamite, etc., for which the speed of detonation, observed by Abel, also increases with the density of the charge.

The ignition point of an explosive gaseous mixture is that temperature to which it must be raised locally so that the flame spreads throughout the whole mixture.

Professor H. B. Dixon and H. F. Coward heated the combustible gases and oxygen, or air, separately to the ignition temperature before the streams of gas and oxygen, or air, were allowed to mix, and found the following ignition ranges --

Gas.	Ignition Range, °C.	
	In Oxygen	*In Air*
Hydrogen	580° 590°	580° 590
Carbon Monoxide (Moist)	637° 658°	644° 658°
Methane	556°-700°	650°-750°
Ethylene	500° 579°	512° 547°
Acetylene	416° 440°	406° 440°

Professor Dixon found that carbon monoxide will not combine with oxygen unless a trace of moisture is present to start the action, although it takes no part in the combination. Professor Baker showed that a mixture of hydrogen and oxygen in the presence of water would not explode without a trace of impurity in the water ; and charcoal would not burn unless a trace of water vapour were present.

Each combustible gas or vapour has a well-defined *range of inflammability*, when mixed with air, which varies according to the nature of the hydrocarbons as well as to the temperature and pressure of the mixture. Combustion ceases when a certain excess of either gas is added to the explosive mixture ; thus 9 volumes in excess of oxygen or 8 volumes in excess of hydrogen, added to the electrolytic mixture, render it non-inflammable, and the only portion of hydrogen burned is that lying in the path of the electric spark ; but on raising the temperature of the new mixture to a certain point, it may be ignited by the electric spark. Hence a mixture of hydrogen and oxygen which will not combine at the ordinary atmospheric temperature and pressure may be made to do so by increasing either the pressure or temperature up to certain definite limits, to be determined by experiment.

The addition of excess of an inert gas like nitrogen to the electrolytic mixture of 2 volumes H_2 to 1 of O_2 also retards combustion.

The same effect is produced by the presence of an excess of the ooled burnt products of a gaseous explosive mixture. In this way the *rate of combustion in the engine cylinder is under control* by : (1) changing the proportion of air to gas in order to determine by experiment the most economical mixture for an engine working at a given compression and speed ; (2) increasing the compression of the charge before ignition, depending upon the nature of the fuel, and enabling weak or dilute mixtures to be burned rapidly and completely ; (3) the time and method of firing the compressed charge in the cylinder ; (4) the shape or design of the combustion chamber ; (5) the temperature of the water jacket and the cooling action of cylinder walls.

Professor W. A. Bone found in his researches* upon the behaviour of hydrocarbons, that their combustion is a complex process, involving slow combustion at temperatures below the ignition point, and that there are clearly defined stages of successive oxidation. Thus, during this slow combustion, the most stable hydrocarbon, methane (CH_4), is oxidized at temperatures between 400° and 500° C., well below its ignition point. In the initial stage *methyl-alcohol* is formed, which breaks up into steam and *formaldehyde* in the second stage. The formaldehyde is subsequently oxidized to *formic acid* (stage 3), which may partly break up into CO and H_2O, to the unstable *carbonic acid* (stage 4), which at once results in CO_2 and H_2O. At temperatures above the ignition point, the *formaldehyde* (stage 2) is so rapidly changed by heat into CO and H_2 that in the final stages of the process these gases are probably oxidized independently.

This view is supported by the presence of formaldehyde found in the condensed steam from the exhaust gases of petrol and oil engines. Also, the "surface effect" in the slow decomposition of methane, according to the reversible reaction $CH_4 \rightleftarrows C + 2H_2$ between 800° and 1200° C., explains the formation of that very hard, lustrous, black carbon "methane carbon," deposited on the vaporizers of some oil engines.

The term **Fuel** is applied to any combustible which is sufficiently abundant, and by its combustion with oxygen generates heat energy rapidly, at a cost low enough to be used for industrial purposes. The fuels mostly in common use are wood, peat, coal, petroleum and gas. Of these, coal is still the chief and cheapest, notwithstanding the enormous waste of the great stores of energy accumulated by Nature in the coalfields.

Solid fuel is formed by the slow decomposition of wood and decayed vegetable debris buried in the earth for long ages, exposed to great pressure and the internal heat of the earth. In peat bogs this long transformation process from wood to anthracite is seen to begin with decayed marsh plants and mosses as spongy peat

* *Journ. Chem. Soc.* (1908), p. 1197 ; and Papers during 1902-6.

near the surface, gradually converted into black dense peat at the bottom of deep bogs.

Lignite, of woody fibrous structure, intermediate between peat and true coal, is found in the tertiary formation of a more recent period than the carboniferous, in which coal occurs.   Lignites vary from earthy brown and friable, to " jet " black, contain 50 to 70 per cent carbon, 20 to 28 per cent oxygen, and up to 80 per cent moisture.

Bituminous coals are chiefly derived from *sigillaroid trees,* gigantic ferns, the debris of primeval forests ; whilst the structure of *cannel coal*, with spore-cases of *Lepidodendra*, point to its origin from vegetable mud and club mosses.   Bituminous, or " flaming " coal yields 30 to 40 per cent volatile matter, and burns with a long smoky flame.   Bituminous caking coals soften when heated, and on distillation in coke ovens at high temperatures, like Durham " coking coal," yield strong coke, used in blast furnaces.

Cannel or *parrot coal*, and other *gas coals*, contain 30 to 40 per cent volatile matter, are chiefly used in the manufacture of coal gas, and yield about 12,000 cub. ft. of gas per ton by high temperature carbonization to 1 100° C., and coke, whilen aphthalene, benzole, toluol, and other products are obtained from the gas tar or oil.

In laboratory tests the products obtained by the *low temperature distillation*, up to 500° or 600° C., of Yorkshire cannel coal and British bituminous coals are crude oil, yielding on fractionation 10 to 15 per cent of their weight of motor spirit, and fuel oil, with 3,000 to 4,000 cub. ft of rich gas ; but Scotch and other cannels yield only from 20 to 40 gallons of crude oil per ton by low temperature carbonization.   After distilling off this volatile matter from the coal, there remains about 70 per cent of the coal as a valuable semi-coke, smokeless in its combustion.

Free burning, non-caking bituminous coals are suitable for making Mond gas, but are likely to give trouble in gas producers owing to the tar formed, unless special precautions are taken to convert the tar into fixed gas or to burn it.

The semi-bituminous *steam coals* yield 15 to 20 per cent volatile matter, and burn with short, practically smokeless, flame.

Anthracites contain 85 to 93 per cent carbon, about 8 to 15 per cent volatile matter, are non-caking, and burn with a short non-luminous flame.   Anthracite and non-caking coals are suitable for small power gas producers, because the gas is practically free from tar and there is little clinkering trouble.

Anthracite appears to be formed from bituminous coal by heat in the earth's crust.

Spontaneous Heating.   Freshly-cut bituminous coals lose occluded gases (as methane, etc.), are liable to absorb moisture, and slowly combine with oxygen from the air, favoured by fineness of division of the coal, and deteriorate 1 to 3 per cent in calorific value and coking properties during the first three weeks after removal

from the pit. This slow oxidation in heaps of coal dust increases rapidly with rise of temperature, and may eventually give rise to spontaneous ignition in the pile, by the cumulative heating effect, as " gob fires " in mines.

In large piles of gas coals, part being broken into fine powder, when the temperature is about 150° C., the rate of oxidation or slow combustion accelerates until the ignition point between 350° and 400° C. is attained.

TABLE II

COMPOSITION AND CALORIFIC VALUE OF SOLID FUELS

FUEL	Ultimate Analysis Percentage							Observed Higher Calorific Value	
	Water	Carbon	Hydrogen	Nitrogen	Sulphur	Ash	Oxygen by difference	C.H.U. per lb	B.Th.U. per lb.
Wood (Ordinary) .	28·9	36·4	4·6	0·1		0·5	29·5	3,310	5,960
Wood (Dried)	6·9	47·1	5·6			0·3	39·8	4,480	8,060
Peat (Poor)	20·8	40·8	3·3			7·7	26·3	3,770	6,790
Peat (Air-dried) .	6·1	53·2	5·5			0·9	34·2	5,190	9,880
Lignite (Dried)		68·4	5·75		0·1		17·8	6,540	11,770
Cannel Coal, Wigan	0·6	78·4	5·1			10·9	5·1	7,760	13,970
Cannel Coal, Scotch	4·0	75·4	6·18	1·28	2·18	2·24	8·72	7,500	13,500
Durh'm CakingCoal	1·14	84·31	5·3	1·73	0·78	2·42	4·29	8,300	14,940
Yorks. Caking Coal	2·2	84·1	4·93	1·67	0·55	1·2	5·35	7,420	13,360
Newc'le Steam Coal	1·2	81·3	5·3	1·3	1·2	3·6	8·6	8,160	14,690
Durh'm Steam Coal	0·8	81·5	4·6	0·9	1·2	5·5	5·1	7,970	14,350
Welsh Steam Coal .		83·8	4·8	1·0	1·4	4·9	4·1	8,050	14,490
Nixon's Navigation Steam Coal	1·0	87·8	4·1			2·2	5·0	8,580	15,440
Slack Coal (Mid.) .	7·3	67·9	4·9	1·3	1·3	7·6	13·7	7,220	13,000
Small Coal (Mid.) .	8·6	62·7	5·0	1·3	1·5	10·4	10·5	6,790	12,220
Welsh Anthracite		91·5	3·5	0·8	0·6	1·0	2·6	8,460	15,220
Ditto (Dried)		87·5	3·3		0·7		3·5	8,140	14,650
Amer. Anthracite .	3·4	86·4	2·0			5·9	2·2	7,480	13,470
Yorkshire Coke	4·8	88·1	1·1	1·6	0·35	1·7	1·75	7,600	13,680

Table II shows the increase of carbon content in coals from lignite to anthracite.

Coal is a very complex material, and its **composition** for industrial purposes is usually given in two ways—

(1) By *proximate analysis*, first heating the powdered sample a little over 100° C., when the moisture is driven off and " dry coal " is left. The dry coal is heated in a closed crucible without air being admitted at 500° to 900° C., and the *volatile combustibles* which escape are burned, leaving a combustible residue of fixed carbon or *coke*. When the coke is completely burned in air, only *ash* remains.

(2) By *ultimate analysis*, giving the percentages by weight of moisture, carbon, hydrogen, nitrogen, and sulphur in a powdered sample.

The *carbon* and *hydrogen* are found by burning a weighed quantity of dry coal in a hard glass combustion tube in a current of dry oxygen, the water and carbon dioxide formed are absorbed by

dehydrated calcium chloride and an aqueous solution of caustic potash respectively.

The *nitrogen* can be found as ammonia by heating the coal to redness or burning it in a current of steam. The *sulphur* is determined separately, the *ash* added to the sum of these elementary constituents, and the amount short of 100 is usually returned as " oxygen by difference," and may also cover errors in the analysis.

The combustion of coal is a complicated interaction, and the products of complete combustion are CO_2 from the carbon, H_2O from hydrogen, and SO_2 from the sulphur. Carbon monoxide shows incomplete combustion of the carbon.

In a boiler furnace the volatile hydrocarbons are first driven off the coal, and the oxygen for their combustion is supplied by air admitted over the top of the fuel bed. The solid carbon or coke left is burnt by the air drawn through the fuel bed. It appears from experiments that, in burning bituminous coal, the fuel bed in a steam boiler need not be more than 6 in. thick, with careful stoking. About half the oxygen required for the complete combustion of the coal is used up within the lower 4 in. of the fuel bed, the other half of the air supply, preferably heated, passes at high velocity over the fuel to mingle with and burn the volatile matter. The highest temperature, 1.200° to 1.500° C, depending on the draught, is usually found about 3 to 5 in. above the grate. An increase in the draught increases the rate of combustion and the supply of air, which is usually 1·5 to twice the theoretical quantity. Large excess of cold air tends to cool down the interacting gases, retards combustion, and gives a low percentage of CO_2 in the flue gas.

Calculation of the Air Supply from an Analysis of the Flue Gas and an Analysis of the Fuel.

The flue gas analysis by volume is conveniently made by means of the Orsat apparatus. A good *average* sample, 100 c.c. of the gas, is collected over mercury in an eudiometer, the steam formed by the combustion of the hydrogen in the fuel condenses to water, and the dry flue gas consists of CO_2, CO, O_2, and nitrogen. These are successively absorbed by reagents in separate flasks. The CO_2 is absorbed by an aqueous solution of caustic potash, the oxygen by an alkaline solution of pyrogallic acid or, more slowly, by sticks of phosphorus, and the CO by an acid solution of cuprous chloride, and the remainder is assumed to be nitrogen.

Then, to calculate the *weight* of each constituent per pound of carbon in the flue gas, the proportional weights are found from the molecular weights. A molecule of CO_2 contains 44 parts by weight; i.e. 12 of carbon and 32 of oxygen ; CO contains 28 parts by weight ; i.e. 12 of carbon and 16 of oxygen.

Example 4. The flue gas analysis by volume in a boiler trial was CO_2, 10 5 per cent ; CO, 1 per cent ; O_2, 8 per cent ; N_2, 80·5 per cent ; and the coal analysis, as burned, was C, 82 per cent ; H_2, 4·2 per cent ; O_2, 4·8 per

cent ; other matter, 9 per cent. Calculate the following items in the heat balance per pound of coal, the rise in temperature in the flue gases being 300° C.—

    (a) Heat carried by products of combustion, average specific heat, 0·24.
    (b) Heat carried by excess air, average specific heat, 0·2375.
    (c) Heat lost by incomplete combustion.
Calorific value of 1 lb. carbon : when burned to CO_2, 8080 C.H.U. ; burned to CO, 2413 C.H.U.   (U.L., B.Sc. (Eng.).)

The quantity of air drawn through the furnace per pound of fuel may be determined with fair accuracy from the total weight of the burnt products, and the carbon content in these when the ultimate analysis of the fuel is given. In this example the volumetric analysis gives the proportional weights—

					lb.		lb.
CO_2 10·5	44	462, contains 10·5	12	=126 C., and 10·5	32	336 O_2
CO 1·0	28	28	,,	1	12 = 12 C., ,,	1	16	16 O_2
O_2 8	32	256	,,				256 O_2
N_2 80·5	28	2254					
Total weight		3000 flue gas contains		138 C., 2254 lb. N_2 and		608 O_2

∴ 1 lb. of carbon is contained in 3000/138 = 21·74 lb. of dry flue gas.

From the fuel analysis, with every pound of carbon burnt, there is $\dfrac{\cdot042}{\cdot82} = 0\cdot0512$ lb. of H_2, which forms $9 \times 0\cdot0512 = 0\cdot461$ lb. of steam. Hence, the total weight of flue gas per pound of *carbon* burnt is $21\cdot74 + 0\cdot461 = 22\cdot2$ lb.

The weight of combustibles burnt to form this flue gas is 1 lb. carbon + 0·0512 lb. of H_2, also the oxygen of the coal in the products of combustion per pound of carbon is $\dfrac{0\cdot048}{0\cdot82} = \cdot0585$, making the total 1·11 lb. Hence the air supply, drawn through the furnace, per pound of carbon in the flue gas is $22\cdot2 - 1\cdot11 = 21\cdot09$ lb. And, by the analysis, 1 lb. of coal burnt has 0·82 lb. of carbon ; therefore, the weight of air supplied per pound of coal burnt is $21\cdot09 \times 0\cdot82 = 17\cdot294$ lb.

Again, the minimum weight of oxygen needed for complete combustion of the coal is—

    C = 0·82 lb. takes $0\cdot82 \times \frac{8}{3} = 2\cdot187$ lb. of oxygen
    $H_2 = 0\cdot042$,, $0\cdot042 \times 8 = 0\cdot336$,,   ,,

    ∴ Total oxygen required   = 2·523 lb.
    And the coal itself supplies O_2 = 0·048 ,,

∴ Oxygen to be supplied by the air   = 2·475 lb.

Air contains 23·1 per cent by weight of oxygen, hence the minimum weight of air needed for complete combustion is

$$\frac{2·475}{0·231} = 10·714 \text{ lb.}$$

Therefore the *excess* air drawn through the furnace is

17·294 – 10·714 = 6·58 lb. per lb. of coal burnt.

Now the products of combustion consist of the air supplied, with the weights of combustibles and oxygen in the fuel. The coal furnishes 0·82 + ·042 + 0·048 = 0·91 lb. Hence the total products of combustion = 17·294 + 0·91 = 18·2 lb. per lb. of coal burnt.

(*a*) Heat carried away by these gases

= 18·2 × 0·24 × 300° = 1310 C.H.U.

(*b*) Heat carried away by excess air   } *Answer*.

= 6·58 × ·2375 × 300° =   469 C.H.U.

(*c*) Incomplete combustion of carbon,

1 lb. carbon burned to CO_2 evolves 8080 C.H.U.

1 lb.   ,,   ,,   CO   ,,   2413 C.H.U.

∴ 1 lb. carbon in CO burned to CO_2 must give 5667 C.H.U.

which is lost when the combustion is incomplete. The proportion by weight of carbon in 10·5 parts by volume of CO_2 is

$$10·5 \times \frac{12}{44} \times 44 = 126 \text{ parts ;}$$

and the carbon in 1 part by volume of CO is 12 parts by weight. Thus the total proportional weight of carbon in the furnace gas is 126 + 12 = 138 parts, and the proportion of carbon burned to CO is $\frac{12}{138}$. The heat lost or not produced in CO per lb. of carbon in dry flue gas = $5667 \times \frac{12}{138}$ C.H.U., and since 1 lb. of coal contains 0·82 lb. carbon, the heat lost by incomplete combustion per pound of coal burned is

$$0·82 \times \frac{12}{138} \times 5667 = \underline{404} \text{ C.H.U.} \qquad Answer.$$

Formulæ for approximate estimate of the weight of air supplied per pound of fuel burned—

(1) Given the *volumetric analysis* of dry flue gas, and *C* per cent by weight of carbon in the fuel   Since air consists of about 77 per cent by weight of nitrogen, *N* parts by weight of nitrogen are contained in $28 \times \frac{100}{77} \times N$ parts by weight of air.

The proportional weights, as above, are found by multiplying the volumes by the molecular weights in 100 parts of flue gas, and the ratio of air to carbon is

$$\frac{N \times \dfrac{100}{77} \times 28}{\left(CO_2 \times \dfrac{12}{44} \times 44\right) + \left(CO \times \dfrac{12}{28} \times 28\right)}$$

and since 1 lb. of the fuel contains $\dfrac{C}{100}$ lb. of carbon, multiply the ratio of air to carbon by $\dfrac{C}{100}$, from which the weight of air per pound of fuel burned is nearly

$$\frac{N}{33(CO_2 + CO)} \times C \text{ lb.} \qquad . \qquad . \qquad . \qquad . \qquad . \quad (1)$$

In example 4, this gives $\dfrac{80 \cdot 5}{33 \times 11 \cdot 5} \times 82 = 17 \cdot 39$ lb. of air.

(2) When the *percentages* of CO_2, CO, and N in the flue gases are given *by weight*, N parts of nitrogen are contained in $\dfrac{100}{77} \times N$ **parts** by weight of air; and in 100 parts by weight of flue gases there **are**

$$\left(CO_2 \times \frac{12}{44}\right) + \left(CO \times \frac{12}{28}\right) \text{parts of carbon.}$$

Then the ratio of air to carbon is

$$\frac{\dfrac{100}{77} N}{\left(CO_2 \times \dfrac{12}{44}\right) + \left(CO \times \dfrac{12}{28}\right)};$$

and 1 lb. of fuel contains $\dfrac{C}{100}$ parts of carbon.

Hence, to find the weight of air supplied per pound of fuel burned, multiply the ratio of air to carbon by $\dfrac{C}{100}$, and the expression becomes

$$\frac{N}{21\ CO_2 + 33\ CO} \times C \text{ lb.} \qquad\qquad (2)$$

Usually the value of N is not directly determined from the flue gases, and is only given by difference, including errors, of the analysis; also small quantities of ammonia formed from the nitrogen of the fuel may be decomposed.

Example 5.   In a trial of a Lancashire boiler with economizer, the following results were obtained---

Volumetric analyses of the flue gases on entering and leaving the economizer.

		Entering	*Leaving*
CO_2 .		8·3	6 2
CO .		0·4	0·3
O .		11·2	13·7
N .		80 1	79·8
	Total	100 0	100 0

Temperature of flue gases on entering and leaving the economizer, 642° F. and 335° F.   Temperatures of feed water on entering and leaving the economizer, 134° F. and 234° F.   Weight of feed water per hour, 7,370 lb. ; weight of coal stoked per hour, 1,000 lb.   Per pound of dry fuel stoked, the carbon burned was 0·735 lb., and the weight of the flue gases, including moisture, entering the economizer was found to be 21 5 lb.   The average specific heat of the gas may be taken as 0·25.

Calculate per pound of fuel stoked :   (*a*) The air leakage into the economizer ; (*b*) the heat lost by the gases in passing through the economizer ;   (*c*) the heat gained by the feed water.   (*I .L., B.Sc. (Eng.)*)

(*a*) By (1) (p. 209), total air leaving the economizer per pound of fuel

$$\frac{79 \cdot 8}{33(6 \cdot 2 + 0 \cdot 3)} \times 73 \cdot 5 = 27 \cdot 34 \text{ lb.}$$

Total weight of gases leaving economizer per pound of fuel

$$= 27 \cdot 34 + 1 = 28 \cdot 34 \text{ lb.}$$

Hence, air leakage into the economizer

$$= 28 \cdot 34 - 21 \cdot 5 = \underline{6 \cdot 84 \text{ lb.}}$$

(*b*) Heat lost by 21·5 lb. of gases

$$= 21 \cdot 5 \times 0 \cdot 25 (642° - 335°) = \underline{1650 \text{ B.Th.U}}$$

(*c*) Feed water per pound of fuel

$$= 7370/1000 = 7 \cdot 37 \text{ lb.}$$

∴ heat gained by the feed water

$$= 7 \cdot 37 (234° - 134°) = \underline{737 \text{ B.Th.U. per lb. of coal.}}$$

Natural Draught in a chimney is produced by the difference of weight between the column of hot gas inside a chimney and a column of the same height of cold air outside.   The *draught* is measured by the difference in height or head in inches of water in a U-shaped water gauge, one limb of which is connected to a tube inserted at

the chimney base and acted on by the pressure of the hot gas, and the other limb is open to the atmospheric pressure in the boiler house outside the ash-pit or below the fire-grate.

A column of water 1 sq. ft. in cross-section and 1 in. high is $\frac{1}{12}$ cub. ft., which, at 17° C., weighs slightly less than $62 \cdot 4 \times \frac{1}{12}$, or $5 \cdot 2$ lb. A draught of 1 in. head of water corresponds to a pressure of $5 \cdot 2$ lb. per sq. ft. If the mean temperature of gas in the chimney is 290° C., and the air outside is 17° C., what height h ft. of chimney 1 sq. ft. in section will produce a draught equal to the pressure of 1 in. of water ?

The difference in weight of a column h cub. ft. of air at 290° and 17° C. is $0 \cdot 0368h$ lb., which must be equal to $5 \cdot 2$ lb., or

$$h = \frac{5 \cdot 2}{\cdot 0368} = 141 \text{ ft.}$$

Take the air pressure p lb. per sq. ft. near the boiler below the fire-grate, and p_1 lb. per sq. ft. pressure of the flue gas, inside the chimney at its base.

Since a head or difference of level of 1 in. water column indicates a pressure of $5 \cdot 2$ lb per sq. ft., the difference of pressure between the air in the ash-pit and at the base of chimney or the draught is $p - p_1 = 5 \cdot 2 \, h$ lb. per sq. ft., which produces the flow of air through the furnace and up the chimney.

If w lb. of air is drawn through the furnace per pound of fuel burnt, the flue gas formed, together with combustibles in the fuel, will be nearly $w + 1$ lb. ; and the volume of these products of combustion is practically equal to the volume of air supplied per pound of fuel under the same pressure and temperature. Therefore, the volume of furnace gas from a given air supply may be calculated by the general formula for air, i.e.,

$$pv = 96 \, wT,$$

where v is the volume in cubic feet of w lb. of air supplied per pound of fuel burnt, and T the absolute temperature degrees C. Also, when the combustion is at atmospheric pressure, 2,116 lb. per sq. ft., the volume of furnace gas is

$$c = \frac{96wT}{2116} = \frac{wT}{22} \text{cub. ft. per lb. of fuel burnt} \qquad . \quad (1)$$

and the density of the air or furnace gas is

$$\frac{w}{v} = \frac{22}{T} \text{ lb. per cub. ft. (approximately)}.$$

Let A be the sectional area of the flue at the chimney base,

where the temperature T is observed, then the total amount of gas entering the chimney in pounds per second is equal to

$$\begin{pmatrix} \text{velocity of flow,} \\ \text{ft. per sec.} \end{pmatrix} \times \begin{pmatrix} \text{Section } A \text{ sq. ft. at} \\ \text{base of chimney} \end{pmatrix} \times \begin{pmatrix} \text{Density of gas,} \\ \text{lb. per cub. ft.} \end{pmatrix}$$

from which the mean velocity of gas flow across that section is

$$\frac{96 \, wTW}{2116A} \text{ ft. per sec.} \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

where W is the pounds weight of fuel burnt per second.

Example 6. A boiler is fired with coal having a percentage composition, carbon 85·1, hydrogen 4·2, oxygen ash, etc., 10·7. The analysis of dry flue gas shows 10·2 per cent of CO_2. Estimate the weight of air supplied to the furnace per pound of fuel fired. If the measured temperature of the flue gas at the base of the chimney is 410° C. when the coal consumption in the boiler is 1,625 lb. per hour, find the mean speed of the flue gas entering the chimney if the cross-sectional area of the chimney is 18 sq. ft.

(U.L., BSc. (Eng.), 1923.)

The oxygen theoretically needed for complete combustion of 1 lb. coal is $0·851 \times \dfrac{8}{3} + 0·042 \times 8 = 2·605$ lb., which is contained in $\dfrac{2·605}{0·231} = 11·28$ lb. of air. From the CO_2 in the dry flue gas the excess air may be estimated as $\left(\dfrac{21 \times 0·851}{10·2} - 1 \right) 100 = 75·2$ cub. ft. at 15° C. and 14·7 lb. per sq. in., and its weight is $\dfrac{75·2}{13·1} = 5·74$ lb. The total weight of air supplied per pound of coal fired is

$$11·28 + 5·74 = 17·02 \text{ lb. (approx.)}$$

The temperature at the base of chimney is $410° + 273° = 683°$ C. (abs.) ; then, from equation (1), the volume of air or flue gas in cubic feet per pound of coal burned is

$$v = \frac{96wT}{2116} = \frac{96 \times 17 \times 683}{2116} = 527 \text{ cub. ft. ;}$$

and the density of the gas at the base of the chimney is

$$\frac{w}{v} = \frac{22}{T} = \frac{22}{683} = 0·0322 \text{ lb. per cub. ft.}$$

The coal fired per second is

$$W = \frac{1625}{60 \times 60} = 0·4514 \text{ lb.}$$

Then, substituting in equation (2), the mean velocity of flue gas entering the chimney is

$$\frac{96wTW}{2116A} = \frac{96 \times 17 \times 683 \times 0·4514}{2116 \times 18} = 13·2 \text{ ft. per sec.}$$

Answer.

Height of Chimney to Produce a Given Draught.

Again, let

$H_{ft.}$ = height of chimney above the fire-grate

H_1 = height of column of gas of density D,

D = mean density of gases in chimney

D_1 = density of outside air

P_1 = pressure of outside air at the chimney top ;

then $\quad P_1 + HD$ lb. per sq ft. is the pressure inside att he base of chimney,

$P_1 + HD_1$ lb. per sq. ft. is the pressure outside at entrance to the ash-pit.

The difference of these pressures which produces the air flow from the outside into the chimney, through the grate, fuel bed, and chimney is '

$$H(D_1 - D) = H_1 D, \quad \therefore H_1 = H \left(\frac{D_1}{D} - 1 \right) \tag{3}$$

The frictional resistance F to flow through the fuel bed is relatively large compared with the head corresponding to the velocity of flow. F is measured by the draught h in. of water column in the U-tube inserted at the chimney base, and is equal to $\dfrac{h \times 62 \cdot 4}{12D}$ ft. of chimney gas $= H_1$, since the density of water is $62 \cdot 4$ lb. per cub. ft. at the atmospheric temperature.

But $D = \dfrac{22}{T}$ lb. per cub. ft. ; and $D_1 = \dfrac{22}{T_1} = 0 \cdot 0764$ lb. per cub ft., where T_1 is the absolute temperature of outside air, and T mean temperature of the chimney gas. Substituting these values in equation (3), draught :

$$h = 4 \cdot 2H \left(\frac{1}{T_1} - \frac{1}{T} \right) \text{in. of water column} \qquad . \qquad . \tag{4}$$

and $\quad H_1 = H \left(\dfrac{T}{T_1} - 1 \right)$ ft. of chimney gas $\qquad . \qquad . \qquad . \tag{5}$

Now $H_1 = F + \dfrac{u^2}{2g}$, where $\dfrac{u^2}{2g}$ is part of the head used to give velocity of flow, u, at the base of the chimney, and

$$= k \sqrt{H \left(\frac{T}{T_1} - 1 \right)} \text{ ft. per sec} \qquad . \qquad . \qquad . \qquad . \tag{6}$$

where the constant k is about 2 when $\dfrac{1}{16}$ of the head is spent in

producing the velocity of flow. Professor Dalby* has found from marine engine trials that the value of k for the chimneys of the ships tested varied from 1·6 to 2·7.

Example 7. The composition by weight of the coal used to fire a boiler is : carbon, 0·863 ; hydrogen, 0·038 ; oxygen, 0·004 ; ash, 0·095. Assuming that the ratio of air used to that theoretically required for combustion is 1·6, find the weight of flue gas per pound of coal fired. If the chimney draught is 1 in. of water, and the temperature of flue gas at the base of the chimney is 400° C., what should be the internal cross-sectional area of the chimney if the boiler is to consume 1½ tons of coal per hour. Assume that one-tenth of the draught is available for giving velocity, the rest of the draught being used in overcoming friction of the air passing through the fuel bed.

(U.L., B.Sc. (Eng.), 1925.)

The oxygen theoretically required for complete combustion of 1 lb. of the coal is $0.863 \times \frac{8}{3} + 0.038 \times 8 = 2.605$ lb., and the coal itself contains 0·004 lb., so that 2·601 lb. is needed in the air supply, by $\frac{2.601}{0.231}$, or 11·26 lb. air, the minimum theoretically required. The air used per pound of coal is $1.6 \times 11.26 = 18$ lb.

The total head to overcome frictional resistance and keep up the velocity of flow is $\frac{h \text{ in. } \times 62.4}{12D}$, where D is the mean density of the chimney gas, and the draught h in. of water. Thus $\frac{5.2h}{D} = H\left(\frac{D_1}{D} - 1\right)$, by equation (3), where $D_1 = 0.0764$ lb. per cub. ft. is the density of the outside air at 15° C. and atmospheric pressure. At 400° C., or 673° C. (abs.), the air or gas in the chimney has density

$$D = \frac{22}{T} = 0.0327 \text{ lb. per cub. ft. ;}$$

also h is given equal to 1 in. of water. Hence

$$5.2 = H(D_1 - D) = H(0.0764 - 0.0327),$$

from which $H = 119$ ft., and one-tenth of this is available for giving velocity. Otherwise, by equation (4), $4.2H\left(\frac{1}{288} - \frac{1}{673}\right) = 1$, hence $H = 12$ ft. The velocity of flow,

$$u^2 = 2g \times 12, \text{ or } u = \sqrt{64.4 \times 12} = 27.8 \text{ ft per sec.}$$

Coal is burned at the rate of 3,360 lb. per hour

$$= \frac{3360}{60 \times 60} = \frac{14}{15} \text{ lb. per second} = W$$

* See *Steam Power*, by Professor W. E. Dalby

Substituting these values in equation (2) (p. 212) gives the internal cross-sectional area of the chimney

$$\frac{96wTW}{2116n} = \frac{96 \times 18 \times 673 \times 14}{2116 \quad 27\cdot8 \times 15} = 18\cdot45 \text{ sq. ft.}$$

Answer.

To check this result : The volume of the flue gas being taken equal to that of the air supplied, and

$$pv = 96wT, \text{ or } v = \frac{96 \times 18 \times 673}{2116} = 549\cdot6 \text{ cub. ft. per lb.}$$

of coal burned ; and, at $\frac{14}{15}$ lb. per sec., the rate of gas flow is $549\cdot6 \times \frac{14}{15} = 512\cdot9$ cub. ft. per sec. This should be equal to the velocity of flow $\times$ sectional area of chimney, or

$$27\cdot8 \times 18\cdot45 = 512\cdot9 \text{ cub. ft. per sec.}$$

The natural draught of a chimney draws sufficient air through the fuel to burn coal completely at the rate of 20 lb. of coal per hour per square foot of grate. The rate of firing is increased by **forced draught.** In a locomotive the draught is produced by the *exhaust steam blast* in the smoke-box, which reduces the pressure of the escaping gases below atmospheric and greatly increases the air flow through the grate, tubes, and up the chimney, at an average rate of burning 80 lb. of coal per hour per square foot of grate in an express locomotive.[*]

In marine boilers the draught is forced by means of fans blowing air into the stoke-hole, which is made air-tight, so that the air can only escape through the ash-pit and fuel bed to the chimney.

In the *Howden system* of forced draught, air driven by a fan passes through heated tubes, before it enters the ash-pit and passes over the grate.

Another way is by **induced draught**, when the fan is placed near the chimney base and draws the air through the furnace, and the hot gases pass through the fan.

The advantages of mechanical draught are : easy control of the draught by regulation of the speed of the fans ; ability to meet sudden demands for increased output of steam ; greater furnace efficiency, because complete combustion is obtained and the furnace gas may be reduced to a lower temperature than if chimney draught alone is used ; natural draught changes with the weather, and forced draught overcomes that difficulty ; also a greater rate of burning at the fire-grate, and cheaper coal can be economically used

The chief disadvantages are the upkeep of the machinery and cost of power in driving the fans ; also increased wear and repairs of the boiler owing to the higher rates of combustion.

[*] See " Trials of an Express Locomotive," *Proc. Inst. C.E.*, Vol. CXXV, 291.

The brake horse-power of the engine driving a fan may be estimated—

Let $v =$ volume, in cubic feet, of air or gas passing through the fan per minute

$h =$ draught in inches of water column

$e =$ efficiency of the fan

$w =$ pounds of air delivered per pound of fuel burnt

$W =$ weight (pounds) of fuel burned per second.

The required brake horse-power $= \dfrac{5 \cdot 2 \ hv}{33,000 \times e}$ \hfill (7)

With *forced draught* $v_f = 60 w W v_o \dfrac{T_f}{273}$

Where $v_o = 12 \cdot 39$ cub. ft., the volume of 1 lb. air at 0° C. and $14 \cdot 7$ lb. per sq. in.

and $T_f =$ absolute temperature of air delivered by the fan.

With *induced draught* there will be $(w + 1)W$ lb. of flue gas delivered per second, and assume density that of air.

$$v_i = 60(w + 1)W v_o \cdot \frac{T_i}{273}$$

$\mathrm{H} = \dfrac{\text{Brake horse-power of induced draught fan}}{\text{Brake horse-power of forced draught fan}}$

$$= \frac{w + 1}{w} \cdot \frac{T_i}{T_f} \qquad . \qquad . \qquad . \qquad . \qquad (8)$$

Heat of Combustion. The quantities of heat evolved by the complete combustion of various hydrocarbons in oxygen, and the subsequent cooling of the final products, water and carbon dioxide to the initial temperature, were determined by skilful experimenters, Andrews, Favre and Silbermann, Berthelot, and others.

Professor Thomas Andrews,[*] of Belfast, mixed the gases to be burned with oxygen in a thin copper vessel, surrounded by a calorimeter containing a known weight of water, kept stirred, and all placed in a large vessel of plate tin, to reduce loss of heat by radiation and convection. The mixture was fired by the electric spark, the *combustion took place rapidly at constant volume*, and the rise in temperature of water in the calorimeter was observed on a very delicate thermometer.

Solid combustibles were reduced to a fine powder and a known weight, placed in a platinum crucible, let down into the copper vessel which was filled with oxygen, and then fired by the electric spark.

[*] *Phil. Mag.* (3), XXXII. pp. 321, 392, and 426.

In the calorimeter used by Favre and Silbermann,* *gradual combustion* took place in a *current of oxygen*, and the products were cooled by passing through a long thin copper coil or worm below the combustion chamber.

Berthelot† measured the heating value of fuels by explosion at constant volume in excess of oxygen, first used by Andrews. The Berthelot bomb calorimeter consists of a strong steel vessel lined with platinum.

Mahler lined the steel bomb with enamel. Mr C. J. Wilson made the bomb of manganese bronze and gilded the inside. Fig. 81 shows
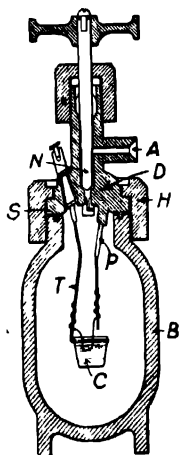
FIG. 81. THE BOMB
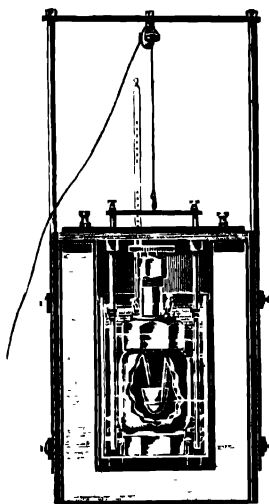
FIG. 82. BOMB CALORIMETER

a section of the bomb made by Mr. C. W. Cook, and Fig. 82 a section of the calorimeter with accessories.

The inner surface of the steel bomb, *B*, is gilt or coated with special enamel to prevent corrosion by the acids formed by combustion of fuel containing sulphur and nitrogen. The cover, *D*, carries a needle valve, *N*, to admit compressed oxygen. A thick platinum wire, *P*, is screwed into the cover, the other wire, *T*, carries the crucible, *C*, at the lower end, which is formed into a ring ; the other end of *T* passes through an insulated plug, protected against the high temperature inside the bomb. A fine platinum or nickel wire, with its end twisted round the wires *P* and *T*, making good electrical contact, dips into the fuel, which is ignited when the fine wire is heated to incandescence by an electric current. One gramme

* *Ann. Chim. Phys.* (3), XXXIV, p. 357 ; XXXVI, p. 5 ; XXXVII, p. 405
† Berthelot's *Mécanique Chimique* and *Annales de Physique et de Chimie*, 1885.

of oil fuel, or finely powdered coal, is accurately weighed in the crucible, which is then placed in the ring and with the ignition wire attached, the cover D is then placed on the bomb. The cover is held firmly in position by a large hexagonal nut H, screwed down tightly, a gas-tight joint being made by the spigot S, fitting into a ring of lead in a groove on the top of the bomb. Oxygen is admitted very gradually through a cone and nut attachment at A, by opening the needle valve N, until the pressure is about $25\frac{1}{2}$ atmospheres, when the valve is closed. Wires from an electric circuit are connected to the terminals of P and T, on the cover. The bomb is then placed in a known weight of water in the copper calorimeter, which is insulated near the top to prevent leakage of heat by conduction, and rests on blocks of cork in the air space of an outer double-walled vessel, shown in Fig. 82. This outer vessel and its lid are well lagged on the outside by a thick layer of felt, and forms a water jacket at the temperature of the room. In order to reduce the correction for radiation, the temperature of the bomb and water in the calorimeter should be as many degrees below the jacket temperature at the start as above it when the maximum temperature is reached by burning the fuel.

A standardized Beckmann thermometer, graduated in $\frac{1}{100}$ ° C., having a range of a little over 5 ° C., is placed in the water surrounding the bomb. Stirring is then started slowly, and continued throughout the experiment, while readings of the thermometer are noted every minute. After 4 or 5 min., the electric circuit is closed and the fuel fired. The heat evolved is absorbed by the bomb and water in the calorimeter, the temperature rises rapidly, and readings are continually taken until 5 to 10 min. after the maximum temperature is attained, in order to estimate the rate of cooling by radiation and the true rise of temperature. The temperature of the outer water jacket is also noted at the start and finish as a check on the radiation correction.

The water equivalent of the bomb, calorimeter and solid parts therein having been previously found by using a fuel of known calorific value, e.g. naphthalene or benzoic acid. The calorific value of benzoic acid is 6,322 C.H.U. per lb. This water equivalent added to the weight of water in the calorimeter and multiplied by the rise in temperature, measures the total heat generated by combustion of the fuel and by subsequent cooling of the products, then, divided by the weight of fuel taken, gives its *higher calorific value*. The steam formed by burning the hydrogen or hydrocarbons present condenses on the walls of the bomb, and its latent heat is included in the quantity measured.

Several corrections are necessary to secure results accurate to 0·2 per cent of the calorific value. Combustion takes place at *constant volume* not at *constant pressure*, but, for the combustion of

bituminous or anthracite coals in oxygen, when the hydrogen content is only 3 to 5 or 6 per cent, there is very little chemical change in volume, and the difference between the two values is about 10 B.Th.U. per lb. With mineral oils which contain about 15 per cent hydrogen, the difference is a little larger. The maximum error in good work, by an experienced observer, with the bomb calorimeter, is $\pm$ 20 gramme calories per gramme, or C H.U. per lb.

In practice, under ordinary furnace conditions, the nitrogen does *not* burn to nitric acid, but escapes as nitrogen ; and the sulphur as sulphur dioxide not as sulphuric acid. Consequently, it is necessary to deduct the excess heat generated in the bomb. For this purpose, 5 c.c. of distilled water are poured into the bomb at the start, and after the experiment, when the bomb has been removed from the calorimeter, and the gases allowed to escape by opening the needle valve, the bomb is washed out with distilled water into a hard glass beaker, from which the chemist estimates the sulphuric acid*

* To the bomb washing add 25 c.c. $N/10$ sodium carbonate solution, and boil down to about 15 c.c. ; filter and wash the filter paper well with hot distilled water. The filtrate when cold is titrated with $N/10$ hydrochloric acid, using methyl orange as indicator. Less than 25 c.c. $N/10$ hydrochloric acid will be required, owing to the acidity present in the bomb washings, which will have neutralized part of the sodium carbonate solution. Now, to estimate the sulphur present, about 5 c.c. of strong hydrochloric acid are added, the liquid brought to the boiling point, and a little warm water containing about 0·5 gramme of barium chloride in solution is added to precipitate the sulphur as barium sulphate. In order to form a granular precipitate, and one easily filtered off, the liquid is boiled gently for half an hour at least, and then allowed to stand 3 or 4 hours before filtering off the precipitate. The filter paper and precipitate must then be well washed with hot distilled water, to remove all soluble salt (e g. $BaCl_2$) from the paper ; after which the paper and precipitate are ignited and burned The residue of barium sulphate is allowed to cool and weighed

The weight of sulphur x grm. weight of $BaSO_4$ 0·1374, and will have yielded $\frac{98}{32}x$ grm. of sulphuric acid (pure) in the bomb, which are contained in

$$\frac{98x}{32} \quad 0·0049 \quad 0·0016 \quad \text{c.c. of } \frac{N}{10} \text{ sulphuric acid present in the bomb}$$

washings. Since 1 grm. of sulphuric acid (pure) is contained in $\frac{1}{0·0049}$ c.c., $N/10$ sulphuric acid, or 4·9 grm per 1000 c.c.

If h c.c. of $\frac{N}{10}$ hydrochloric acid are used, the total acidity due to sulphuric acid and nitric acid is $(25 \quad h)$ c.c $\frac{N}{10}$ acid ; and of this $\frac{x}{0·0016}$ c.c. is sulphuric acid $\left(\frac{N}{10}\right)$, therefore $\left(25 - h - \frac{x}{0·0016}\right)$ c.c. is due to nitric acid $\left(\frac{N}{10}\right)$

The heat of formation of nitric acid is 1·43 calories per 1 c.c. of $\frac{N}{10}$ acid

$\therefore$ the nitric acid correction is $\left(25 - h \quad \frac{x}{0·0016}\right)$ 1·43.

The sulphur of the coal, instead of burning to SO_2, as in air, burns to H_2SO_4, and the (excess) heat of the reaction $SO_2 + O + H_2O - H_2SO_4$ is allowed for by deducting $S \times 2250$ calories from the bomb result.

and nitric acid formed, and the excess heat evolved in the combination.

The **higher or gross calorific value** of a fuel is the amount of heat given out in the complete combustion of unit weight of the fuel in oxygen, when the products of combustion are cooled down to the initial atmospheric temperature (15° C. or 60° F.) at which the oxygen is supplied.

For comparative purposes, the higher calorific value of coal is usually the value found, either by experiment or calculation, on a sample of " dry coal," from which moisture has been driven off by heating the sample to 105° C. until its weight is constant, since the amount of moisture in coal changes with atmospheric conditions and the time of exposure. " Pure dry coal," or " dry ash-free coal," denotes the combustibles left in the coal after the coal is dried and the ash content also subtracted. The calorific value on a " moist coal " basis, i.e. on a sample, as received and used, is the actual value generally required by the engineer.

Moreover, in practice, the furnace gas rejected to the chimney, and the exhaust products of combustion from the cylinder of the internal combustion engine, contain steam not condensed to water, and the exhaust gases are not cooled to the ordinary temperature before leaving the engine.

The **lower or net calorific value** is the heat obtained by the complete combustion of unit weight of the fuel in oxygen when the products are cooled down to 100° C., and the steam is not condensed to water at 100° C.

The latent heat per pound of steam at 100° C. is 539 C.H.U., or 970 B.Th. U., and in cooling the condensed water from 100° C. to 15° C., gives 85 C.H U. The heat given out by 1 lb. of steam at atmospheric pressure in condensing and cooling to 15° C. is 539 + 85, or 624 C.H.U. Since 1 lb. of hydrogen when burnt forms 9 lb. of steam, the difference between the higher and lower calorific values of hydrogen is 9 × 624, or 5.616 C.H.U. Therefore, the

For instance, in an experiment when burning 1·034 grm of coal in the bomb calorimeter, the amount of $N/10$ sodium carbonate solution added = 25 c.c., and the amount of $N/10$ hydrochloric acid required = 10 c.c.

Weight of barium sulphate = 0·082 grm., gives 0·0113 grm of sulphur.

Sulphuric acid correction = 0·0113 × 2250 = 25·4 calories.

Nitric acid correction = $\left(25 - 10 - \dfrac{0\cdot0113}{0\cdot0016}\right) \times 1\cdot43 = 11\cdot4$ calories.

these corrections = 36·8 calories to be deducted.

The allowance for the heat from ignition wire or thread is 16·5 calories, making the total corrections 53·3 calories. The gross heat given to the calorimeter by combustion of 1 034 grm of coal, etc. = 7403 calories, so that the heat from the ordinary combustion of the coal alone is 7403 – 53, or 7350 calories. Therefore, the higher calorific value is $\dfrac{7350}{1\cdot034}$ = 7108 grm.-calories per grm. (C.H.U. per lb.), or, 7108 × 1·8 = 12,795 B.Th.U. per lb.

lower calorific value of hydrogen is 34,500 – 5616, or 28,884, which may be taken in round numbers as 28,900 or 29,000 C.H.U. per lb., corresponding to 29,000 × 1·8, or 52,000 B.Th.U. per lb.

The mean specific heat of carbon dioxide is 0·21 (p. 192), and 1 lb. of carbon completely burnt gives 11/3 lb. carbon dioxide, which, in cooling from 100° to 15° C., gives out $\frac{11}{3}$ × 0·21 × 85°, or 65·45 C.H.U Thus the lower calorific value of carbon is 8,080 · 65, or 8,015 C.H.U. per lb. This correction for carbon is usually neglected.

Experiment shows that, when carbon is burned in oxygen to carbon dioxide, the amount of heat evolved per unit weight depends on the variety of carbon taken, whether diamond, graphite, wood charcoal, or amorphous carbon, and the higher calorific values vary accordingly, from 7,800 to 8,140 C.H.U. per lb.

The heat given out in converting solid carbon into carbon dioxide is less than that evolved in converting the same weight of carbon in a liquid or gaseous hydrocarbon into the same product, because the heat used up in changing the physical condition of the solid carbon is greater than that required to separate it from the hydrogen.

The calorific value of a simple hydrocarbon, like CH_4 or C_2H_4, is different from the sum of the calorific values of its chemical constituents when burned separately, and depends upon the chemical constitution, or, on the way in which the carbon and hydrogen are combined in the fuel. Thus 1 lb of marsh gas, CH_4, consists of 0·75 lb. of carbon and 0·25 lb. of hydrogen. The total amount of heat given out by burning these weights of each constituent separately in oxygen, when free, is 0·75 × 8080 + 0·25 × 34,500, equal to 14,685 C H.U The actual value found by burning marsh gas in a calorimeter is 13,100 C.H.U. per lb. The difference, 1,585 C.H.U. per lb., must be used up in tearing the constituents in the compound apart in the process of combustion, which is found to be very complicated (pp. 203 and 242).

The heat energy spent in doing internal work, tearing the particles of carbon and hydrogen asunder, must necessarily vary with the arrangement or internal combination of the particles in the molecule. Liquid hydrocarbon fuels consisting of light volatile paraffins are chemically unstable compounds, of low ignition point, and their normal rate of burning is very rapid. The aromatic hydrocarbons, benzene and toluene, have greater chemical stability and high ignition temperature.

Again, 1 lb. of acetylene, C_2H_2, consists of 0·923 lb. of carbon and 0·077 lb. of hydrogen. The heat evolved by burning each constituent separately in oxygen is 10,100 C.H.U. per lb. of acetylene. But, in burning 1 lb. of acetylene in a calorimeter, the heat evolved is 12,000 C.H.U.; that is, 1,900 C.H.U. more than the constituents give out. Berthelot found that acetylene is an *endothermic*

compound, that is to say, it can be formed by direct combination of carbon and hydrogen, with *absorption* of heat at intensely high temperature, as in the electric arc between pencils of retort carbon in an atmosphere of hydrogen. This is an example of the direct *synthesis*, or building up, of a hydrocarbon from its elements. The same amount of heat absorbed in forming the compound is evolved in the direct decomposition of the molecule into carbon and hydrogen. Professor Vivian B. Lewes first observed at 800° C. the luminous " flash," during instantaneous decomposition of about 75 per cent of acetylene gas at this temperature.

Further, two substances, *isomers*, which the chemist finds by ultimate analysis to be made up of exactly the same proportions of carbon and hydrogen, have the same molecular density, yet differ in physical and chemical properties.

Cellulose, $C_6H_{10}O_5$, appears to form the basis of wood or vegetable fibre, and different fuels have been formed by its gradual transformation, until, in bituminous coal, the proportions are C_9H_6O

Crude petroleum from the oil wells contains many different compounds, and is a complex mixture of solid, liquid and gaseous hydrocarbons, with one common characteristic, that the various compounds consist only of carbon and hydrogen. Their complicated combinations during combustion must baffle the wit of man, and ultimate analysis affords uncertain help, but the final products of complete combustion are carbon dioxide and water.

The *calorific value of a fuel can only be determined with accuracy by direct experiment in the calorimeter.*

It is obvious, from the results of experiment, that a *general formula* given for calculating the calorific value of any kind of fuel from the ultimate analysis can only give approximate results. In the formula deduced by Rankine from Dulong's experiments, and commonly used, it is assumed that all the oxygen present in a fuel is combined with one-eighth its weight of hydrogen in the proportion to form water, and only the *surplus* $\left(H - \dfrac{O}{8}\right)$ is " available " for combustion—an unjustifiable assumption. According to this formula (which necessitates ultimate analysis of the fuel), the higher calorific value in C.H.U. per pound is

$$8080C + 34{,}500\left(H - \frac{O}{8}\right) + 2220S$$

where C, H, O, S are the weights of these constituents in 1 lb. of any fuel, the remainder being nitrogen, ash, and other impurities. The constants denote the calorific values of the combustibles.

When compared with the results obtained by actual combustion of the fuel in a calorimeter, the values given by this formula are too

low.   If the whole of the carbon and hydrogen in a fuel are assumed entirely uncombined and available for combustion, the calorific value obtained with the above constants is usually too high.   Moreover, the percentage of oxygen in the usual fuel analysis is practically a fictitious figure, found by difference, and covers all the errors of the analysis,* and, if correct, its relation to the heat absorbed in the decomposition of the fuel is not apparent.   Besides, the same formula will not give equally accurate values for both solid and liquid fuels.

When the oxygen is less than 5 per cent, the calculated values are in closer agreement with those found by the bomb calorimeter than when the oxygen is higher and the term $\dfrac{O}{8}$ then comparatively large.

The " carbon value " of a dried fuel is found by dividing the calorific value of the dried fuel by 8,080 C.H.U., the calorific value of pure carbon when completely burned to carbon dioxide.

Various formulæ have been proposed for the calculation of the calorific value from the proximate analysis of coals. Goutal's formula (*Comptes Rendus*, September, 1902) is $82C + aV$ for the calorific value of dry coal, in calories per grm , where C is percentage of fixed carbon in the dry coal, V the volatiles in it, and a varies with V_1, the percentage of volatiles on a dry ash-free basis, thus--

V_1	5	10	15	20	25	30	35
a	145	130	117	109	103	98	94

The calorific value† of most of the coals of the Notts. and Derbyshire coalfield, expressed in B.Th U. per pound $= 100 \left(145 - \dfrac{10A}{6} \right)$, where A is the percentage of ash in the dry coal.   This formula is liable to an error of less than 2 per cent, when the volatiles exceed 38·5 per cent on a dry, ash-free basis. The inorganic mineral matter in these coals is about 1·15 times the ash. With coals yielding less than 38·5 per cent volatiles on a dry mineral matter free basis, the calculated calorific values are low by 1 to 3 per cent.

Example 8.   The volumetric analysis of the flue gas of a locomotive boiler was CO_2, 15·0 ; CO, 2·2 ; O_2, 1·6 ; N_2, 81 2 per cent.   The coal had a calorific value of 8,250 C.H.U per lb. and contained 85 per cent of carbon.   The weight of cinders and ashes collected per pound of coal fired was 0·18 lb., and these contained 62 per cent of carbon.   Determine the percentage of the calorific value of the coal which was actually produced as heat in the furnace.

* " Report of the Committee on Steam Engine and Boiler Trials," *Proc. Inst. C.E.*

† See Paper by J. W. Whitaker, B.Sc., *Inst. Mining Engrs.* (*Midland*), March, 1924.

If the efficiency of heat transmission through the tubes was 75 per cent, what was the evaporation from and at 100° C. per lb. of coal fired ?   One pound of carbon burnt to CO gives 2,420 and to CO_2 8,080 C.H.U.

(U.L., B.Sc. (Eng.), 1924)

The calorific value of cinders and ashes

$$\frac{62}{100} \times 8080 \text{ C.H.U. per lb.}$$

∴ the heat lost in cinders and ashes per pound of coal fired

$$= 0.18 \times 0.62 \times 8080 \text{ C.H.U.} = 901.73 \text{ C.H.U.}$$

The proportion by weight of carbon in 15 parts by volume of CO_2 is $15 \times 12 = 180$, and the carbon in 2·2 parts by volume of $CO = 2.2 \times 12 = 26.4$.

The total weight of carbon is 206·4, and the fraction of this burnt to CO is $\dfrac{26.4}{206.4} = \dfrac{1.1}{8.6}$ But the heat lost or unproduced by 1 lb. of carbon burnt to CO is $8080 - 2420 = 5660$ C.H.U.

∴ the heat lost per pound of carbon in furnace gas

$$= 5660 \times \frac{1.1}{8.6} = 724 \text{ C.H.U.}$$

∴ loss per pound of coal fired, by incomplete combustion, is

$$0.85(1 - 0.18)724 = 504.6 \text{ C.H.U.}$$

∴ the heat actually produced in the furnace per pound of coal fired is $8250 - 901.73 - 504.6 = 6844 \text{ C.H.U.},$
and the percentage of the calorific value actually produced is

$$\frac{6844}{8250} \cdot 100 = \underline{83} \text{ per cent.} \qquad \textit{Answer.}$$

Again, the transmission of heat through the tubes is

$$6844 \times .75 = 5133 \text{ C.H.U.},$$

and, since the latent heat of evaporation is 539 C.H.U. per lb. of water, the equivalent evaporation from and at 100° C. per pound of coal fired is

$$\frac{5133}{539} = 9.523 \text{ lb.} \qquad \textit{Answer}$$

Crude Petroleum oil, issuing from wells in America, Russia and other countries, varies in physical and chemical properties, and is a mixture of many solid, liquid, and gaseous hydrocarbons which have different boiling points and properties.   Methane and other gaseous hydrocarbons are dissolved or held in solution, and the lighter constituents are extremely volatile, some of them evaporating

rapidly when the crude oil is exposed to the air. The light oils, gasoline, naphtha or petroleum spirit, and benzoline are dangerous because, at ordinary atmospheric temperature, they rapidly change to vapour, which is much heavier than air The vapour of pentane (C_5H_{12}), present in American crude petroleum and in gasoline, is 2·5 times heavier than air, and that of heptane (C_7H_{16}) is 3·4 times the weight of air.

These liquids ignite when a flame is brought near, without touching the liquid, since the vapour given off forms, with the air, a highly explosive mixture. Special precautions are necessary for safety in the transport, storage, and use of such inflammable hydrocarbons.

Crude petroleum is separated by fractional distillation and various refining processes into the following commercial products : (1) *Light oils*, as petroleum ethers, naphthas, and petroleum spirits or petrols ; (2) *burning oils*, or kerosene ; (3) *intermediate* gas oils, for making oil gas or used as fuel in oil engines ; (4) *lubricating oils* ; (5) *paraffin wax* and *vaseline* ; (6) *heavy oil refuse* or fuel, known as residuum (America) and ostatki (Russia).

Shale Oil. Bituminous shales are found in many countries, large deposits occur in Great Britain, New South Wales, New Zealand, Nova Scotia, Spain, France, and Serbia.

In 1847, Dr. James Young, of Renfrewshire, first extracted a light burning oil, heavy lubricating oil, and paraffin wax from oil found in the Riddings Colliery, at Alfreton, in Derbyshire. In 1850 Dr. Young discovered that **paraffin oil** could be obtained by the slow distillation of bituminous coal and oil shale The shale oil industry was started soon afterwards in America and Scotland.

Oil shale is a dark grey mineral and has a laminated fracture. The Scotch oil shales are found in great abundance in Midlothian and Linlithgow. One of the richest seams is at Broxburn, about 12 miles west of Edinburgh.

Crude Shale Oil is distilled from shale heated in the cast-iron part of the retorts at 900° F., and superheated steam is passed through the retort to carry over the paraffin oil vapours and ammonia without dissociation. The spent shale passes down and is used as fuel in the brickwork part of the retorts, kept at about 1.300° F. Each ton of Broxburn shale yields 32 gallons of crude oil, 44 lb. of sulphate of ammonia, and about 2,000 cub. ft. of gas. Other shales yield about 20 gallons of crude oil and 70 lb. of ammonia sulphate per ton.

This crude shale oil is a dark green, viscous fluid, of specific gravity 0·806 to 0·89, and resembles crude natural petroleum obtained direct from oil wells in America. It contains 70 to 80 per cent of hydrocarbons, mainly of the paraffin and less of the olefine series. At the refinery the crude oil is fractionated into light and heavy oils, and the various distillates, after refining by

treatment with sulphuric acid, soda lye, or solution of caustic soda, and washing with water, give an average yield as follows—

COMMERCIAL PRODUCTS FROM SCOTCH SHALE OIL

PRODUCT	Percentage by Volume		Specific Gravity	
Gasoline and Naphtha .	3 to	6	0·665 to 0 730	
Paraffin Oils (Burning) .	30 ,,	40	·790	·820
Intermediate or Gas Oils	4	10	·850 ,,	·880
Lubricating Oils . .	12	18	About 0 885	
Paraffin Wax .	10	12	0 87 to 0·91	
Coke, Gas, and Loss	30	20		

The fractionation varies in detail at different refineries. The products comprise gasoline of specific gravity 0 64 to 0·66, shale petrol spirit or " autoline " (0 68), naphtha (0 72 to 0·73), No. 1 burning oil (0 802 to 0·804), No. 2 burning oil (0·808 to 0 810), and lighthouse oil (810 to 820). The intermediate oils are used as fuel in many heavy-oil engines

American petroleum consists chiefly of the paraffin series of hydrocarbons, having the general formula $C_nH_{2n\ 2}$, of which methane, CH_4, is the lightest member, and hexadecane $(C_{16}H_{34})$ one of the heaviest liquid products, and solid paraffin wax, also obtained from Scotch shale oil.

These heavy paraffins, when distilled under pressure at a temperature higher than their normal boiling points, are decomposed or **cracked** into lighter paraffins of lower boiling points and olefines (C_nH_{2n}) (all containing C 85·7 and H 14·3 per cent), while at the same time gas is evolved and a little solid carbon deposited. For example, by regulating the temperature and pressure, the heavy paraffin, $C_{15}H_{32}$, be decomposed into $C_{10}H_{22} + C_5H_{10}$, or $C_7H_{16} + C_8H_{16}$, or $C_5H_{12} + C_{10}H_{20}$, etc.; and the product obtained is a mixture of several paraffins and olefines

This process of breaking up heavy oils into lighter oils of lower boiling point is called **cracking,** and is taken advantage of by petroleum distillers to obtain a much larger yield of the lighter oils, both kerosene and petrols, than the crude petroleum would yield by ordinary fractional distillation. Cracking also takes place in the distillation of heavy oil at the high temperature necessary to force complete evaporation. Some of the heavy vapours driven off are readily condensed and fall back into the hotter liquid, which cracks or decomposes them.

The cracking process patented by Sir Boverton Redwood and Sir James Dewar in 1889 appears similar to that of Burton, used at present by the Standard Oil Company to distil heavy petroleum

residue at about 650° F., and at a pressure of 4 or 5 atmospheres, for the production of low boiling point hydrocarbons.

In the Hall process, heavy oil is passed up and down long tubes heated at temperatures and pressures to suit the desired product. The vapours are expanded down to atmospheric pressure at the exit into larger tubes, passed through dephlegmators, and cooled to 100° C. before entering the compressors. The highest temperature and rate of feed have to be kept constant within 5 per cent, and the yield of motor spirit obtained is 50 to 70 per cent of the original oil.

In some oil engines, compression of the heavy oil vapours checks evaporation and causes partial condensation ; then the high temperature of inflammation may result in cracking some oils, and in time form a hard carbon deposit, like *methane carbon* —jet-black from refined oils, while that from crude oil is grey, like gas retort carbon.

In order to avoid cracking, *superheated steam* is blown through the petroleum residue, after the lighter hydrocarbons are distilled *in vacuo*, and carries over the heavy vapours to the condenser, giving distillates to be used in the manufacture of lubricating oils and paraffin wax.

On still further raising the temperature to a *bright cherry-red* heat in a retort, in the absence of air, drops of oil may be partly converted into real *oil gas* and tarry products, which are usually washed or scrubbed out in the process of *making oil gas*, but cause trouble in the oil engine cylinder. It is, therefore, necessary to regulate the temperature of the vaporizer in these engines below this *gas-making* stage to prevent the formation of tar and the deposit of carbon.

Experience shows that, with the residual burnt products in the clearance space of the engine cylinder, the excess of air added holds the heavier hydrocarbon vapours in suspension, like a cloud, which is readily ignited and gives complete combustion of the whole of the oil.

Gumming of lubricating oils is due chiefly to the removal of the more volatile hydrocarbons by evaporation, and the continued heating or stewing of the oil at a high temperature thickens the tarry matters and forms a sticky or gummy residue, which clogs the valves and working parts, and causes trouble in the practical running of the engine. Excessive quantity of lubricating oil in the high pressure cylinder of a three-stage air compressor, and too high a temperature, have caused trouble by carbonization of valves, gumming of the high pressure piston rings, and occasionally explosion in the air reservoir and pipe line.

All animal and vegetable *fats*, when heated up to 150° F., become *fixed oils*, which differ from mineral oils in not being volatile or capable of evaporating without decomposition. Fixed oils are unsuitable for use in the internal combustion engine because, at the high temperatures in the motor cylinder, they are partially

decomposed with the production of free fatty acids, as oleic and stearic acid, which corrode the metal of the cylinder, forming oxides of iron and metallic soaps, whilst the pitch and carbonaceous deposits gradually accumulate and eventually block up the valves and ports.

Liquid Fuel. The Author and his assistant, the late Mr. W. Inchley, found that, for many liquid fuels tested, the *higher calorific values,* calculated by the formulæ

$$7500C + 33,830H \text{ gramme calories per gramme or C.H.U. per lb.,}$$

or $13,500C + 60,890H$ B.Th.U. per lb.   .   .   .   .   (1)

are all in tolerably close agreement with those obtained by actual combustion in the bomb calorimeter.

TABLE III

COMPOSITION AND CALORIFIC VALUES OF OIL FUELS

Description of Petroleum Fuel	Specific Gravity at 60° F.	Carbon per cent	Hydrogen per cent	Higher Calorific Value		
				Observed Calories per gramme	Calculated Calories per gramme	Actual B.Th.U. per lb
Amer "Royal Daylight"	·797	85·7	14·2	11,167	11,231	20,100
American Kerosene	·780	85·05	14·4	11,163	11,250	20,100
Russian Refined (Baku)	·825	86·0	14·0	11,270	11,186	20,286
" Russolene " (H V O)	·890	85·95	13·5	10,907	11,013	19,600
American Crude Oil		86·89	13·11	10,912	10,951	19,640
Canadian Crude Oil	·859	86·92	12·87	10,797	10,873	19,440
Texas Crude Petroleum	·917	86·62	14·80	10,517	10,488	18,930
Russian Crude Oil	·875	86·90	13·10	10,833	10,918	19,500
Ditto (Caucasian)		84·9	14·63	10,328	10,301	18,590
Java Crude Petroleum	·867	87·10	12·7	10,654	10,828	19,180
Solar Oil	·896	86·61	12·6	10,783	10,758	19,410
Coal Oil	·917	83·2	11·87	10,233	10,256	18,400

The author obtained the following results by fractional distillation of average typical samples of refined American and Russian kerosines, and the higher calorific values of the fractions in the bomb calorimeter, also the lower calorific values by combustion in air at constant pressure in the Junker calorimeter.

Description of Fuel	American Kerosene "White Rose"	American "Royal Daylight"	Russian Kerosene "Russolene"
Specific Gravity	0·784	0·797	0·825
Flash Point (Abel Close Test)	105° F.	81° F.	88° F.
Vapour Temperature at which Distillation begins	145° C	125° C.	115° C.
Distillates	Per Cent	Per Cent	Per Cent
Spirit below 150° C.	7	23	20
Kerosene, 150° to 270° C.	85	54	68
Distillate, above 270° C.	5	10	9
Residue, at 360° C.	3	13	3
	100	100	100

The sample of *kerosene,* " White Rose," obtained direct from America, is of fairly uniform composition, 85 per cent distilling between 150° C. and 270 C°. Another American burning oil, "Water White," although of low specific gravity, 0·78, has a high flash point, 108° F. ; the first drop is distilled at liquid temperature 175° C., and below 215° C., 55 per cent by volume comes over. The other samples consist of mixtures in larger proportions of light and heavier hydrocarbons.*

HIGHER CALORIFIC VALUE OF " ROYAL DAYLIGHT " AND FRACTIONS

Kerosene and Distillates	Volume per Cent	Specific Gravity at 60° F	Weight of 1 gal. of Oil lb.	Calorific Value, B.Th.U. per lb	Heat Evolved by each fraction, B Th.U.	Calorific Value of Oil, B.Th.U. per lb.
"Royal Daylight"		0·797	7·97	20,100		20,100
Fractions —						
Below 150° C.	23	0·748	1·72	20,286	34,890	
150° to 270° C.	54	0·800	4·32	20,124	86,940	
Above 270°	23	0 840	1·93	19,410	37,520	
Total Distillates	100		7·97		159,350	20,000
Carbon in 1 lb. oil			0 857	13,500 = 11,569		
Hydrogen .. ,,			0·142	60,890 — 8,646		20,215

The higher calorific value of the original sample of " Royal Daylight," taking the average of several tests in the bomb calorimeter, is 20,100 B.Th.U. per lb. ; and the separate distillates, allowing for the proportion by weight of each present in the sample, give a total heating value of 20,000 B.Th.U. per lb of the mixture.

The lower calorific value of " Royal Daylight " by combustion in air, at constant pressure, in the Junker calorimeter is 18,640 B.Th.U. per lb. This low value is probably due to the method of vaporizing and burning the oil in the *primus* coil lamp at constant pressure.

Calculation, from the ultimate analysis, by the formula (1) gives 20,215 B.Th.U. per lb., which is slightly too high. Every pound of oil burned produces $9 \times 0·142 = 1·278$ lb. of steam, which, in condensing at 212° F. and cooling as water to 60° F., gives out $1·278 \times 1122 = 1,434$ B.Th.U. The difference, $20,100 - 1,434 = 18,666$ B.Th.U. per lb., is the lower calorific value, which is in close agreement with the value obtained in the Junker calorimeter. The heat actually measured in the calorimeters includes the latent heat of the steam formed, which is condensed and the water cooled to the ordinary temperature 60° F. In the Junker this water is collected

* *Journal Soc. Arts,* 29th April, 1891 ; also *ibid.,* " Cantor Lectures on ' Uses of Petroleum in Prime Movers,' " 1892.

and the heat given out in cooling deducted to determine the lower calorific value. All these determinations by the bomb calorimeter differ by less than half of 1 per cent from the average value.

HIGHER CALORIFIC VALUE OF "RUSSOLENE" AND FRACTIONS

Oil and Distillates	Volume per Cent	Specific Gravity at 60° F.	Weight of 1 gal. of Oil, lb	Calorific Value, B.Th.U. per lb.	Heat Evolved by each Fraction, B.Th.U.	Calorific Value of Oil, B.Th.U. per lb.
Refined Russian		0·82	8·25 × 20,286			20,286
Fractions —						
Below 150° C.	20	0·793	1·59	19,840	31,550	
150° to 270° C.	68	0·825	5·61	20,385	114,360	
Above 270° C	12	0·866	1·04	19,969	20,770	
Total Distillates	100		8·24		166,680	20,228
Carbon in 1 lb. oil			0·86	13,500	11,610	
Hydrogen ,, ,,			0 14	60,890	8,525	20,135

The total calorific value of the distillates burned in the bomb agrees closely with that found by burning the oil itself. The *lower calorific value* of the sample of "Russolene" burned in air at constant pressure in the Junker calorimeter is 18,870 B.Th.U. per lb.

Calculation, from the ultimate analysis, by formula (1) gives 20,135 B.Th.U. per lb. Every pound of oil burned forms $9 \times \cdot14 = 1\cdot26$ lb. of steam, which, in condensing at 212° F. and cooling to 60° F., gives out $1\cdot26 \times 1,122$, or 1,414 B.Th.U. The difference, $20,286 - 1,414 = 18,872$ B.Th.U., the *lower calorific value*, is in remarkable agreement with the value obtained by direct measurement in the Junker calorimeter.

Example 9. The analysis by weight of petrol, of specific gravity 0·72, is C, 84·6 ; H_2, 15·4 per cent. Calculate, (*a*) the higher and lower calorific values of the petrol ; (*b*) the cubic feet of air required for complete combustion of 1 lb. of petrol ; (*c*) the heat energy evolved by the combustion of 1 cub. ft. of the explosive mixture, which contains 2 per cent by volume of petrol vapour with air, in the engine cylinder. Given : the lower calorific value, in B.Th.U. per pound of carbon, and hydrogen in a liquid hydrocarbon is 13,500 and 52,200 respectively. The heat needed to vaporize the petrol is 140 B.Th.U. per pound ; and 1 lb. of air occupies 13 cub. ft. at the atmospheric pressure and temperature ; also the density of petrol vapour is 3 25 times that of air.

(*a*) The net heat given out by 1 lb. of *liquid* petrol in burning to CO_2, the steam formed being cooled only to 100° C. is- -

Carbon $0\cdot846 \times 13,500$ - 11,421 B.Th.U.

Hydrogen $0\cdot154 \times 52,200$ -: 8,039   ,,

and therefore the lower calorific value -- 19,460 B.Th.U. per lb,

The steam formed from the hydrogen is $0.154 \times 9 = 1.386$ lb. The latent heat of the steam at 212° F. is 970 B.Th.U. per lb., and the condensed water cooled to 60° F. has 212 – 60, or 152 B.Th.U. abstracted; a total of 1,122 B.Th.U. per lb., that is, for the amount of steam formed, $1.386 \times 1,122 = 1.555$ B.Th.U. This heat is usually deducted from the higher calorific value to give the lower value; therefore, the higher calorific value is $19,460 + 1,555 = 21,015$ B.Th.U. per lb.

Since petrol vapour at 60° F. is 3·25 times heavier than air, 1 lb. of the vapour occupies $\dfrac{13}{3.25} = 4$ cub. ft.; and 1 lb. of liquid petrol, of specific gravity 0·72 at 60° F., occupies 0·0223 cub. ft. The calorific values determined by experiment in the bomb calorimeter are given from the liquid state. If the petrol were all in the state of vapour before combustion in the explosion engine cylinder, its lower calorific value would be increased by that part of the heat of combustion which is needed to vaporize the liquid petrol, and which does not appear as heat in the determination by the calorimeter. Hence the latent heat of evaporation should be added to the lower calorific value of the liquid to give the **available heat** of combustion of 1 lb. of petrol vapour: that is, $19,460 + 140 = 19,600$ B.Th.U. More correctly, the latent heat of the fuel at *constant volume* only should be added, which is less than the latent heat at constant atmospheric pressure by the equivalent of the work done in the expansion or change in volume from liquid to vapour, namely, $140 - \dfrac{14.7 \times 144}{778} (4 - 0.0223) = 130$ B.Th U. nearly, or about $\dfrac{14}{15}$ of 140 B.Th.U. at constant pressure.

(b) Again, the oxygen required for the complete combustion of the carbon and hydrogen in 1 lb. of petrol is

$$0.846 \times \frac{8}{3} + 0.154 \times 8 = 2.256 + 1.232, \text{ or } 3.488 \text{ lb.}$$

Air contains 23·1 per cent by weight of oxygen, and the air required to supply 3·488 lb. of oxygen is $\dfrac{3.488}{0.231} = 15.1$ lb., i.e. the ratio of air to petrol by weight is 15·1 to 1 lb., also the volume of this weight of air, at 60° F. and 14·7 lb. per sq. in., is $13 \times 15.1 = 196$ cub. ft. This air is 98 per cent of the correct explosive mixture, since the oil vapour occupies 2 per cent, and, therefore, the total volume of explosive mixture of 1 lb. petrol vapour with air is $196 \times \dfrac{100}{98}$, or 200 cub. ft. at ordinary temperature and pressure. Hence the volume of 1 lb. petrol vapour is 200 – 196, or 4 cub. ft., which agrees with the value found above.

(c) The heat energy evolved by burning 1 cub. ft. of this mixture in the engine cylinder is $\dfrac{19,600}{200} = 98$ B.Th.U., or 76,244 ft.-lb. If the mixture were at 32° F. or 492° F. (abs.), instead of 60° F., and the same pressure, the heat energy liberated by burning 1 cub. ft would be $98 \times \dfrac{520}{492}$, or 103·57 B.Th.U ; and the heat energy per cubic inch of the mixture is equivalent to $\dfrac{103\cdot57 \times 778}{1728} = 46\cdot6$ ft.-lb.

Further, experiment shows that (at normal temperature and pressure) the specific volume of the products of combustion of petrol vapour and air is 1·05 times that of the mixture before combustion, and the total energy available by combustion becomes $46\cdot6 \times 1\cdot05$, or 48·9 ft.-lb. per cub. in. of the mixture.

Mr. H. R. Ricardo carried out an extensive experimental investigation* into the behaviour of various hydrocarbons and their influence as fuels on the performance of internal combustion engines. Table IV gives the heat values of different hydrocarbons, which are constituents of most of the available volatile hydrocarbon fuels.

The heat of combustion of the fuel and air mixture is expressed in the equivalent foot-pounds of heat energy evolved by the combustion of 1 cub. in. of a mixture of fuel and air, measured at standard temperature and pressure, and in the proportion for complete combustion when all the oxygen in the air is combined. In hydrocarbon fuels like " petrol " (containing hexane, heptane, and octane), the specific volume of the mixture after complete combustion is greater than before it. There is thus an increase in the number of molecules in the working fluid mixture during combustion and the total internal energy of the burnt mixture is increased, since, at the same temperature, the pressure will be proportional to the number of molecules ; and the energy will be equal to the product of the heat of combustion of the fuel, when burnt with the necessary quantity of air, and the ratio of the specific volume after combustion to that before it. In the case of benzene and the mixture of the aromatic group (Table IV), known as " benzol," there is only a slight increase in specific volume, about 2 per cent after combustion with the theoretical amount of air, about 5½ per cent with the paraffins, and nearly 6 per cent increase in the alcohol and air mixture ; whereas for carbon monoxide and hydrogen the decrease is 15 per cent, and the mixture of coal gas and air also decreases in specific volume (pp. 198 and 199).

Owing to the greater specific gravity of benzene and other aromatic hydrocarbons, their heat values per gallon are greater, and the percentage of the heat converted into work is also greater, than those of the paraffin series, although their calorific values per pound are lower.

* *The Automobile Engineer*, 1921.

TABLE IV. OIL FUEL CHARACTERISTICS (H. R. Ricardo)

Name of Fuel	Specific Gravity at 15°C	Ratio Air/Fuel by Weight for Complete Combustion	Self-ignition Temperature by Adiabatic Compression with Air	Lower Calorific Value B Th U per lb of Liquid	Latent Heat of Evaporation B Th U per lb	Heat of Combustion ft lb per cub in of mixture at N.T.P	Increase of Specific Volume after Combustion	Total Energy Liberated by Combustion ft-lb. per cub in of Mixture at N.T.P.
Paraffin Series—								
Hexane, C_6H_{14} . . .	0·670	15·2	366° C.	19,390	156	46 0	1·053	48·33
Heptane, C_7H_{16} . .	·688	15·1	330°	19,420	133	46·06	1·056	48·64
Octane, C_8H_{18} . .	·704	15·05	—	19,210	128	46·06	1·058	48·73
Decane, $C_{10}H_{22}$.	·757	15·0	—	19,060	108	46·06	1·060	48·82
Aromatic Series—								
Benzene (pure), C_6H_6	·884	13·2	419°	17,300	172	46·9	1·013	47·51
Toluene (99%), C_7H_8	·870	13·4	422°	17,520	151	46·9	1·023	47·98
Xylene (91%), C_8H_{10}	·866	13·6	—	17,800	145	46·85	1·029	48·26
Naphthene Series—								
Cyclohexane (93%) . .	·786	14·7	387°	18,800	156	46 08	1·045	48·11
Hexahydrotoluene (80%)	·770	14·7	378°	18,760	138	46·04	1 049	48·32
Olefine Series—								
Heptylene, C_7H_{14} . .	·70	14·7	—	19,170	167	46·8	1·049	49·1
Decylene, $C_{10}H_{20}$ . .	·76	14·7	—	19,170	—	47·2	—	—
Alcohol Group—								
Ethyl Alcohol, (98%)C_2H_6O	·798	8·95	514°	11,840	397	44 5	1·065	47·4
Methyl Alcohol (Wood Naphtha), $C_2H_4O_2$	·829	6·4	457°	9,630	512	42·46	1·061	45·05
Methylated Spirits . .	·821	8·0	—	10,200	450	44	1·064	46·82

233

The last column gives the remarkable result, confirmed by many experiments, that, provided the fuel is completely evaporated and burned at suitable pressure in the engine cylinder, the energy content and the maximum power obtainable from all the various volatile hydrocarbon fuels, or from any combination of such fuels, is practically the same, within narrow limits, except only the alcohols. In a case where the total internal energy is lower, the latent heat is generally higher, consequently the fuel air mixture taken into the engine cylinder will be at a lower temperature, allowing a greater weight of charge to be admitted, with greater volumetric efficiency, sufficient to make up for the lower internal energy.

For instance, the energy liberated by the combustion of a cubic inch, at standard temperature and pressure, of benzene–air mixture is a little lower than that of the paraffins which form the greater proportion of petrols. But the latent heat of benzene is much greater, i.e. 172, and the mixture called motor benzole 164, B.Th.U. per lb. Take the specific heat of the fuel 0·5, and that of air 0·2375. The heat capacity or water equivalent of the air-fuel mixture is

$$13 \cdot 2 \times 0 \cdot 2375 + 1 \times 0 \cdot 5 = 3 \cdot 635 \text{ lb.}$$

If the latent heat of evaporation of benzene were taken from the mixture, its fall in temperature would be $\dfrac{172}{3 \cdot 635} = 47 \cdot 2^{\circ}$ F. ; whereas with the octane–air mixture the drop in temperature is only 31·4° F.

TABLE V

OIL FUELS   EXPERIMENTAL RESULTS   (*H. R. Ricardo*)

FUEL	Specific Gravity at 15° C	Approximate Composition Percentage by Weight			Ratio Air by Fuel Weight for complete combustion	Lower Calorific Value, including its Latent Heat of evaporation B Th.U. per lb.	Latent Heat of evaporation, B Th.U. per lb
		Par- affins	Aro- matics	Naph- thenes			
Aromatic Free Petrol	0 718	63	1 7	35	15 05	19,200	133
" A " Petrol .	·782	26	39 0	35	14 3	18,580	142
" B " ,, .	·723	62	14 9	23	14 7	19,020	140
" C " ,,	727	61	8·5	30 5	14·8	19,120	135
" D " ,, .	760	38	11 6	47	14 6	18,890	132
" E " ,, .	719	68	11 3	20	14 9	19,090	133
" F " ,, .	704	80	4 3	15 2	15	19,250	134
" H " ,, .	·707	10	1 8	85	14 7	18,920	145
Heavy Aromatics	0 885		71 5		13 5	18,030	136
Kerosene . .	813				15 0	19,100	108
Paraffin Series							
Hexane (80% pure)	0·685				15·2	19,390	156
Heptane (97% pure)	·691	100	up to 0 2		15·1	19,420	133
Alcohol Group		Water					
Ethyl Alcohol, 98 5%	0 798	1 5 to 2			8 95	11,840	
Ditto (95 vol %)	·815				8 1	11,125	
Methyl Alcohol (Wood Naphtha) . .	·829				6·5	10,030	500 app.
Methylated Spirits .	·821				8 0	10,580	450 app.
Ether (50% with Petrol) . . .	727		50% by vol. pet'l aromatic free		13 0	16,830	180 app.

The light and more volatile fractions of ordinary petrol consist of either hexane, cyclohexane, or benzene, and these render possible the starting of an engine from cold, by " flooding " the carburetter to give an over-rich mixture (p. 316).

It will be seen by the Tables IV and V that, of these volatile constituents in petrol, the air to fuel ratio is about 14·7 : 1, to form mixtures giving complete combustion. For ethyl alcohol, ⅑ of the weight of air in the mixture must consist of fuel vapour. The average latent heat of evaporation, in B.Th.U. per pound, of petrols is about 135, motor benzole 164, and ethyl alcohol 397. The latent heat absorbed in vaporizing 1 lb. of alcohol is 2·5 times that taken by 1 lb. of petrol ; and 1·6 times the weight of alcohol is required to give an inflammable mixture with a given cylinder full of air, so that 4 times the quantity of heat is required to give a combustible vapour mixture with alcohol as compared with petrol.

With petrol " F," a light aircraft spirit of boiling range 60° to 150° C., which has a vapour pressure of 68 m.m. mercury at 0° C., an engine is easily started from cold.

The *available energy* of each fuel when burnt in the *state of vapour* in the engine cylinder is taken equal to its lower calorific value in the liquid state as burnt in the bomb calorimeter, plus its latent heat of evaporation, because this amount of heat is added in vaporizing the liquid fuel.

Gas Calorimeter. Instead of making a chemical analysis of liquid or gaseous fuel to estimate its calorific value by calculation from the combustible constituents burnt separately, **a far simpler, better, and more practical way** is to determine the calorific value of the gas directly during the engine trial by burning it with air in a **gas calorimeter** like that devised by Junker or Professor C. V. Boys.

The Junker calorimeter, Figs. 83 and 84, consists of a combustion chamber, C, surrounded by a water jacket, fitted with a large number of thin copper tubes shown in sectional plan, Fig. 84. A current of cold water enters the water jacket at the bottom by an annular space, with perforated ring to distribute and steady the flow. The water surrounds the tubes and *flows upwards* at a steady rate under a " head," kept constant by the overflow at the funnels in the supply and discharge vessels. There are also baffle plates at the top of the water jacket. The rate of flow is regulated by means of a cock with a pointer and graduated dial at the inlet to the calorimeter, in order to adjust the rise of temperature in the calorimeter.

The products of combustion of the gas or oil vapour, from the burner in the combustion chamber, enter the copper tubes at the top, as shown by the arrows, *pass downward*, and escape by a butterfly valve. The heat evolved by the combustion passes through the large surface of thin copper to the cooling water, and the burnt

products are reduced practically to atmospheric temperature by the coldest water around the lower end of the tubes. The steam, formed by burning the hydrogen in the hydrocarbons, is condensed inside the tubes, and after their surface is all wetted, the water drips down

Fig. 84. Cross-section
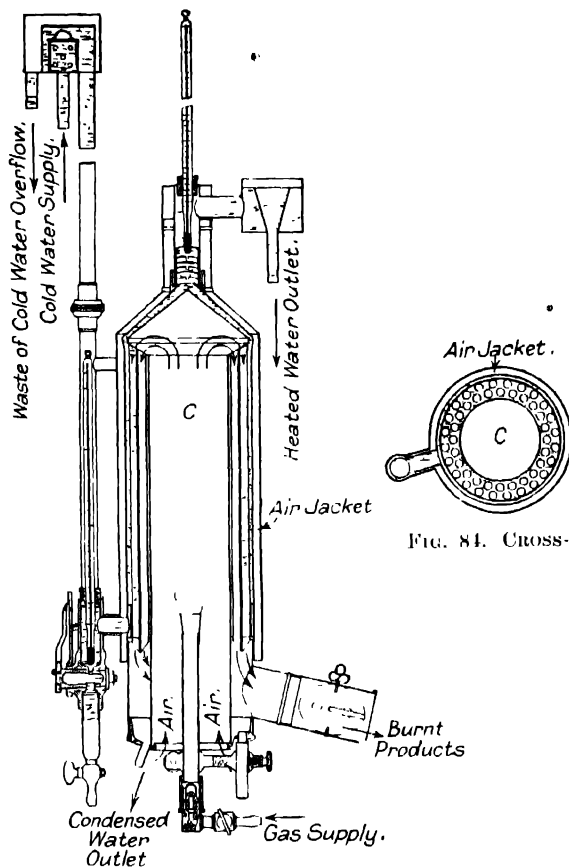
Fig. 83. Junker Gas and Oil Calorimeter

and is collected in a small measuring glass. The latent heat of this steam, given to the circulating water and to be subtracted from the total heat, is determined from the amount of condensed water formed per cubic foot of gas burned. This water is also a measure of the hydrogen content of the gas or oil fuel. The water jacket is surrounded by an annular air space, closed at the top, which, with the polished nickel-plated outer surface, greatly reduces loss by radiation.

The gas to be burned is passed through a small meter, reading to the hundredth of a cubic foot, and fitted with a gas governor or regulator to ensure constant pressure, which is measured by a U-shaped water gauge manometer tube. The barometer reading must also be taken to obtain the absolute pressure of the gas supply. The temperature of the gas leaving the meter is observed so that the cubic feet of gas burned may be reduced to standard temperature and pressure. The size of the flame and its position in the combustion chamber can be adjusted. The flame can be seen during a test by a mirror placed below the combustion chamber. When the flow of water is adjusted, the temperature of the outlet water is constant, and the drip of the condensed water steady, measurements are to be made of the cubic feet of gas burned, and the cooling water passed through the calorimeter in the same time is collected and weighed; the outlet and inlet temperatures are observed every minute to obtain the mean rise in temperature, and the condensed water collected and weighed. The latter may be collected from the gas burned for a longer period. The temperature and pressure of the gas supply is noted ; also the reading of the barometer. In this way the higher and lower calorific values are determined without the trouble of collecting samples and without any chemical analysis.

Example 10.   The gas used in a gas engine trial was tested in a Junker ·alorimeter and the following results were obtained : Gas burned in calorimeter, 2·13 cub. ft. ; pressure of gas supplied, 2·1 in. of water ; temperature of gas, 11·7° C ; weight of water heated by gas, 50·3 lb. ; mean temperature of water at inlet, 8·6° C., and at outlet, 22·4° C. ; steam condensed during test, 0·116 lb. ; and temperature of steam condensed during test, 25° C.

Determine the higher and lower calorific values of the gas per cubic foot at 15·6° C., and barometric pressure 30 in. of mercury. (Specific gravity of mercury · 13·6.)   (*U.L., B.Sc (Eng.).*)

Barometer, 30 in. of mercury = 14·7 lb. per sq. in. ; 1 in. of mercury is $\dfrac{14·7}{30} = 0·49$ lb. per sq in ; and 1 in. of water column· $= \dfrac{·49}{13·6} = 0·036$ lb per sq. in. ; so that 2·1 in. of water = ·036 × 2·1, or 0·0756 lb. per sq. in. Therefore, absolute pressure of gas supply is 14·7756 lb. per sq. in.

The temperature of gas 11·7° C. + 273° = 284·7° C. (abs.), and the volume 2·13 cub. ft. is to be reduced to that at 15·6° + 273 or 288·6° C. (abs.).

Let p_1, v_1, T_1 and p, v, T be the observed and standard pressure, volume, and temperature respectively, then

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1}, \text{ or } v = v_1 \times \frac{p_1 T}{p T_1}$$

$$\frac{2·13 \times 14·7756 \times 288·6}{14·7 \times 284·7} = 2·17 \text{ cub. ft.}$$

Heat given to raise 50·3 lb. water from 8·6° C. to 22·4° C. is 50·3(22·4 − 8·6) = 694·14 C.H U. This includes the latent heat of steam, 539 C.H.U. per lb. at atmospheric pressure, and cooling of condensed water from 100° C. to 25° C. = 75 C.H.U. per lb., or a total 614 C.H.U. per lb. Hence the condensed steam gives out 0·116 × 614 = 71·22 C.H.U. The heat available in the engine cylinder from the gas burned = 694·14 − 71·22, or 622·92 C.H.U., when the steam in the burnt products is cooled only to 100° C.

Therefore, higher calorific value of the gas is

$$\frac{694·14}{2·17} = 320 \text{ C.H.U. per cub. ft.,}$$

and the lower calorific value of the gas is

$$\frac{622·92}{2·17} = 287 \text{ C.H.U. per cub. ft.} \qquad \textit{Answer.}$$

These results, multiplied by 1·8, give the calorific values in B.Th.U. per cubic foot.

In order to determine the *calorific value of liquid fuels*, when burned *with air at constant pressure*, a *Primus* lamp, fitted with a vaporizing coil and special burner, may be used. The oil to be tested is poured into the cistern of the coil lamp and, when the coil is heated by burning a little methylated spirit under the coil to start the lamp, air is pumped into the cistern. When the flame is steady and indicating complete combustion, the lamp is clamped to one arm of a special balance with counterpoise weight. The coil burner hangs freely in the combustion chamber of the Junker calorimeter, which is raised on longer legs than when a Bunsen burner is used for gaseous fuels.

The weights in the scale pan are adjusted for balance when the heated water from the calorimeter has attained a constant temperature. At the instant when the long pointer of the balance is passing zero on the dial, remove a weight from the scale pan and collect the water flowing through the calorimeter at a steady rate, observe its mean rise of temperature until the pointer is again passing the zero, balance having been restored by the weight of oil burned. The heat given to the water is thus measured for a known weight of liquid fuel burned, the usual readings having been taken as when gas is burned.

Again, after adjustment of the flame and balance, another weight may be taken off the scale pan, and further readings noted to verify the previous determination. The steam formed by burning the liquid fuel is condensed in the water jacket and collected in order to obtain the lower calorific value of the fuel.

Heavy oil fuels, that are viscous and difficult to evaporate, are mixed with petrol or other light oil of known calorific value, and the calorific value of the mixture obtained, care being taken to

avoid partial volatilization of the petrol or light oil in the mixture before combustion.

Liquid Fuel for Power Purposes. Liquid fuel must be pulverized, sprayed, or broken up into extremely fine particles, like fog or mist, and intimately mixed with the necessary air, heated to the ignition point, to ensure complete combustion. The particles of liquid are changed at once into vapour and burn rapidly.

Many types of burners are used to break up and inject heavy liquid fuel in a fine spray cloud under steam boilers, by an air blast or a steam jet.

The Holden burner,* Fig. 85, a common type used on *locomotives*, acts like an injector. Superheated steam enters around *CD* and, passing through the annular space towards the nozzle, not only
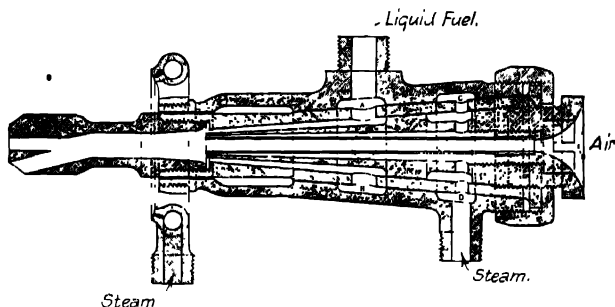
FIG. 85   HOLDEN LIQUID-FUEL INJECTOR

heats the liquid fuel in the conical passage around it and injects it as a fine spray, but also induces a strong current of air in the central cone of the burner. The three jets converge to and mix at the nozzle, where they are broken up and projected into the fire-box, just above the surface of the solid fire   Here the sprayed fuel air mixture is played on and formed into fog by inclined jets of steam, from holes drilled in the ring blower around the nozzle, which converge at a point about 6 in. in front of the nozzle. The fire-grate is covered with a layer 3 or 4 in. deep of broken fire-bricks kept incandescent. The air pipe to the fuel injector is connected with heaters fitted in the smoke-box, so that the air enters the injector at about 200° C.

The oil is filtered through a wire-gauze strainer into the oil tank carried on the tender, and in cold weather the thick, heavy oil is kept thin and less viscous by means of a warming coil heated by exhaust steam.

The fire-box is arranged to burn either coal, liquid hydrocarbons, or both combined. The actual equivalent evaporation per pound

* *Journal of the Society of Arts*, 1891 ; and *Proc. Inst. C.E.*, 1910–11, Vol. 185, Part 3, p. 340.

of fuel in the boiler fired with Russian ostatki is 14 lb. of water from and at 212° F., and for economy the maximum price allowable for liquid fuel may be a little higher than twice the price of coal.

In another type of oil fuel burner used in *marine boilers*, a conical pointed piston acts as a regulating valve, having holes drilled in it inclined to the face through which the oil is forced by a pump, and enters the boiler furnace in a whirling vortex, to be intimately mixed with a vortex of air blown through an annular channel, forming a fog or cloud which is completely burned in the furnace.

Gaseous Fuel has several well-recognized advantages over solid fuels : (1) Gas can be made in bulk from coal, and economically distributed through pipes ; (2) complete combustion is obtained under ideal conditions with a minimum excess of air, and without smoke or dust nuisance ; (3) the rate of combustion of gas is completely under control, and can be varied at will ; (4) higher heat intensity or temperature is attained with gas than with solid fuel, and the cost of fuel for a regenerative gas-fired steel furnace is much less than that of a coal-fired furnace, owing to the quality and consumption of coal required.

Natural gas consists mainly of methane or marsh gas, formed by the decomposition of organic matter, and may be disengaged in any stagnant pool by stirring the mud   The escaping gas contains nearly equal parts by volume of methane and nitrogen, with 3 per cent carbon dioxide. Coal seams contain occluded gases, and methane in a " blower " is known to the coal miner as *fire damp*.

The occluded gas in some freshly-won steam coals contains : CH_4, 60 to 80 per cent ; other hydrocarbons, 6 to 8 per cent ; and CO_2, 1 to 12 per cent by volume ; also CO.

Natural gas dissolves in petroleum, and escapes therefrom, carrying with it part of the oil vapour, which condenses when the gas is compressed.

The paraffin constituents, chiefly methane and ethane, in the Pittsburg natural gas contain by weight, carbon 75·4 per cent, and hydrogen 24·6 per cent. An average analysis is : methane, 87 ; ethane, 6·5 ; ethylene and carbon monoxide, each 0·2 per cent ; helium, 0·1 per cent ; the balance being made up of H_2, O_2, and N_2. The specific gravity is about 0·61, and the calorific value 493 C.H.U. per cub. ft.   Many tests in the Junker calorimeter of natural gas containing CH_4 up to 95 per cent, give, in round numbers, 555 C.H.U., or 1,000 B.Th.U per cub. ft. at N.T.P.

The escape of natural gas from the wells reduces the pressure, and pumps are used to compress the gas, which is conveyed in pipes from 16 to 6 in. in diameter.   The loss of pressure by skin frictional resistance to the flow of the gas in an 8 in. pipe is about 2·5 lb. per sq. in. per mile, and in a 6 in. pipe 5 lb per sq. in.   Thus, in a pipe-line 8 in. diameter and 70 miles in length, the pressure of the gas leaving the pumps is 210 lb. per sq. in., and at the delivery supply

end 41 lb. per sq. in. (abs.) ; the rate of flow being 221,000 cub' ft. per hour.

Coal gas is obtained by the distillation or carbonization of bituminous gas-coal in closed retorts, heated externally at high temperature to distil the hydrocarbons from the coal out of contact with the air, and yield crude coal gas which, when purified, is distributed in towns for lighting, heating, and power purposes. The quality and quantity of the coal gas per ton of coal vary greatly with the *composition* of the coal carbonized, as well as with the *temperature* of the retorts, and the *time* the distillation is allowed to continue.

Low temperature carbonization at 450° to 600° C. yields 2,000 to 4,000 cub. ft. of gas per ton of coal ; and high temperature, 900° to 1.100° C. yields 10,000 to 13,000 cub. ft. of gas per ton of coal.

The gas formed at low temperature is rich in hydrocarbons, and the semi-coke contains up to 15 per cent volatile matter. The gas made at 950° C. consists of the combustibles H_2, 40 to 50 ; CH_4, 25 to 35 ; other hydrocarbons, 2 to 5 ; and CO, 5 to 10 per cent, and has calorific value 500 to 600 B.Th.U. per cub ft.

The scarcity of cannel and rich gas coal for admixture with ordinary bituminous coal in the retorts, led to the use of hydrocarbons from oils to enrich the simple " straight " coal gas. The latter is also mixed with carburetted *water gas*, produced by the action of steam on incandescent coke and enriched by the products from the vaporization of cheap hydrocarbon oils. This mixed gas contains a larger percentage of carbon monoxide. Table VI gives examples of coal gas used in special research work on gas engines.

TABLE VI

COMPOSITION AND CALORIFIC VALUE OF COAL GAS

	Hydrogen (H)	Methane (CH_4)	Carbon Monoxide (CO)	Heavy Hydrocarbon	Carbon Dioxide (CO_2)	Oxygen (O)	Nitrogen (N)	Calorific Value B Th U per cub. ft. at 60° F. and 14 7 lb. per sq. in.	
								Higher	Lower
London Coal Gas	42 8	27·8	11·3	5 4	—	0·1	12 0	645	580
London Mixed Gas	46 73	24 57	14 46	4·91	1 45	0 73	7·15	574	513
Cambridge	47·2	35 2	7 15	4 8	—	0 25	5 4	620	563
Birmingham	43·4	33 67	9 46	4·48	0·16	0 13	8·7	—	553
Ashton-under-Lyne	41·29	33·73	7 13	4·74	2 62	0 27	10 22	622	561
Newcastle	48·49	35 9	6 61	3 83	0 12	· ·	5 05	—	—
Carburetted Water Gas	29·35	20·49	33·19	11·32	—	—	5·66	—	570

The **calorific value of gaseous fuel** is determined by burning the gas in a calorimeter (see p. 235). In order to *calculate* the *calorific value* of a gaseous fuel, average samples of the gas are taken, and

complete chemical analysis made to find its composition. By reckoning the sum of the calorific values of all the constituent combustible gases when burned separately as free gas, an estimate is made of the amount of heat evolved by the complete combustion of a cubic foot of the gaseous mixture, and in the subsequent cooling of the burnt products. The complete analysis requires skilful chemical manipulation in the separation and determination of the quantity of the various *heavy hydrocarbons* present. The chemical analysis gives the volume or weight of each gas present in the compound or mixture, but affords no clue as to the chemical constitution, or the way in which the elements are held together, or how the constituent gases interact during combustion at the high temperatures in the cylinder of the internal combustion engine.

The late Professor Vivian B. Lewes pointed out that our knowledge of the specific hydrocarbons present in coal gas is so limited, and the exact determination of the quantity of each present so fraught with difficulty, that no exact calculation of the calorific value from the analysis of the gas is possible, and direct calorimetric methods become necessary for accuracy.

An example or two will illustrate the method of calculation—

Example 11.   A coal gas gives the following volumetric analysis : H_2, 0·48 ; CH_4, 0·31 ; CO, 0·110 ; N_2, 0·020 ; CO_2, 0·035 ; C_2H_4, 0·025 ; O_2, 0·020. Calculate the higher calorific value of this gas, employing the usual carbon and hydrogen figures, viz. 1 lb. C, burning to CO, evolves 2,420 C.H.U. ; burning to CO_2, 8,080 C.H.U. ; 1 lb. H_2, burning to H_2O, evolves 34,000 C.H.U.

State the assumptions made in such a calculation. Compare the result with the value obtained when the following calorific values for the different constituents are used. Calorific values, C.H U. per cubic foot. H_2, 191 ; CH_4, 575 ; CO, 188 ; C_2H_4, 930.

(*U.L., B.Sc. (Eng.),* 1921.)

Given the calorific values of C and H_2 per pound in order to estimate the calorific values of the hydrocarbons present, it is assumed that the calorific values of CH_4 and C_2H_4 are equal to the sum of the calorific values of the carbon and hydrogen present, as *free* elements, when burned separately, without knowing either the amount of heat energy used up in the decomposition of the gases or the interaction that may take place between the different gases in the process of combustion (pp. 203 and 221).

Taking the density of hydrogen at normal temperature and pressure equal to 0·00559 lb. per cub. ft., the weight of 1 cub. ft. of any gas is equal to half its molecular weight multiplied by 0·00559 lb.

Hence the density of $CH_4 = \dfrac{12 + 4}{2} \times 0.00559$, or 0·0447 lb per cub ft. ; and of $C_2H_4 = \dfrac{2 \times 12 + 4}{2} \times 0.00559 = 0.07826$ lb. per cub. ft. ; also $CO = \dfrac{12 + 16}{2} \times 0.00559 = 0.07826$ lb. per cub. ft.

Now 1 lb. of CH_4 consists of C $= \frac{12}{16}$ or $\frac{3}{4}$ lb., and hydrogen $\frac{1}{4}$ lb.,

so that these constituents, burning separately, evolve

$$\tfrac{3}{4} \times 8080 + \tfrac{1}{4} \times 34,000 \text{ C.H.U.} = 6060 + 8500,$$

or 14,560 C.H.U   Therefore the calorific value of 1 cub. ft. of CH_4, which weighs 0·0447 lb., is 14,560 $\times$ 0·0447 = 650 C H.U.

Similarly, 1 lb. of C_2H_4 contains carbon, $\frac{24}{28}$ or $\frac{6}{7}$ lb., and hydrogen $\frac{1}{7}$ lb., which, in burning, evolve

$$\tfrac{6}{7} \times 8080 + \tfrac{1}{7} \times 34,000 = 6926 + 4857,$$

or 11,783 C.H U., and 1 cub. ft. of C_2H_4 has calorific value

$$0·07826 \times 11,783 = 922 \text{ C.H U.}$$

Again, the weight of C in 1 cub. ft. of CO $= 0·07826 \times \frac{12}{28} = 0·03354$ lb.

∴ calorific value of CO

$$= ·03354(8080 - 2420) = 190 \text{ C H.U. per cub. ft.}$$

The calorific value of hydrogen

$$= 0·00559 \times 34,000 = 190 \text{ C.H U. per cub. ft.}$$

Hence the calorific value of this coal gas

$$= 0·48 \times 190 + 0·31 \times 650 + 0·11 \times 190 + ·025 \times 922$$
$$= 91·2 + 201·5 + 20·9 + 23·0$$
$$= 336·6 \text{ C.H.U. per cub. ft., at N.T.P.}$$

Using the calorific values given for the combustible constituents, the calorific value of the coal gas

$$= 0·48 \times 191 + 0·31 \times 575 + 0·11 \times 188 + 0·025 \times 930$$
$$= 91·68 + 178·25 + 20·68 + 23·25$$
$$= 313·86, \text{ or nearly } 314 \text{ C.H.U. per cub. ft.}$$
$$= 314 \times 1·8, \text{ or } 565 \text{ B.Th.U. per cub. ft.} \qquad Answer.$$

Example 12. A producer gas has the following percentage analysis by volume : Hydrogen, 16 ; carbon monoxide, 20 ; carbon dioxide, 6 ; nitrogen, 58. Determine, (*a*) its higher and lower calorific value per cubic foot at standard temperature and pressure ; (*b*) the minimum amount of air for complete combustion : (*c*) the volumetric analysis of the products, if combustion is complete. Calorific value of 1 lb. of carbon, burning to CO_2, is 14,500 ; burning to CO, 4,400 ; of hydrogen, 62,000 B.Th.U. Composition of air by volume : Oxygen, 21 ; nitrogen, 79 per cent. Volume occupied by 2 lb. of hydrogen is 357 cub ft at standard temperature and pressure.

(*U.L., B.Sc. (Eng.).*)

(*a*) Given the weight of 1 cub. ft of hydrogen $= \frac{2}{357} = 0·0056$ lb.

Density of CO is $\left(\frac{12 + 16}{2}\right) \times 0·0056 = 0·0784$ lb. per cub. ft.

Weight of carbon in 1 cub. ft. CO is $0·0784 \times \frac{12}{28} = 0·0336$ lb.

The calorific value of 1 lb. carbon burning from CO to CO_2

$$= 14,500 - 4400 = 10,100 \text{ B.Th.U.,}$$

and the calorific value of 1 cub. ft. of CO

$$= 0.0336 \times 10,100 = 339.36 \text{ B.Th.U.}$$

Again, the higher calorific value of H_2 is

$$0.0056 \times 62,000 = 347.2 \text{ B.Th.U. per cub. ft.}$$

Now 1 lb. H_2 forms 9 lb. steam, which gives out 970 B.Th.U. per lb. in condensing at 212° F. (100° C.), and, in cooling as water from 212° to 60° F., 152 B.Th.U., that is, 1,122 B.Th.U. per lb., hence 9 lb. of steam gives out $1122 \times 9 = 10.098$ B.Th.U.

∴ the lower calorific value of hydrogen

$$= 62,000 - 10,098 = 51,902 \text{ B.Th.U. per lb.}$$

and the lower calorific value of hydrogen

$$= 51,902 \times 0.0056 = 290.65 \text{ B.Th.U. per cub. ft.}$$

Hence the lower calorific value of the producer gas, by burning its content of H and CO.

$$0.16 \text{ cub. ft.} \quad 290.65 + 0.2 \times 339.36 = 46.5 + 67.87$$
$$= 114.37 \text{ B.Th.U. per cub ft}$$

and the higher calorific value is

$$0.16 \times 347.2 + 0.2 \times 339.36 = 123.4 \text{ B.Th.U. per cub. ft.}$$
Answer.

(b) $2H_2 + O_2 = 2H_2O$ and heat produced

2 volumes H takes 1 volume O to form 2 volumes of steam,

∴ 0.16 cub. ft. H takes 0.08 cub. ft. O to form 0.16 cub. ft. of steam.

Also $2CO + O_2 = 2CO_2$ and heat evolved.

2 cub. ft. H takes 1 cub. ft. O to form 2 cub. ft. CO_2

∴ 0.2 cub. ft. H takes 0.1 cub. ft. O to form 0.2 cub. ft. CO_2

Hence the oxygen required for complete combustion of 1 cub. ft producer gas is

$$0.08 + 0.1 = 0.18 \text{ cub. ft.} ;$$

and the minimum amount of air necessary to supply this oxygen is

$$0.18 \times \frac{100}{21} = \frac{6}{7} \text{ cub. ft.} \qquad\qquad \textit{Answer.}$$

(c) The products from the combustion of 1 cub. ft. of producer gas are—

		cub. ft.			%
Steam (H_2O)		$= 0.16$	. .	$\dfrac{.16}{1.677}$: 100 $=$	9.53
$CO_2 = 0.2$ cub. ft. 0.06 cub. ft. in gas		$\Big\} = .0.26$	. .	$\dfrac{.26}{1.677}$ $\times$ 100 $=$	15.51
N_2 in air supply $= \dfrac{6}{7} \times \dfrac{79}{100} = 0.677$ N_2 in gas itself $= 0.58$		$\Big\}$ 1.257	. .	$\dfrac{1.257}{1.677}$ $\times$ 100 $=$	74.96
$\therefore$ Total Products		$= 1.677$			100.

Answer.

A **Gas Producer** converts solid fuel into combustible gas. When 1 lb. of carbon is burnt in air or oxygen to CO, the heat liberated is 2,420 C.H.U., and, by completely burning this gas to CO_2, additional heat, 5,660 C.H.U., is liberated, making the total heat of combustion of 1 lb. solid carbon 8,080 C.H.U. Thus the heat available in the CO gas, and evolved by burning it, is 70 per cent of the total heat obtained by completely burning the solid carbon. The remaining 30 per cent of the total heat of the carbon is evolved and goes to raise the temperature in the fuel-bed.

Rhead and Wheeler showed that when dry air or oxygen is passed over purified wood-charcoal or incandescent carbon, the primary product is a complex of carbon and oxygen, which rapidly decomposes into a mixture of both CO and CO_2 in proportions which depend upon the temperature. Karl Wendt found by drilling holes through the wall of a gas producer, working with an air blast upon a fuel-bed of coke 7 ft. 6 in. in depth, that the gas at the fire-grate consisted of CO_2, 15 ; CO, 9.7 ; and N_2, 75.3 per cent, and at 10 in. above the grate the temperature was 1400° C., and the gas at that level consisted of CO, 34.1 ; CO_2, 0.2 ; and N_2, 65.7 per cent.

There are two ways by which solid carbon may be converted into CO gas.

With excess of carbon above 1200° C., or insufficient oxygen, by the reaction—

$$2C + O_2 = 2CO \text{ and heat liberated} \qquad . \qquad . \qquad . \qquad . \qquad (1)$$
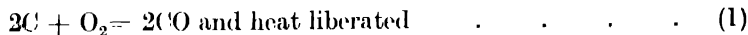
that is, 24 lb. of C and 32 lb. of O_2 form 56 lb. of CO, and the heat evolved is 24 × 2420 C.H.U.

Thus, 1 volume of oxygen combines with carbon to form 2 volumes of CO gas. If all the oxygen in the air is used up in forming CO, the nitrogen in the air with 1 volume of oxygen is $\dfrac{79.1}{20.9}$, or 3.785 volumes, and the producer gas will consist of 2 volumes of CO

mixed with 3·785 volumes of nitrogen. The composition of the gas is—

$$N_2 = 3\cdot785 = 65\cdot4 \text{ per cent by volume}$$
$$CO = 2\cdot0 \quad 34\cdot6 \quad ,, \qquad ,,$$
$$\overline{} \quad \overline{}$$
$$5\cdot785 \quad 100\cdot0 \quad ,, \qquad ,,$$

The composition by weight will be the same as by volume, since CO and N_2 are of the same density

For the complete combustion of carbon with free oxygen in the lower layers of the fuel bed, the reaction is

$$C + O_2 = CO_2 \text{ and heat liberated} \qquad . \qquad . \qquad . \qquad . \qquad (2)$$
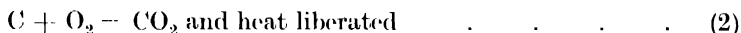
that is, 12 lb. of carbon combined with 32 lb. of O_2 give 44 lb. CO_2 and liberate 12×8080 C.H.U.

Again, this CO_2 may be reduced by carbon above 1200° C.,

$$CO_2 + C = 2CO \text{ and heat absorbed } 39,000 \text{ C.H.U.} \qquad . \qquad . \qquad (3)$$
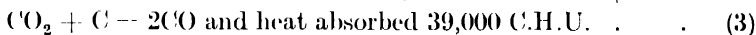
This double process gives the final result, from 24 lb. of carbon,

$$C + O_2 + C = 2CO + (97,000 - 39,000) \text{ or } 58,000 \text{ C H U},$$
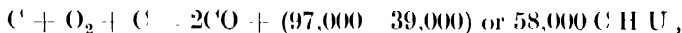
as in equation (1).

All the oxygen in the air blast is quickly used up near the fire-bars and the mixture of CO_2 and CO formed in contact with the carbon, follows the reversible reaction, depending on the temperature,

$$C + CO_2 \rightleftharpoons 2CO$$

From 850° C. upwards, CO_2 is reduced to CO, and very little CO_2 remains above 1200° C.; whereas, from 250° to 500° C., the reaction is reversed and the products are CO_2 and C to the left.

By the conservation of energy, the amount of energy required to decompose a compound substance is equal to the energy evolved in its formation.

The 30 per cent (2,420 C.H.U. per lb) of the total heat of the carbon evolved in its partial combustion to form CO, goes to keep the fire alight and maintain the high temperature necessary. About 8 per cent is lost by radiation and conduction, and a large proportion will raise the sensible heat of the gas. But the high temperature in the producer may become excessive, fuse the ash and slag, and form clinker, which may cause serious trouble by blocking the air passages. In order to avoid too high a temperature and to increase the efficiency of the producer by reducing the 30 per cent loss, part of the sensible heat may be used to generate steam to be added to the air blast. By the admixture of steam with the air passed into the producer, heat is absorbed in the decomposition of the steam, the oxygen of the steam combines with the carbon, and free hydrogen is added to the gas, while the temperature of the producer is under control by the proportion of steam in the blast.

The gas formed in a producer burning carbon in a mixed blast of air and steam will consist of a greater proportion of the combustible gases H_2 and CO to the total volume of the producer gas than when the blast is air alone, and therefore the calorific value of gas will be higher.

Water gas is produced by the interaction of superheated steam with incandescent carbon, and consists of a mixture, in nearly equal parts by volume, of the combustible gases hydrogen and carbon monoxide. Experiments by Dr Bunte give the proportion of steam decomposed and the composition by volume of the gases formed at various temperatures—

Temperature Degree C.	Percentage of Steam Decomposed	Percentage Composition by Volume of Gas Formed		
		Hydrogen (H_2)	Carbon Monoxide (CO)	Carbon Dioxide (CO_2)
674°	8 8		4·9	29·8
758°	25·3	5·2	7·8	27·0
838°	41·0	1·9	15 1	22·9
954°	70 2	3 3	39·3	6 8
1010°	94 0	48·8	49·7	1·5
1060°	95 0	0 7	48·0	1·3
1125°	99·4	·0·9	48·5	0·6

The reaction between steam and carbon commences about 500° C. and below 600° C only 8 per cent of steam is decomposed, and the products are principally carbon dioxide and hydrogen, i.e. the reaction is

$$C + 2H_2O = CO_2 + 2H_2 + 12 \times 8080 - 4 \times 29{,}000 \text{ C.H.U.} \quad (4)$$
$$= CO_2 + 2H_2 - 19{,}040 \text{ C.H.U. absorbed}$$
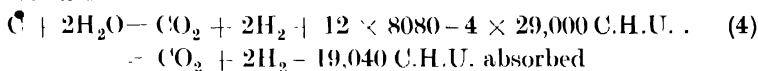
12 lb. C. decompose 36 lb. steam, absorbing heat.

As the temperature is raised, the CO_2 is reduced in the presence of carbon to CO, until above 1000° C., the ideal reaction of carbon and steam in the manufacture of *water gas* gives CO and H_2 in equal volumes, thus—

$$C + H_2O = CO + H_2 + 12 \times 2420 - 2 \times 29{,}000 \text{ C.H.U.} \quad (5)$$
$$= CO + H_2 - 28{,}960 \text{ C.H.U.}$$
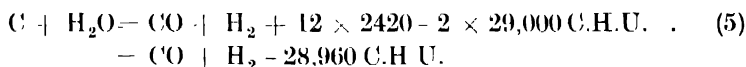
12 lb. C decompose 18 lb. steam, absorbing nearly 29,000 C.H.U from the heat of the incandescent fuel in the producer.

The mixture of gases formed by the reactions (4) and (5) also interact, thus—

above 1000° C, $CO_2 + H_2 = CO + H_2O - 10{,}000$ C.H.U. absorbed.

at 500° C, $CO + H_2O = CO_2 + H_2 + 10{,}000$ C.H.U liberated,

and the reaction is reversible—

$$CO + H_2O \rightleftharpoons CO_2 + H_2 \pm 10{,}000 \text{ C.H.U.}$$
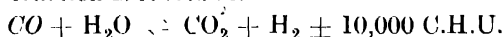
In the practical working of a gas producer, these gases undergo a chemical change, depending on the temperature, which results in a mixture of all four gases. Generally, increase of temperature favours the formation of CO, giving a richer gas and higher efficiency, while at low temperatures a higher percentage of CO_2 is obtained.

At any temperature above $500°$ C. the ratio $\dfrac{CO \times H_2O}{CO_2 \times H_2} = K$, the "equilibrium constant," increases with the temperature.

Oscar Hahn found the values of K to change from about 1 to 2·49 at various temperatures between $800°$ C. and $1400°$ C.

The manufacture of *water gas* from coke is *intermittent*; by alternate air blast or blow during 1 min. to raise the temperature of the coke to incandescence, followed by the superheated " steam blow " about 4 min., when the ideal gas by reaction (5) is collected until the temperature is reduced too low. Then the " air blow " is repeated, and the " steam blow " passed alternately up and down through the incandescent fuel.

Carburetted Water Gas. In the *Lowe* plant, the process is to produce water gas from coke in the gas *generator*, to carry illuminating hydrocarbon gases from heavy oil heated in the *carburetter*, and to gasify or make permanent gas of the oil vapour mixture in a *superheater*.

Dowson Gas. To Mr. J Emerson Dowson is due the introduction, in 1878, of a compact plant for making producer gas from anthracite or coke, cleaning and cooling the gas produced to drive gas engines. In 1881, at the York Meeting of the British Association, Mr. Dowson showed his gas producer plant working a 3 h p. Crossley-Otto gas engine.

Dowson gas is made by a jet of high pressure superheated steam which, acting as an injector, draws in a current of air, and the mixture of air and steam is forced up through red-hot anthracite or coke in a generator. Not only is the steam decomposed, but a sufficiently high temperature is maintained to carry on the process *rapidly* and *continuously*. The gas passes off near the top of the generator into the cooler, washer, coke scrubber, and sawdust purifier to the gas holder. *Pressure* producer gas made by this process led to the rapid development of the gas engine.

Producer gas is cooled before being supplied to the internal combustion engine, part of the sensible heat in the hot gas as it leaves the producer being utilized to generate the steam which is added to the air supplied to the producer. In this case the effective *efficiency of the producer* may be taken as the ratio of the net or lower calorific value of the cold gas obtained to that of the fuel from which the gas is made. When the gas producer is considered alone as a means of converting solid fuel into combustible gas, the ratio of the higher calorific value of the gas as it leaves the producer to that of the solid fuel, including the fuel used to raise the necessary

steam added to the gas supply, is the *thermal efficiency* of the producer.

Theoretical Weight of Steam Required per Pound of Carbon in a producer with mixed air and steam blast. The richest gas and maximum efficiency will be obtained by the ideal reactions (1) and (5). Assume no heat loss by radiation, and that all the sensible heat of the gases leaving the producer are used in raising steam utilized in the reactions, so that all the heat energy of the fuel is available. Then the amount of heat absorbed by the interaction of steam and carbon will be equal to the heat evolved by the formation of CO from carbon with the oxygen in the blast.

By reaction (1) in the direct combination of 24 lb of carbon with oxygen to form CO, the amount of heat liberated is $24 \times 2,420$. or 58,080 C.H.U.

By (5), above 1000° C., every 12 lb of carbon decompose 18 lb. of steam, and the heat absorbed is 28,960 C.H.U. Since H_2O is supplied to the producer as water, not as steam, and taking the temperature of water supply 17° C., the amount of heat required to raise 1 lb. of this feed water to 100° C., and generate 1 lb. of steam at atmospheric pressure, is $(100 - 17) + 539 = 622$ C.H.U.

Therefore 18 lb. of steam require 18×622, or 11,200 C.H.U

Hence the reaction of steam with 12 lb. of carbon absorbs $28,960 + 11,200$, or 40,160 C.H.U. ; and the weight of water needed to take up the heat evolved in reaction (1) is $18 \times \dfrac{58,080}{40,160} = 26$ lb

The carbon required for this weight of H_2O will be, by reaction (1) 24 lb., and by reaction (5) $12 \times \dfrac{26}{18} = 17\cdot34$, or the total carbon, 41·34 lb.

∴ weight of water to form the steam decomposed per lb. of carbon is $\dfrac{26}{41\cdot34} = 0\cdot63$ lb.

In case it is necessary to work at the lower temperature, for the recovery of the nitrogen in the fuel as ammonia in the Mond type of gas producer, by reaction (4), 12 lb. of carbon, in decomposing 36 lb steam, absorb 19,040 C.H.U. + heat of formation of steam from water at 17° C., that is 36×622, or 22,390 C.H.U., or total heat 41,430 C.H.U.

∴ Weight of water required is $36 \times \dfrac{58,080}{41,430} = 50\cdot47$ lb. and the total weight of carbon is

$$24 + 12 \times \frac{50\cdot47}{36} = 24 + 16\cdot82 \text{ or } 40\cdot82 \text{ lb.}$$

∴ Weight of steam decomposed per lb. of carbon is

$$\frac{50\cdot47}{40\cdot82} = 1\cdot236 \text{ lb.}$$

Composition of the Ideal Producer Gas. By reaction (1), 24 lb. of carbon yield 56 lb. of CO gas. The weight of 1 cub. ft. of CO gas is 14×0.00559, or 0.07826 lb. ; hence 24 of carbon give

$$\frac{56}{.07826} = 716 \text{ cub ft. of CO at N.T.P.}$$

From reaction (5), 17.34 lb. carbon give

$$\frac{28 \times 17.34}{12 \times 0.07826} = 517 \text{ cub. ft. of CO at N.T.P.}$$

making the total yield of CO, 1.233 cub. ft.

Also, from the steam and 17.34 lb. C, the yield of hydrogen is

$$\frac{2 \times 17.34}{12 \times 0.00559} = 517 \text{ cub. ft. at N.T.P.}$$

Again, by (1), the total oxygen supplied is 32 lb.

$$\text{or,} \qquad \frac{32}{16 \times .00559} = 357.8 \text{ cub. ft. at N.T.P.}$$

and, since air contains 79.1 per cent by volume of nitrogen, the total nitrogen in the air with 357.8 cub. ft. of oxygen is

$$357.8 \times \frac{79.1}{20.9} = 1.354 \text{ cub. ft. at N.T.P.}$$

The gas formed will be made up of

CO	1,233 cub. ft.	39.73 per cent	Combustible gases
H_2	517 ,,	16 65 ,,	56.4 per cent
N_2	1,354 ,,	43.62 ,,	
Total	3,104	100.00	

having higher calorific value $= 107.3$ C.H.U. per cub. ft.
and its lower calorific value $= 101.7$ C.H.U. per cub. ft.

The proportion of steam to the air supply controls the temperature of these reactions, as well as the yield of gas and its composition.

In each case there is a certain proportion of air and steam in the blast which gives the best thermal efficiency. In the *Dowson pressure type of gas producer*, with superheated steam drawing in a suitable proportion of air, the steam decomposed may vary from 0.5 to 0.75 lb. per lb. of fuel gasified. If the steam supply is reduced below 0.45 lb. per lb. of carbon, the temperature may rise too high and tend to form clinker, which adheres to the fire-brick lining and fire-grate of the gas generator, giving bad working conditions and increase of heat losses. On the other hand, when the proportion of steam in the air blast is increased much above 0.75 lb. per lb. of carbon gasified, the temperature falls too low, so that the steam is not all decomposed, and the thermal efficiency falls.

Karl Wendt determined the temperature and actual composition of the gas at different levels of the fuel-bed, 7 ft. 6 in. in depth, in a gas producer with coke and a mixed air and steam blast saturated at 60° C, as follows—

RESULTS OF EXPERIMENTS WITH MIXED AIR AND STEAM BLAST
IN A GAS PRODUCER (*Professor Bone*)

	Height above Tuyères Inches	Temperature ° C.	Percentage Composition of Gas				
			CO_2	CO	CH_4	H_2	N_2
Interaction of Air and Steam with incandescent coke	0	–	11·4	—	– ·	—	79·1
	10	1110°	9·3	22·0	0·4	10·6	57·7
	20	· ·	5·5	28·0	0·9	13·7	51·9
Distillation of the Coal	30	925°	3·0	32·7	1·2	17·9	45·2
	40	· ·	5·0	28·7	5·0	21·8	39·5
	50	810°	6·0	28·3	4·8	20·7	40·2
	60	· ·	5·3	28·0	4·1	19·0	43·6
Gas Outlet		440°	5·5	26·8	3·4	14·6	49·7

At 0″ above the grate there was 9·5 per cent of free oxygen. These results show the effects of steam mixed with air in the blast, compared with air alone (p. 245) ; at the same levels : the temperature of the fuel is lower, the proportion of CO_2 and H_2 increased, while CO is decreased.

A gas producer consists of a cylindrical vessel, lined with fire-brick, with an outer casing of iron, and containing the fuel, which is kept incandescent by an upward draught of mixed air and steam forced through it. The fuel usually rests at the bottom, on the fire-grate, and is fed into a bell hopper fitted on the top, and the gas outlet is at one side near the top of the producer.

The conditions of working vary with the depth of the fuel-bed, which must be sufficient to allow the necessary reactions to take place, and this depends on the nature of the fuel and size of the lumps used ; also upon the velocity of the blast to keep up the desired temperature. Anthracite, clean gas coke, and non-caking bituminous coal are most suitable, and are taken in small pieces, like " washed nuts," which expose a large surface, so that the gases are brought into intimate contact with the fuel and there is uniform distribution of the blast through the fuel bed, and not along channels.

The chief cause of difference between the ideal theoretical results and those obtained in the actual working of producers, assuming no loss of heat, is that the fuel contains not only carbon but also volatile substances which are driven off as vapours when the fuel is heated. A fresh charge of coal in the top layers below the hopper of a producer first undergoes a process of distillation evolving H_2, CH_4, tarry hydrocarbons, steam, etc. The carbonized coal or coke

sinks down gradually to the hot zone. The blast, on entering the bottom layers, soon loses its oxygen, and the interactions between CO_2, CO, steam, and the incandescent carbon take place in the hot zone, while the gases formed mix with the volatile contituents higher up and are enriched.

The crude producer gas is washed, cooled, and cleaned by different kinds of scrubbers, wet coke and sawdust, to get rid of tar, before it is fit for use in the gas engine.

Mond Gas. Dr. Ludwig Mond overcame the difficulties in making gas suitable for gas engines from non-caking bituminous slack, and recovered a valuable by-product, ammonium sulphate. The main distinctive features of the Mond process* are—

A large excess of superheated steam in the air blast keeps the producer at a working temperature, low enough to check the formation of clinker and recover the ammonia, while the fuel is completely burned.

The crude hot gas and undecomposed steam from the producer pass into the inner concentric pipes of a *regenerator* to heat the incoming mixture of air and steam blown in the opposite direction through the outer pipes. The air and steam are further heated by passing down the annular space around the producer, between the fire-brick lining and the outer casing, to the fire-bars. The sensible and latent heat of the steam in the hot gas is also partially recovered.

The fuel fed from a large hopper undergoes distillation, and the volatile vapours driven off pass through the upper part of the hot zone of fuel to reduce the tarry products to fixed gas, and carbonizes the bituminous slack, which sinks into the body of the producer.

Professor Bone and Dr. R. V. Wheeler made a thorough investigation† of a slightly modified Mond plant, comprising two producers making gas equivalent to 2,500 B.H.P., with superheated blast, and saturated with steam at any desired temperature.

Ten trials were made, and each lasted throughout a full working week. The non-caking bituminous coal used was " Collins Green washed nuts," screened through a 1 in mesh, and as fed into the producers average samples gave, by analysis, 3 to 7 per cent moisture. The dry coal contained—

Carbon	. .	78·0
Hydrogen	. .	5 4	Volatiles at 900° C. — 36·0 per cent
Nitrogen	. .	1·4
Sulphur	. .	1·0	Calorific value ⎰ 13,900 gross
Oxygen	. .	10·0	B Th.U. per lb. ⎱ 13,375 net
Ash	. . . .	4·2

100·0

* See " Power Gas and Large Gas Engines for Central Stations," by H. A. Humphrey, *Proc. Inst. Mech. Engrs.*, 1901, Part I, p. 41.
† *Journal, Iron and Steel Institute*, 1907, I, p. 126 ; 1908, II, p. 206.

RESULTS OF TESTS ON MOND GAS PRODUCERS (*Bone and Wheeler*)

Average Depth of Incandescent Fuel	3 ft. 6 in.			7 ft.	
Average Rate of Gasification per hour	22·5 cwt.			11·5 cwt.	
Steam Saturation Temperature of Blast	45° C.	55° C.	60° C.	60° C.	80° C.
Percentage Composition of the Gas —					
Carbon Dioxide . . .	2·35	4·40	5·10	5·25	13·25
Carbon Monoxide . . .	31·60	28·10	27·30	27·30	16·05
Hydrogen	11·6	15·45	15·50	16·60	22 65
Methane	3·05	3 00	3·05	3·35	3·50
Nitrogen	51·40	49·05	49·05	47·50	44·55
Total Combustibles . .	..	46·55	45·85	47·25	42·20
Calorific Values of the Gas } Gross	180	180·8	178 7	185·6	169·5
B.Th.U. per cub. ft at 0°C. and 760 m.m. } Net .	170 5	169 1	166 9	173 0	154·3
Yield of Gas, cub. ft. at 0°C. and 760 m.m. per ton of Coal	133,700	132,700	135,000	138,250	147,500
Steam added to Blast, lb. per lb coal	0·2	0·32	0·45	0·45	1·55
Percentage Steam Decomposed	All	All	76·0	87·4	40·0
Thermal efficiency, including Steam for Blower Engine .	0·73	0·722	0·725	0·778	0·665

Conclusions from these Trials. With ordinary bituminous producer coal, 92·25 per cent of the total carbon in the fuel is converted into gas ; 6·5 per cent leaves the producer, either as dense tarry hydrocarbons or as soot ; and the remaining 1·25 per cent is lost in the ashes. With steam saturation temperatures of blast up to 55° C., the whole of the steam may be decomposed in passing through the incandescent fuel bed. As the blast saturation temperature is gradually raised beyond the limit of 55° C., a decreasing proportion of the steam introduced is decomposed.

Successive increments in the proportions of steam introduced in the blast, by reason of their increasing cooling effects upon the fuel bed, profoundly affect the reactions occurring therein, and, consequently, also the composition of the resulting gas. For the lower the temperatures throughout the fuel bed, the more will the interaction $C + 2H_2O = CO_2 + 2H_2$ displace the $C + H_2O = CO + H_2$ change, and the more also will the equilibrium in the two reversible systems $2CO \rightleftharpoons C + CO_2$ and $CO + H_2O \rightleftharpoons CO_2 + H_2$ shift towards the right hand.

The ratio of the net calorific value of the cold gas obtained to

that of the coal gasified did not vary very much with the blast-steam saturation temperature throughout the investigation. This ratio worked out between the limits of 0·744 and 0·773, with a mean value of 0·76.

The yield of ammonia as sulphate is reduced from 71·8 lb. to 39 lb. per ton of coal gasified, as the steam saturation temperature is reduced from 80° C. to 60° C., and the steam added to the air-blast through the fuel is changed from 1·55 lb. to 0·45 lb. per pound of coal, while both the calorific value of the gas and the thermal efficiency of the producer are increased.

The best furnace gas is obtained with a blast-steam saturation temperature of about 50° C.

The Mond plant was primarily designed to make " furnace " gas from bituminous coal at a low price, and the recovery of the ammonium sulphate on a large scale, for which it is admirably adapted. But for use in the gas engine, the gas must be free from tarry matter, and special scrubbing and rotary tar extractors are absolutely necessary.

Suction Gas Producer. The pioneer of the suction gas power plant, M. Léon Bénier, was the first to dispense with the steam boiler, gas holder, and air blower in making and supplying producer gas to the engine. Anthracite or coke is fed into the producer Steam is generated in the hollow circular fire-grate of the producer. There is an annular space between the inner fire brick lining and the outer casing around the producer for the mixture of steam and air to pass and get heated, on its way to the bottom of the fire.

A suction pump draws gas from the producer and delivers it into the gas engine cylinder, after air has been passed in to clear out the burnt products through the exhaust ports, which are uncovered by the motor piston at five-sixths of its working or explosion stroke The suction of gas from the producer reduces the pressure below atmospheric, and air flows in to mix with the steam or water vapour before it enters the fuel. The quantity of gas admitted to the pump is regulated by the engine governor.

In 1894, Professor Aimé Witz tested a Bénier suction gas plant. The engine was only 15 B H.P., and the producer suitable for 25 H.P. with anthracite. In one trial, with English anthracite of good quality, the calorific value of the gas was only 129 B.Th.U. per cub. ft. The engine developed 27·6 I.H.P., of which 13 H.P. was spent by the pump, leaving only 14·6 B.H.P., while the fuel used was 1·5 lb. per (metric) brake horse-power hour.

The idea was ingenious, and the next important advance was to do away with the Bénier suction pump, and to use the suction stroke of the engine piston to draw its charge of gas from the gas plant through an expansion box.

In a modern suction gas plant, up to 200 H.P., the **essential parts** are : (1) A cylindrical gas *producer*, lined with fire-brick, in which

the anthracite or gas coke is made into gas by the action of air and steam drawn through it by the suction stroke of the engine. (2) A *vaporizer*, around the upper part of the producer, evaporates the regulated water supply, which may be pre-heated in a Field tube by the hot gas leaving the producer, on its way to (3) a water-sprayed *coke scrubber*, for washing and cooling the gas. (4) An *expansion box*, which serves as a gas reservoir and reduces the pulsations due to the suction stroke. (5) *Water trap seals* and *hydraulic main* to prevent the gas getting back, and to form a safety valve.

Since the working pressure in the producer is below atmospheric, any leakage is of air inwards, and the only danger is loss of efficiency.

A suction gas plant is compact, simple, and easy to work, requires little attention and labour, and is economical in fuel consumption even for small powers of 20 or 30 H.P.

In a series of careful tests by Professor A. H. Gibson upon a " National " suction gas producer of rated capacity 30 H.P., and a National gas engine, with a varying degree of saturation of the air supply to the gas producer, the fuel used was gas coke containing moisture 9·1 to 12·5 per cent. The mean result of analysis of the dry fuel was : C, 81·75 ; H_2, 1·19 ; O_2, N and S, 8·79 ; and ash 8·27 per cent. The higher calorific value of the dried coke is 12,739, and the lower value 12,635 B.Th.U. per lb. On the average, 96·3 per cent of the carbon in the fuel was converted into permanent gas, and 3·7 per cent lost in the form of tar, soot, or ash. With saturation temperatures up to about 125° F (52° C.), the whole of the water vapour supplied to the furnace was decomposed, corresponding to 0·4 lb. of water vapour per lb. of dry fuel. Air saturated with steam at 51·3° C. gave the richest gas, the maximum hydrogen content 13·1 per cent, and of highest calorific value, also the maximum thermal efficiency, 78·6 per cent actually obtained.

The composition of this suction gas was : CO_2, 5·10 ; O_2, 0·75 ; CO, 25·45 ; CH_4, 0·28 ; H_2, 13·10 ; N_2, 55·32 ; and its lower calorific value 127 B Th.U. per cub. ft. at 0° C. and 760 m m.

The mean negative pressure on the piston during the suction stroke varied from 0·5 to 1·0 lb. per sq. in., and the average power absorbed in drawing the gas through the producer is 0·59 I.H.P., adding to this a correction for loss of available heat in the pre-heated vaporizer water reduces the maximum effective efficiency of the gas producer to 76·6 per cent. As the saturation temperature is further increased, the weight of vapour decomposed is increased to 0·457 lb. per lb. of dry fuel, but the thermal efficiency of the producer decreases.

So long as the amount of excess air is the same, the composition and calorific value of the gas has practically no effect on the thermal efficiency of the engine, which reaches a maximum with about 160 per cent of excess air.

The overall thermal efficiency attains a maximum when the vapour supply amounts to about 0·5 lb. of dry fuel, being greater than the weight, 0·4 lb. per lb., for maximum efficiency of the gas producer. The lowest coke consumption per brake horse-power hour is 1·11 lb., of lower calorific value 12,635 B.Th.U. per lb., and gives an overall combined thermal efficiency of about 18 per cent.

The author made a series of tests upon a Hornsby-Stockport suction gas plant of 96 H.P., and rated at a daily working load of 85 B.H.P. The *higher* calorific value of the fuel, as used, was determined in the Mahler-Cook bomb calorimeter ; that of ordinary gas coke was 12,500 B.Th.U. per lb. The anthracite was " washed peas," from the Great Mountain Colliery Co., of South Wales, and gave a higher heating value of 14,580 B.Th.U. per lb.

In each test the plant worked 8 hours, under ordinary conditions, and during the last 6 hours the fuel consumption, indicated and brake horse-power, were determined. When anthracite was used in the producer the engine developed 102 B.H.P., with a considerable reserve of power or overload capacity, and the fuel consumption was 0·726 lb. per brake horse-power hour. The thermal efficiency of gas producer and engine combined was 27·5 per cent. The mechanical efficiency of the engine was 87·3 per cent ; so that ·873 × 0·275 — 0·24, or 24 per cent of the heat energy of the fuel was converted into work on the brake.

When coke was used in the same producer, the engine running at 87·3 B.H.P., the fuel consumption was 0·92 lb. per brake horse-power hour ; that is, an overall combined efficiency of about 22 per cent on the brake.

Suction Gas Producer using Non-caking Bituminous Coal. Various devices are adopted to free the gas from tarry products and burn bituminous coal completely in the producer.

The Dowson producer of this type has a down draught of air in the top part of the fuel bed, and an up draught of air and steam through the fire-grate in the lower part. The gas outlet is about half-way down. The fresh coal is fed in at the top, where the fire burns downwards. The hydrocarbon vapours, distilled off, pass down through incandescent fuel, and are either burnt with the air or converted into fixed gases. The carbonized fuel formed in the upper part sinks slowly down and is converted into producer gas by the upward current of steam and air. The hot gas carries no tar when it leaves the producer and is passed through a tubular vaporizer to cool the gas and generate the steam required. The gas is further cooled and cleaned by passing through scrubbers before passing to the engine.

Trials, by the author, of a 500 H.P. Dowson suction gas producer plant using bituminous Kingsbury coal, at quarter, half, and full load, extended over a week. The usual tarry matter was effectively removed, and the gas leaving the producer was as clean as that from

anthracite. This suction gas plant gave excellent results as to steadiness of running and reliability under the various loads.

Dowson suction producer gas, from bituminous coal in a 700 H.P. plant, gave percentage analysis by volume : H_2. 16 ; CH_4, 1·0 ; CO, 23·9 ; CO_2, 7·2 ; N_2, 51·9 ; of lower calorific value 138 B.Th.U. per cub. ft.

The composition of producer gas from anthracite and gas coke is given as follows—

ANALYSIS BY VOLUME OF DOWSON PRODUCER GAS

	Pressure Gas* Mean of 7 Plants from Anthracite (Hot Start) (Dowson and Larter)	Suction Gas† from	
		Anthracite (Hot Start) (M. A Adam)	Gas Coke (Hot Start) (A. T. Larter)
	Per Cent	Per Cent	Per Cent
Hydrogen (H_2) . . .	17·36	15 64	13 2
Methane (CH_4) . . .	1 20	1·16	0·35
Carbon Monoxide (CO)	25·55	20·13	25 3
Carbon Dioxide (CO_2) .	5·77	6 09	5·4
Oxygen (O_2) . . .	0·30	0·74	0·6
Nitrogen (N_2)	49·82	56·24	55·15
Calorific Value Higher	161	135·3	136·2
B Th.U per cub. ft. Lower	145	124·8	127·5
Yield of Gas, cub. ft. per ton Fuel	180,000	208,000	182,700
Air required for Combustion of 1 cub. ft. of Gas, in cub. ft.	1·1222	0·927	0·921

The Welsh anthracite used by Mr. Adam was "washed peas," and had calorific value in the bomb calorimeter 13,890 B.Th.U. The calorific value of the coke was 12,477 B.Th.U. per lb.

The **Cambridge** bituminous suction gas producer is of the down draught type and is fitted with an air jacket around the upper part, from which heated air is drawn into the fire through adjustable air ports about half-way down. The fresh fuel fed into the producer at the top is partly burned in the upper part of the fuel-bed, giving off its volatile constituents. The extra air drawn in through the ports gives an intensely hot zone through which the volatile matter from the fuel is drawn by the suction of the gas engine, and the tar converted into useful fixed gas.

The revolving crusher fire-grate prevents clinker deposit, and there is a water bottom below the grate for drawing out the ashes and supplying water vapour. A water-sprayed coke scrubber is used to wash and cool the gas.

* *Proc. Inst. C.E.* (1901), Vol. CXLIV, p. 282.
† *Ibid.* (1904), Vol. CLVIII, p. 322.

Bituminous coal, lignite, brown coal, and wood fuels are converted into good, clean gas by this plant in Australia and New Zealand. A trial run with lignite of percentage analysis : fixed carbon, 31·8 ; hydrocarbon, 44·82 ; water, 23·15 ; ash, 3·2 ; sulphur, 0·41 : gave fuel consumption about 1·1 lb. per brake horse-power hour. Using Green Island nuts (lignite), the coal consumed is about 1·35 lb. per brake horse-power hour. The chief feature of this suction gas plant is that it makes gas, suitable for power purposes, from various sorts of waste wood in shavings and blocks.

Example 13. A gas engine uses producer gas which has a volumetric analysis · CO, 11 , H_2, 29 ; CH_4, 1 8 ; CO_2, 16 1 ; N_2, 42·1. Calculate the volume of air required to completely burn a cubic foot of this gas.

If the air supplied to the gas engine be 50 per cent in excess of this quantity, what is the calorific value of a cubic foot of cylinder mixture ? Calorific values of CO, H_2, and CH_4 are 190, 162, and 535 C.H.U. per cubic foot respectively. What is the gas consumption per horse-power hour if the engine efficiency is 23 6 per cent ?

<div align="right">(C.L , B Sc. (Eng.), 1922.)</div>

The oxygen required for complete combustion of 1 cub. ft. of producer gas is—

For $CO = \dfrac{0 \cdot 11}{2}$ $= 0 \cdot 055$ cub. ft. (p. 198)

,, $H_2 = \dfrac{0 \cdot 29}{2}$ $= 0 \cdot 145$,,

,, $CH_4 = 2 \times 0 \cdot 018 = 0 \cdot 036$,,

    $\therefore$ total oxygen $= 0 \cdot 236$ cub. ft.,

and air contains 20·9 per cent by volume of oxygen

$\therefore$ air required $= \dfrac{\cdot 236 \times 100}{20 \cdot 9} = 1 \cdot 1292$ cub. ft.   *Answer.*

   and 50 per cent excess air $= 0 \cdot 5646$,, ,,

     making a total of $1 \cdot 6938$ cub. ft

The mixture of 1 cub. ft. of producer gas with excess air occupies 2·6938 cub. ft. in the engine cylinder, and the combustible content evolves in burning,

   $0 \cdot 11 \times 190 + 0 \cdot 29 \times 162 + 0 \cdot 018 \times 535$ C.H.U.

     $= 20 \cdot 9 + 46 \cdot 98 + 9 \cdot 63 = 77 \cdot 51$ C.H.U.

Hence the calorific value of a cubic foot of cylinder mixture is

$\dfrac{77 \cdot 51}{2 \cdot 6938} = 28 \cdot 77$ C.H.U   *Answer.*

Again, 1 H.P. hour $= 33,000 \times 60$ ft.-lb., and its thermal equivalent

$$= \frac{33,000 \times 60}{1400} = \frac{9900}{7} \text{ C.H.U}$$

If the thermal efficiency of engine is

$$\frac{23 \cdot 6}{100} = \frac{\text{Heat converted into work}}{\text{Heat supplied}}$$

$$= \frac{9900}{7} \times \frac{1}{77 \cdot 51 \times \text{cub. ft. gas}}$$

∴ gas consumption per horse-power hour is

$$\frac{9900 \times 100}{7 \times 77 \cdot 51 \times 23 \cdot 6} = \underline{77 \cdot 32} \text{ cub. ft} \qquad \textit{Answer.}$$

EXAMPLES VII

1. In a boiler trial the dry coal contained 84 per cent of carbon and 3 per cent of free hydrogen. The flue gas analysis gave 11·5 per cent CO_2, 8·4 per cent O_2, and 80·1 per cent of nitrogen. Calculate per pound of dry fuel, the weight of necessary air, and the weight of excess air.
(*U.L., B.Sc. (Eng.).*)

2. The volumetric analysis of a producer gas is : H_2, 14 per cent ; CH_4, 2 per cent ; CO, 22 per cent ; CO_2, 5 per cent ; O_2, 2 per cent ; N_2, 55 per cent. Find the air required for the perfect combustion of 1 cub. ft. of the gas. If 40 per cent excess air is supplied, find the volume analysis of the dry products. Air contains O_2, 20·9 per cent ; N_2, 79·1 per cent by volume.
(*U.L., B.Sc (Eng.).*)

3. The volumetric analysis of a producer gas supplied to a gas engine is : H_2, 20·2 ; CH_4, 2·8 ; CO, 22·2 ; CO_2, 7·7 ; N_2, 47·1 per cent. Calculate the volume of air necessary for complete combustion of 1 cub. ft. of the gas. If 50 per cent excess air is admitted, find the actual percentage contraction in volume after combustion in the engine cylinder, and the volumetric analysis of the dry products of combustion.

4. The ultimate analysis by weight of a petrol is : C, 0·835 ; H_2, 0·147 ; O_2, 0·018. Calculate the ratio of air to petrol consumption by weight, if the volumetric analysis of the dry exhaust gas is : CO_2, 12·1 ; CO, 1·7 ; O_2, 0·8 ; N_2, 85·4 per cent. (*U.L., B.Sc. (Eng.), 1925.*)

5. Analysis of coal in a boiler trial was : C, 88 ; H, 3·6 ; O_2, 4·8 per cent. ; and other matters 3·6 per cent. Flue gas by volume : CO_2, 10·9 ; CO, 1·0 ; O_2, 7·1 ; N_2, 81 per cent. Find (*a*) the proportion of C burned to CO ; and (*b*) air required per pound of fuel for combustion as it actually occurred ; also (*c*) the weight of flue gas per pound of fuel burned.
(*U.L., B.Sc. (Eng.).*)

6. In a boiler trial the percentage analysis of dry coal as burned was : C, 83 ; H_2, 4 ; O_2, 8 ; ash, etc, 5 ; and the volumetric analysis of the flue gas was : CO_2, 10 ; CO, 1·7 ; O_2, 8·1 ; N_2, 80 per cent. The rise in temperature of the flue gases was 290° C. Calculate the following items : (*a*) the proportion of C burned to CO, and the heat lost through incomplete combustion, expressing the latter as a percentage of the heat in the fuel ; (*b*) the heat carried away in the flue gas per pound of coal burnt, average specific heat 0·24 ; (*c*) heat carried away in excess air, average specific heat, 0·2375. Calorific value of 1 lb. carbon, when burnt to CO_2, 8,080 C.H.U., when burned to CO, 2,420 C.H.U. (*U.L., B.Sc. (Eng.).*)

7. During a boiler trial, the average analysis of the flue gases gave the following results : CO_2, 10·7 ; CO, 1·1 ; O_2, 6·85 ; N_2, 81·35 per cent. The chemical analysis of the coal as fired gave : C, 87·6 ; H_2, 3·4 ; O_2, 4·5 ; ash, etc., 4·5 per cent. Determine the number of pounds of air supplied to the boiler furnace per pound of coal ; also calculate the weight of flue gas per pound of coal.　　　　　　　　　　　　　　　　　　　　(U.L., B.Sc. (Eng.).)

8. An oil fuel of specific gravity 0·875 contained carbon 86·9 per cent, and hydrogen 13·1 per cent, tested in a bomb calorimeter, the following data were obtained : Weight of oil taken, 0·9034 gramme ; total weight of water, including water equivalent of bomb and calorimeter, etc., 2,800 grammes. Observed rise of temperature by combustion in the bomb calorimeter, 3·47° C., and loss by radiation, 0·025° C. Calculate the higher and lower calorific values of the oil fuel, (a) in C H U. per pound ; also (b) in B Th U. per pound, and (c) per gallon.

9. During an engine trial, the gas, as used, was tested in a calorimeter and gave the following results : Gas burned, 0·75 cub. ft., at pressure 14 8 lb per sq. in. and 18° C, which raised the temperature of 24·5 lb. of water from 15° C. to 23·5° C ; and the steam formed was condensed to 0·04 lb. of water at 18° C. Calculate the higher and lower calorific values per cubic foot of the gas at N T.P. Take the latent heat of steam as 539 C.H.U. per lb. at atmospheric pressure

10. In a gas producer 1 ton of coal, having calorific value 14,000 B.Th.U. per lb , yields 208,000 cub. ft. of gas of calorific value 125 B.Th.U. per cubic foot. A gas engine takes 67 8 cub ft. of this gas per brake horse-power hour. Calculate : (a) The thermal efficiency of the gas producer ; and (b) the brake thermal efficiency of the gas engine ; (c) the pounds weight of coal consumed per brake horse-power hour ; and (d) the overall thermal efficiency of the gas plant.

11. In a gas producer fed with bituminous coal of lower calorific value 13,400 B.Th.U. per lb., the yield of gas per ton of coal is 134,000 cub. ft. at N.T.P. of lower calorific value 172 B.Th U. per cubic foot. The percentage analysis by volume of the gas is : H_2, 18·3 ; CH_4, 3 4 ; CO, 25·4 ; CO_2, 6·9 ; N_2, 46 ; and a gas engine takes 49 9 cub ft. of this gas per brake horse-power hour. Calculate : (a) The cubic feet of air necessary for the complete combustion of 1 cub ft. of the gas ; (b) the thermal efficiency of the producer from the lower calorific value of the gas ; (c) the weight in pounds of coal used per brake horse-power hour ; (d) the brake thermal efficiency of the gas engine ; and (e) the overall efficiency of the power plant.

12. Find the maximum efficiency of a suction gas producer, the composition of the gas produced, and its calorific value per cubic foot, assuming that the fuel is carbon, and that only dry air is passed through the fuel. Given that 1 lb. of H_2 occupies 178·8 cub. ft , that the calorific value of CO is 342·4 B.Th.U. (190 2 C.H.U.) per cubic foot ; and that the calorific value of 1 lb. C. is 14,544 B.Th.U. (8,080 C.H.U.).

What is the effect of admitting steam in addition to the air (a) on the working ; (b) on the efficiency of the producer ?

　　　　　　　　　　　　　　　　　　　　　　　(U.L., B.Sc. (Eng.).)

13. In a test of a suction gas producer and gas engine under ordinary working conditions, the following results were obtained : Duration of trial, 6 hours ; gas engine cylinder, 17 in. diameter by 24 in. stroke ; average speed of engine, 209·5 r.p.m. ; explosions per minute, 102 ; mean effective pressure from indicator cards, 83 4 lb. per sq. in. Brake horse-power, 102·1. Total coal consumed, 444·7 lb., of higher calorific value 14,580 B.Th U. per lb. Work out : (a) Fuel per brake horse-power hour ; (b) indicated horse-power ; (c) mechanical efficiency of engine ; (d) thermal efficiency of gas producer and engine combined ; (e) percentage heat of fuel converted into work on the brake.

CHAPTER VIII

THE term internal combustion engine includes *gas, petrol,* and *heavy oil engines,* according to the fuel that is mixed with a suitable proportion of air, usually in excess, to form the charge in the cylinder. When the mixture is compressed and fired, the heat developed by the explosion in the cylinder raises the temperature and pressure of the products of combustion, with the excess air, which expand, driving the piston and doing useful work. The highest temperature reached by explosion of the charge in the *gas engine cylinder* has been estimated with a normal load at not-less than 1800° C., and usually 2250° C. to 2500° C in ordinary practice. Thus in the internal combustion engine the working substance receives heat by its own combustion up to a very high temperature, and although the heat rejected is also at a high temperature, the actual working range is so great that a gas or oil engine can convert into useful work about 30 to 35 per cent of the heat energy of the fuel.

From 25 to 35 per cent of the heat developed passes into the cylinder walls, pistons, and valves. This heat given to the metal has to be conveyed away as fast as it is generated within the cylinder, hence the necessity for the circulation of cooling water through the jacket around the cylinder, or other means of cooling. The rate of heat flow in the gas depends mainly upon its temperature, density, and turbulence. The temperature of the metal at any point is determined by the mean rate of heat flow into the surface per unit area, the conductivity for heat of the material, and the distance that the heat has to flow to the jacket water or place where it is conveyed away from the cylinder.

The heat flow from any point is maintained by a suitable temperature gradient, which may be 60° C. per inch. High local temperatures may be produced at the centre of the piston or exhaust valve in large gas engines, because the heat has farther to flow to the water jacket. The heat flow through the cylinder wall is partly radial to the jacket, also in an axial direction to cooler parts. The difference of temperature at places only a few inches apart may be 200° to 400° C. with heavy charges, and this produces stresses* in the metal at the top of the cylinder, close to the head, due to unequal expansion, which form a serious problem of great practical importance to the designer of large engines, and is mainly metallurgical. Also, if the temperature at any point in contact

* See *Proc. I. Mech. E.,* 1923, p. 1045; also 1925, p. 19.

with the combustible mixture becomes too high, there is the risk of premature ignition (pre-ignition) of the charge, which may stop the engine.

The most common type of internal combustion engine works on the **four-stroke constant volume cycle,** invented by **Beau de Rochas** in 1862, adopted and made a practical success by Dr. N. A. Otto in 1876, and first developed in this country by Messrs. Crossley Bros., Manchester, and later by many others. In a single-acting engine the cycle consists of four operations in the working cylinder on one side of the piston, during four consecutive strokes, giving one
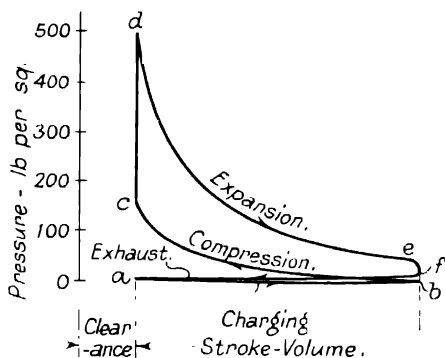
Fig. 86. Gas Engine Indicator Diagram

explosion or working stroke for every two revolutions of the crank-shaft. The action is shown by the indicator diagram, Fig. 86, from an engine of 40 B H.P.

Suction. During the first forward or *outstroke, ab,* of the piston a mixture of gas with excess of air is drawn into the cylinder, so that the cylinder is filled with the explosive mixture at atmospheric pressure.

Compression. During the return or *instroke, bc,* the mixture is compressed into the clearance space at the end of the cylinder.

Explosion and Expansion. At *c* the compressed charge is ignited just before the crank passes the dead centre, and the pressure due to the heat energy developed by the combustion at *constant volume* rises so rapidly that the maximum pressure is reached before the piston has moved appreciably on its second outstroke. This is the *working stroke, cde,* and the piston is driven forward by the expanding gas.

Exhaust (efa). The exhaust valve is opened at *e* to give " release," and during the second back-stroke the burnt products are discharged from the cylinder, except what remains in the clearance space. These burnt products mix with the next incoming charge of gas and air, and the cycle is repeated, except when the *governor* cuts off

the supply of gas ; then air is admitted, compressed, and expanded, without explosion, and discharged.

In some modern gas engines, *scavenging air* under slight pressure is admitted to the cylinder towards the end of the exhaust stroke, to sweep out the burnt products from the clearance space through the exhaust pipe, to cool the exhaust valve and leave the clearance space full of pure air.

Scavenging is specially desirable when working with gas of low heating value, such as blast furnace gas, containing a small proportion of combustibles which require to be well mixed with air to ensure complete combustion, retarded by the inert gases present. Usually there is no special arrangement for scavenging.
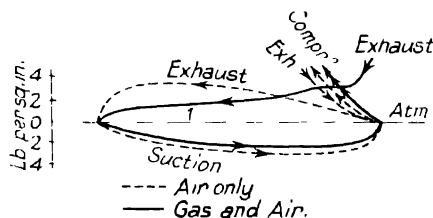
Fig. 87. Light Spring Diagrams of Pumping Strokes

Beau de Rochas first gave the **chief conditions** necessary to obtain the best results and greatest economy from the elastic force of gas in the engine cylinder—

(1) Largest ratio of cylinder volume to boundary surface, that is, reduce to a minimum all cooling surfaces in contact with hot gases, or make the combustion space as nearly spherical as possible.

(2) Piston speed as high as possible.

(3) Long range of expansion, within practical working limits, in order to have the heat converted into work rapidly.

(4) Highest possible compression of charge before ignition, which is only limited in practice by pre-ignition or spontaneous combustion.

The outstanding disadvantage of this cycle is, only one impulse for every two revolutions of the crankshaft, when the motor piston acts as a pump during the suction and exhaust strokes, although this type is most convenient for small powers and tends to keep the cylinder comparatively cool with good mechanical and volumetric efficiencies.

The light spring indicator diagrams, Fig. 87, taken from the same engine as Fig. 86, shows the bottom loop for the pumping strokes during exhaust and suction. The full line is the negative part of the diagram, Fig. 86, in which gas and air are taken ; and the mean pressure of this full load suction loop is 2·9 lb. per sq. in., corresponding to 1·4 H.P. at 180 r.p.m. The dotted lines form the diagram of the pumping strokes when the governor stops the gas

supply and only air is admitted to the cylinder. In this case the mean pressure of the loop is 5 lb. per sq. in., taking 2·5 H.P.; that is, more power than is required for the exhaust and suction strokes after an explosion. (See pp. 50 and 51.)

When the four-stroke cycle engine was introduced by Otto, inventors tried to overcome its defects by the **two-stroke compression cycle,** giving an explosion every revolution of the crankshaft, or one to and fro movement of the piston in the engine cylinder at full load.

The first invention or pioneer engine of this type appears to be that described in Patent No. 2334 of 1877, by James Robson, which was improved by his Patent No 4501 of 1879. In this engine the single cylinder is closed at both ends. The front end of the piston next the crankshaft is used as a pump to draw in and compress the mixture of gas and air, at 6 lb. per sq. in., into an intermediate reservoir below the cylinder, from which it flows through an inlet valve into the power end of the cylinder, and displaces the exhaust gases of the previous explosion through ports, when these are uncovered by the back end of the piston, near the completion of its working stroke. On the return stroke the piston covers the ports and then compresses the charge in that end of the cylinder, while a fresh charge is drawn into the front end. During the next outstroke, when the charge, compressed to 35 lb. per sq. in. is ignited on the back side of the piston, the next charge is being compressed by the front side of the piston into the intermediate reservoir

There is thus a compression stroke before firing the charge, which is followed by an explosion or expansion stroke, in this cycle, during one revolution of the crankshaft.

Only one engine of the 1877 type was made and used for driving machine tools in Robson's workshop at North Shields, until replaced by the 1879 engine

In 1881 the Robson " two-stroke cycle " invention was developed and introduced by Messrs. Tangye, Birmingham, and, in a slightly modified form, used in many small engines.

An experimental engine of the two-stroke cycle type was designed by Sir Dugald Clerk in 1878, and exhibited in 1879. There were separate motor and pump cylinders. The mixture of gas and air was compressed by the pump into a reservoir at 70 lb. per sq. in. above atmospheric pressure, and supplied to the motor cylinder through a slide valve. One serious difficulty was back ignition into the reservoir ; another difficulty was shock in the motor cylinder.

In 1886, Clerk built another engine* of this type, having no reservoir between the pump and motor, which gave better results.

The well-known type of Clerk engine, shown by drawings of the

* See Sir Dugald Clerk's book on *The Gas, Petrol, and Oil Engine*, Vol. II, p. 195.

Patent No. 1089 of 1881, had two separate cylinders, i.e. the pump
to draw in a mixture of gas and air, and deliver it into the motor
cylinder, where it was compressed by the motor piston before
ignition. The conical clearance space in the motor cylinder was
designed so that the flow of the fresh charge of gas and air entering
the cylinder gradually pushed the burnt gases through the exhaust
ports, when these were uncovered by the motor piston near the
end of the explosion stroke. But it was " hardly possible to avoid
direct loss of explosive mixture through the exhaust before com-
pression in the smaller engines."

FIG. 88. INDICATOR DIAGRAM. MOTOR CYLINDER

FIG. 89. INDICATOR DIAGRAM. PUMP CYLINDER CLERK
GAS ENGINE : 6 H.P. FULL LOAD

At the Crystal Palace Exhibition of 1882 a Clerk engine of 6 H.P.
was tested.* The indicator diagrams, Fig. 88 and Fig. 89, were taken
during the full load trial The chief results are—

Pump and motor cylinders, 6 in. diameter ; stroke of pump, 18 in., and of
motor piston, 12 in. ; 144·7 r.p.m. ; pressure before ignition 41 lb. per sq. in.
(gauge) ; mean effective pressure, 60·72 lb. per sq. in. ; net indicated horse-
power, 6·82 ; power spent in pump and engine friction, 3·04 ; brake horse-
power, 3·77 ; mechanical efficiency, 55·4 per cent ; gas used per brake horse-
power hour, 64·8 cub. ft. of calorific value 633 B.Th.U. per cub. ft. Heat
turned into useful work, 6·2 per cent.

Of the gross indicated power, 0·71 H.P. spent in the pump was
deducted to give the net available indicated power. Exhaust or
back firing was noted, as well as pre-ignitions. The loss in engine
friction was abnormal ; and the engine had given better results
beforehand in Glasgow with richer gas.

This type of two-stroke cycle appears to be best adapted for

* Inaugural address by Professor W. Grylls Adams, as President of the
Society of Telegraph Engineers and Electricians, 1884 ; also more details in
" State Papers—America—Information from Abroad- -Report on the exhibits
at the Crystal Palace Electrical Exhibition, 1882," by Frank J. Sprague,
issued by the Naval Department, U.S., Washington, 1883-4.

modern gas and oil engines of large output, as the Koerting ; and in the small improved Dolphin or Ricardo petrol engines, when the work taken to drive the pump is a much smaller proportion of the total generated, and the other difficulties are more easily overcome.

Ideal Air Standard. A Committee of the Institution of Civil Engineers, on the efficiency of internal combustion engines, recommended an ideal standard engine of comparison, having a cycle approximating to that of actual internal combustion motors, which should satisfy the following conditions—

(1) The reception and rejection of heat should take place as nearly as may be in the same way as in the actual engine.

(2) There should be no heat losses due to conduction, radiation, leakage, or imperfect combustion.

(3) The data for the numerical evaluation of the standard should be ascertainable by simple measurement.

(4) The expression for the efficiency of the standard should be a simple one.

(5) The standard engine should work with a perfect gas of the same density of air, and having no change of specific volume ; the specific heat to be assumed constant for all working temperatures, and the value of the ratio of the specific heats of the ideal air at constant pressure and constant volume to be taken as $\gamma = 1\cdot4$.

It is known that the properties of the working fluid are different from those assumed for the ideal air, and the specific heat of air does increase at the high temperatures obtained in the engine cylinder, while that of the working fluid after combustion changes still more. Until the true law of variation of the specific heat with temperature is more exactly known, the air standard enables the efficiency of different engines to be compared, but does not show how close the performance of the actual engine is to perfection.

For internal combustion engines employing adiabatic compression and expansion, there are three symmetrical types of thermodynamic cycles which are each cycles of maximum efficiency for the conditions assumed

(1) Constant temperature, or the Carnot cycle.

(2) Constant volume, as the Beau de Rochas cycle

(3) Constant pressure type.

(1) **The Carnot or Constant Temperature Cycle.** Heat is received and rejected by the working fluid at the constant higher and lower temperatures. By adiabatic compression, unit weight of the air is changed from state T_0, v_0 to T_1, v_1, Fig. 90, then an amount of heat Q_1 is given to the air during isothermal expansion to v_2 ; followed by adiabatic expansion to v_3, T_0.

During isothermal compression from v_3 to v_0, heat Q_2 is rejected.

Thermal efficiency is $\dfrac{Q_1 - Q_2}{Q_1} = \dfrac{T_1 - T_0}{T_1} = 1 - \dfrac{T_0}{T_1}$ (see p. 96).

In the adiabatic compression, $\dfrac{T_0}{T_1} = \left(\dfrac{v_1}{v_0}\right)^{\gamma-1} = \left(\dfrac{1}{r}\right)^{\gamma-1}$

where the compression ratio, before heat reception, is $r = \dfrac{v_0}{v_1}$.

Hence the ideal air cycle efficiency is $1 - \left(\dfrac{1}{r}\right)^{\gamma-1}$

When drawn to scale, the adiabatic and isothermal curves, Fig. 90, are close together, and the area of the ideal indicator diagram represents an exceedingly small amount of work for large range of pressure and volume.

A very bulky and heavy engine would be required to withstand
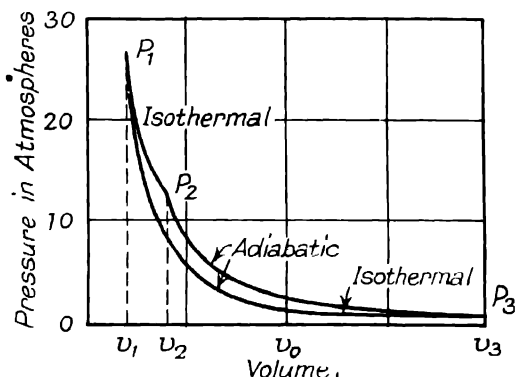
FIG. 90. CARNOT CYCLE

the enormous compression necessary to give only small power. The cycle is obviously unsuitable for an internal combustion engine.

In 1892, Herr Rudolph Diesel patented (No. 7421) a *Rational Heat Motor* to work on the ideal Carnot cycle, described in his book of 1893. The proposed " rational " motor was to use all kinds of fuel—solid, liquid, and gas—and to operate as follows : (*a*) Isothermal compression of excess air, the heat produced being carried off by water spray ; (*b*) adiabatic compression of the air to the highest temperature of the cycle far greater than the ignition temperature of the fuel, to 250 atmospheres in a pulverized coal engine, modified to " only " 90 atmospheres (1,300 lb. per sq. in.) ; (*c*) gradual and regulated injection of the fuel, such as pulverized coal, into the hot air to produce gradual *isothermal combustion*, so that the temperature will not exceed the compression temperature during expansion at *constant pressure* ; and (*d*) after the fuel cut-off *adiabatic expansion* of the surplus air and products of combustion would cool the gases to the lower limit of temperature, without artificial cooling of the cylinder walls, but, on the contrary, they should be

" lagged to protect against loss by radiation of heat. By special arrangement the exhaust temperature can be made lower than the atmospheric temperature, and can be used for refrigerating purposes." " The combustion, in opposition to all hitherto known processes of combustion, does not produce any increase of temperature, or at least only an unessential one, the highest temperature in the cylinder is produced by compression of air."

This first Diesel engine patent was based on a practically impossible thermodynamic cycle, and the difficulties were found too great without a water jacket. In 1895, air injection of paraffin oil
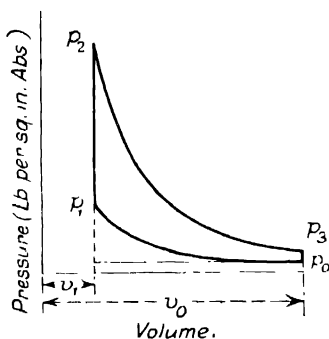
FIG. 91. CONSTANT VOLUME CYCLE.   FIG 91A
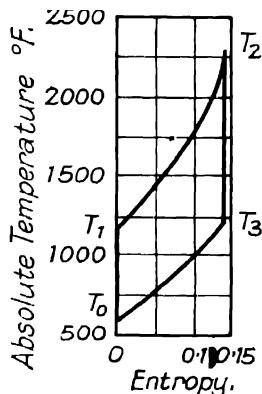
was adopted, at a pressure higher than that of adiabatic compression, which was reduced to 35 or 40 atmospheres, the isothermal compression by water spray cooling was abandoned, and external cooling by water jacket introduced (p. 275).

(2) **Constant Volume Cycle.** The ideal (pv) indicator diagram for this air standard cycle is shown in Fig. 91, with the state points marked 0, 1, 2, and 3 for pressure, volume, and temperature. Fig. 91A is the temperature entropy ($T\phi$) diagram for this ideal constant volume cycle. The value of $\dfrac{pv}{T_0}$ is found from the pv indicator diagram just before compression begins, and the temperature at any point of the pv diagram is calculated on the assumption that the specific heats of the ideal perfect gas remain constant throughout the cycle.

The entropy of the charge at T_0, reckoned from 32° F. or 492° F. (abs.), at constant pressure is $C_p \log_\varepsilon \dfrac{T_1}{T_0}$. The change of entropy, at different points on the compression and expansion curves, may be calculated by equation (18) (p 116). The entropy at points on

the constant volume lines, during explosion and exhaust, is calculated* by equation (16).

Take 1 lb. of working fluid, the heat supplied or generated by the explosion of the charge at *constant volume*, after adiabatic compression, is $C_v(T_2 - T_1)$, and the heat rejected at *constant volume* is $C_v(T_3 - T_0)$; the specific heat is assumed to be constant.

Hence, the thermal equivalent of the work done during the cycle is—

Heat received – heat rejected $= C_v(T_2 - T_1) - C_v(T_3 - T_0)$;
and the *ideal air standard efficiency* is

$$\frac{C_v(T_2 - T_1) - C_v(T_3 - T_0)}{C_v(T_2 - T_1)} = 1 - \frac{T_3 - T_0}{T_2 - T_1}.$$

Since the compression and expansion are assumed to be adiabatic

$$\frac{T_0}{T_1} = \frac{T_3}{T_2} = \left(\frac{v_1}{v_0}\right)^{\gamma-1}, \quad \text{from which} \quad \frac{T_3 - T_0}{T_2 - T_1} = \left(\frac{1}{r}\right)^{\gamma-1}$$

and the efficiency is $1 - \left(\dfrac{1}{r}\right)^{\gamma-1}$

where r = ratio of compression

$$= \frac{v_0}{v_1} = \frac{\text{Piston displacement + clearance volume}}{\text{Clearance volume}}$$

and γ is taken as 1·4, the value of the ratio $\dfrac{C_p}{C_v}$ for air.

The value of γ for the actual mixture in ordinary gas engines varies from 1·3 to 1·38, and is usually taken as 1·37 for the mixture of coal gas and air at the ordinary temperature; and becomes less for a mixture richer in gas.

The *ideal air cycle efficiency is reduced by an increase of the specific heats of the gases in the cylinder.*

Since $C_p - C_v = R$, and $\gamma = \dfrac{C_p}{C_v}$, $\gamma - 1 = \dfrac{R}{C_v}$

and the efficiency $e = 1 - \left(\dfrac{1}{r}\right)^{\frac{R}{C_v}}$,

that is, $1 - e = \left(\dfrac{1}{r}\right)^{\frac{R}{C_v}}$, or $\dfrac{1}{1 - e} = r^{\frac{R}{C_v}}$.

Differentiate with respect to C_v, and we have

$$-\frac{1}{1 - e} \cdot \frac{de}{dC_v} = \frac{R}{C_v^2} \cdot \log_\varepsilon r$$

$$\therefore \quad \frac{de}{dC_v} = -\frac{R(1 - e)}{C_v^2} \cdot \log_\varepsilon r = -\frac{R}{C_v^2}\left(\frac{1}{r}\right)^{\frac{R}{C_v}} \cdot \log_\varepsilon r .$$

* Graphical methods are given in *Proc. Inst. Mech. Eng.*, May, 1906, and February, 1908.

Hence $\quad de = -\dfrac{dC_v}{C_v{}^2}\left\{R(1-e)\log_\varepsilon r\right\}$

that is, $\quad \dfrac{de}{e} = \dfrac{-dC_v}{C_v}\left\{\dfrac{R}{C_v} \cdot \left(\dfrac{1-e}{e}\right) \cdot \log_t r\right\}$

$$= -\dfrac{dC_v}{C_v}\left\{(\gamma-1)_i\left(\dfrac{1-e}{e}\right)\log_\varepsilon r\right\}.$$

Suppose $\gamma = 1\cdot4$, and compression ratio $r = 5$, what is the fractional change of efficiency for 1 per cent increase in C_v? We

have, $\quad e = 1-\left(\dfrac{1}{5}\right)^{0\cdot4} = 0\cdot475,$

Then, $\quad \dfrac{de}{e} = -\dfrac{1}{100}\left\{0\cdot4 \times \dfrac{1-0\cdot475}{0\cdot475} \times 2\cdot303\log_{10}5\right\}$

$$= -\dfrac{1}{100}\left\{\dfrac{0\cdot4 \times 0\cdot525}{0\cdot475} \times 2\cdot303 \times 0\cdot699\right\},$$

$$= -\dfrac{1}{100}(0\cdot711) = -0\cdot711 \text{ per cent}$$

i.e. the efficiency would decrease $0\cdot711$ per cent when C_v is increased 1 per cent.

Greater compression of the charge before ignition increases the ideal thermal efficiency of the gas engine cycle, giving higher temperature and pressure, as well as greater expansion. The effect of increased compression is at once seen by the numerical values of

the ideal air standard efficiency $= 1 - \left(\dfrac{1}{r}\right)^{0\cdot4}$; also of those with

the lower values of $\gamma = 1\cdot37$ for the specific heats, supposed constant, of a mixture of gas and air, which are still more reduced when allowance is made for the increase of specific heat of the working mixture at the high temperatures reached in the gas engine cylinder.

THERMAL EFFICIENCY OF AIR-STANDARD CYCLE

Ratio of Compression	$\gamma = 1\cdot4$	$\gamma = 1\cdot37$	$1\cdot3$
3	0·356	0·334	0·281
4	0·426	0·401	0·340
5	0·475	0·449	0·383
6	0·512	0·485	0·416
7	0·540	0·513	0·442
8	0·565	0·537	0·464
10	0·602	0·573	0·500
12	0·630	0·601	0·526
13	0·642	0·612	0·537
14	0·652	0·623	0·547

The curves, Fig. 92, also show the relation of the ideal efficiency for various values of the ratio of compression. The lowest curve gives the actual thermal efficiency obtained in the gas engine research* by Professor F. W. Burstall, which is about four-fifths of the ideal standard. In these trials the limit reached for most economical compression was 175 lb. ner sq. in. before explosion, using producer gas of lower calorific value 162 B.Th.U. per cub. ft. in the Premier engine, 16 in. diameter by 24 in. stroke, which was provided with water-cooling of the piston, valves, and ignition plug, as well as the jacket around the cylinder.
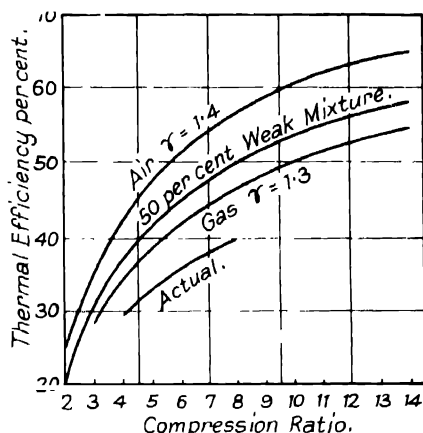
FIG. 92. EFFICIENCY OF AIR STANDARD AND ACTUAL CYCLES

With higher compression the efficiency apparently begins to fall off in these experiments.

In ordinary engines, using a mixed charge, the risk of pre-ignition practically limits the compression, which has been gradually raised in the development of the gas engine. In practice, the limit of the ratio of compression also depends upon the nature of the fuel ; a suitable compression ratio in engines using coal gas is from 4 to 7 or 8 ; in petrol engines it varies from 3 to 5 or 6. High compression makes a weak mixture, such as that of producer gas or blast furnace gas and air, more easily ignited, and increases the rapidity of combustion ; while the lower mean temperature in the cylinder, due to the large proportion of the inert gases in the mixture, reduces the heat losses during explosion and expansion.

Super-compression. The mean effective pressure may be increased about 40 per cent, and the maximum temperature in the cylinder reduced by adding a certain proportion of air or cooled exhaust products at 3 or 4 lb. per square inch above atmospheric pressure to the charge near the piston at the end of suction stroke. The

Proc. Inst. Mech. E., 1908.

ordinary rich charge in the combustion chamber near the charge inlet valve ensures ignition, while the inert gas reduces both the maximum and mean temperatures.

Maximum thermal efficiency is attained with rather weak mixtures of gas and air. The temperature in the cylinder during the explosion rises nearly in proportion to the compression ratio and the strength of the gas mixture. Excessively high temperatures are produced by highly compressed rich mixtures. The specific heats of the products of combustion. carbon dioxide and steam, in the cylinder increase greatly with the temperature, and, neglecting losses, the rise of temperature would be equal to the heat generated divided by the average specific heat of the gas over that range of temperature. When using weak mixtures the lower temperature, combined with the corresponding reduction of heat loss, during explosion and expansion, at high speed and higher compression ratio, gives an increase of thermal efficiency.

Messrs. Tizard and Pye have proved* that the actual thermal efficiency with a 50 per cent weak mixture, Fig. 92, approaches nearer to that of the air standard as the compression ratio is increased. Similar experimental results have been obtained by A. F. Burstall† on a suitably designed very high speed gas engine. at piston speed up to 2,000 ft. per minute, giving higher power and efficiency, and can work efficiently with mixtures even 75 per cent weaker than the true explosive mixture.

S. W. Sparrow has also shown ‡ that increasing the compression ratio up to 14 : 1, gives an increase in the brake horse-power and a reduction of fuel consumption per brake horse-power hour at high speeds, due to the smaller heat loss from the combustion chamber, provided serious pre-ignition can be avoided.

A well-designed gas engine can also be adjusted, by changing the proportion of combustible gas to air in the charge, to give greater economy and steady running with a weak mixture, producing lower mean temperature and lower mean pressure during expansion than the maximum pressure possible.

(3) **Constant Pressure Cycle.** The ideal pressure-volume and temperature-entropy diagrams for this cycle are shown in Figs. 93 and 94. Heat is received at the higher constant pressure and rejected at the lower pressure. No internal combustion engine works on this cycle. In the Brayton engine, heat was received by the combustion, at constant pressure, of the compressed mixture of fuel and air as it entered the motor cylinder until cut-off, followed by expansion and exhaust. Only so far did it approximate to the

---

* *Report of the Empire Motor Fuels Committee.*
† "Experiments on the Power and Efficiency of the High-speed Gas Engine," by A. F. Burstall, *Institution of Automobile Engineers*, 1925.
‡ *Report No. 205 of the National Advisory Committee for Aeronautics* (Washington. U.S A)

constant pressure cycle. Separate motor and pump cylinders were employed to compress the charge.

The ideal operations are : (*a*) Adiabatic compression from p_0, v_0, T_0 to p_1, v_1, T_1 ; (*b*) heat is received at constant pressure, the temperature rising to T_2 ; (*c*) adiabatic expansion from pressure $p_2 = p_1$ to p_3, the temperature falling to T_3 ; (*d*) heat is rejected at pressure $p_3 = p_0$, the temperature falling to T_0.

The specific heat of the air is assumed to be constant throughout the cycle.

The heat taken in is $C_p(T_2 - T_1)$, the heat rejected is $C_p(T_3 - T_0)$, and the ideal efficiency

$$\frac{C_p(T_2 - T_1) - C_p(T_3 - T_0)}{C_p(T_2 - T_1)} = 1 - \frac{T_3 - T_0}{T_2 - T_1}$$
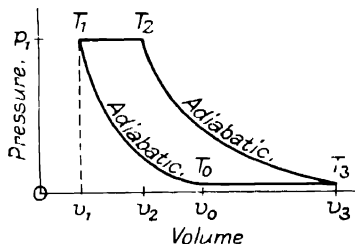
FIG. 93. IDEAL CONSTANT
PRESSURE AIR CYCLE
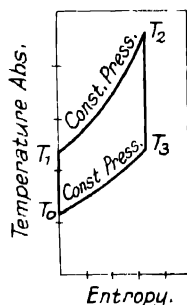
FIG. 94
$T\varphi$ DIAGRAM

The ratio of adiabatic expansion is equal to the ratio of adiabatic compression, $r = \dfrac{v_0}{v_1}$,

since $\qquad \dfrac{T_0}{T_1} = \dfrac{T_3}{T_2} = \left(\dfrac{v_1}{v_0}\right)^{\gamma-1} = \left(\dfrac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

from which $\quad \dfrac{T_3 - T_0}{T_2 - T_1} = \left(\dfrac{1}{r}\right)^{\gamma-1}$

Hence the air standard efficiency of this cycle becomes $1 - \left(\dfrac{1}{r}\right)^{\gamma-1}$

Thus the three ideal types of engine have the same thermal efficiency, provided the adiabatic compression ratios are equal before the reception of heat. Yet the ranges of temperature would be very different, and for any given ideal efficiency the Carnot cycle has the least range of temperature.

The Diesel oil engine receives heat at nearly constant pressure after adiabatic compression of excess air, although a considerable

proportion of the heat is actually received at constant volume. Adiabatic expansion ends when the volume is the same as that at which compression begins, and heat is rejected at *constant volume*, as shown by the ideal diagram, Fig. 95.

On these assumptions, the heat received is $C_p(T_3 - T_2)$ and the heat rejected is $C_v(T_4 - T_1)$. The efficiency of this ideal air engine of comparison is

$$\frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2} \qquad . \qquad . \qquad . \qquad . \qquad (1)$$
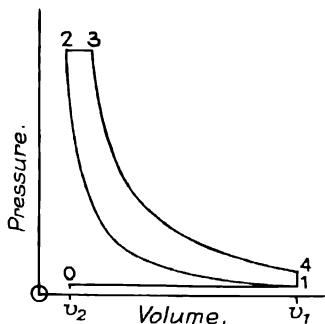
FIG. 95. IDEAL CYCLE FOR DIESAL ENGINE

To express this efficiency in terms of the volume ratios—

Let ratio of compression $r = \dfrac{v_1}{v_2}$, and $\rho = \dfrac{v_3}{v_2} = \dfrac{\text{Volume at cut-off}}{\text{Volume of clearance}}$

then, during the adiabatic operation, $T_2 = T_1 . r^{\gamma-1}$; at constant pressure $\dfrac{T_3}{T_2} = \dfrac{v_3}{v_2} = \rho$, and $T_3 = \rho . T_2 = \rho . T_1 r^{\gamma-1}$;

also $\qquad T_4 = \rho^\gamma . T_1$

Substituting these values, $\dfrac{T_4 - T_1}{T_3 - T_2} = \dfrac{T_1(\rho^\gamma - 1)}{T_1 r^{\gamma-1}(\rho - 1)}$

and equation (1) becomes

$$\text{Ideal efficiency} = 1 - \frac{1}{r^{\gamma-1}} \left\{ \frac{\rho^\gamma - 1}{\gamma(\rho - 1)} \right\} \qquad . \qquad . \qquad . \qquad (2)$$

which is independent of the temperature.

The factor $\dfrac{\rho^\gamma - 1}{\gamma(\rho - 1)}$ depends upon the cut-off, and is greater than unity; hence the ideal efficiency of this constant pressure cycle, for a given compression ratio, is less than $1 - \left(\dfrac{1}{r}\right)^{\gamma-1}$, but increases

as $\rho = \dfrac{v_1}{v_2}$ is reduced. In the limit, when $\rho = 1$, this ideal efficiency becomes equal to the air standard constant volume efficiency for the same value of r, the ratio of compression. The earlier the cut-off the more economical the Diesel engine becomes, and the lower the temperature to which the products of combustion are reduced at the end of expansion, and therefore the higher the indicated thermal efficiency.

A reliable Diesel oil engine was built in 1897, after four years of experimental work in the Maschinenfabrik Augsburg-Nurnberg (M.A.N.).

In this engine the cylinder is thoroughly water jacketed. An excess of air is drawn into the motor cylinder and compressed to
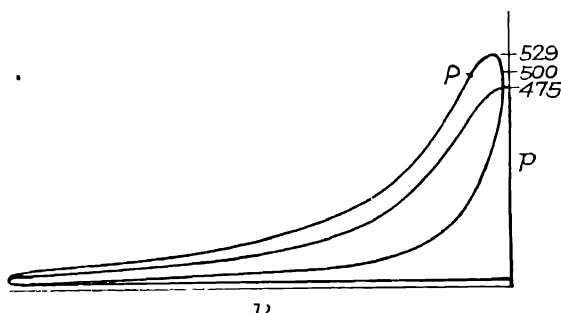
FIG. 96. INDICATOR DIAGRAMS FROM DIESEL OIL ENGINE

500 or 600 lb. per sq. in. Another supply of air is raised by an air compressor to about 1,000 lb. per sq. in., and stored in a reservoir for starting the engine, and for the blast or injection air which pulverizes the oil fuel by forcing it, through finely perforated disks fitted around a fluted needle valve, into the compressed air in the motor cylinder. The air blast, at a much higher pressure than that in the cylinder, causes turbulence in the combustion chamber. The expansion or drop in pressure cools the air blast 50° or 60° C., owing to the lower pressure of the compressed air. The finely divided oil mist ignites at once in contact with the hot air, and the rate of admission is so regulated that, about full load, the burning takes place at a slightly rising pressure, and when the load is decreased the burning pressure gradually decreases, although the temperature rises, during the first part of the working stroke. After cut-off the air and products of combustion continue to expand, driving the piston forward until release at constant volume, when the exhaust gases are pushed out during the return stroke in the four-stroke cycle.

Fig. 96 shows two superimposed indicator diagrams taken by Professor Dalby with his optical indicator, in trials of a Diesel engine, having compression ratio $r = 14.3$, and cut-off at $\rho = 1.56$.

The compression curve is practically the same for each diagram, and admission of blast at 475 lb. per sq. in. Expansion begins at the point P, after the oil spray is cut off. (See Fig. 10, p. 19.)

Some of the results obtained are as follows—

	1/3 Load	0·83 Full Load
Pressure in Air Storage Bottle by gauge, lb. per sq. in.	1080	1110
Maximum Pressure in Engine Cylinder, lb. per sq. in.	478	529
Blast Pressure by gauge, lb. per sq in.	575	700
Mean Effective Pressure, lb. per sq. in.	61·6	102
Thermal Efficiency from I.H.P. (per cent) . . .	46 4	40·6
Thermal Efficiency from B.H.P. (per cent) . . .	21·1	29
Mechanical Efficiency, per cent	45·5	71·4

If $\gamma = 1·4$, the ideal air standard efficiency for this cycle is 0·62, and the thermal efficiency from the indicator diagram at one-third load is 75 per cent of this ideal ; and for 0·83 full load this *efficiency ratio* is 65 per cent, while the actual results on the brake are much lower.

Sir Dugald Clerk pointed out* the fact that the work done by the blast air pump in compressing the air increases the work done in the motor cylinder and the area of the motor indicator diagram, in the same way that additional compression in the power cylinder would do with more air added to the charge ; and, therefore, the work shown by the blast air pump diagram should be deducted from that of the power cylinder diagram in calculating the indicated horse-power. This necessary correction reduces the very high incorrect indicated efficiency, usually given, of the Diesel oil engine to about 39 or 40 per cent, and increases the low mechanical efficiency, without changing the brake thermal efficiency when the air blast compressor is driven by the engine.

Thus the compressed charge does not simply receive heat by the combustion of the oil fuel, but more air at much higher pressure from the air compressor, added in the blast with the fuel, increases the pressure and weight of the charge in the power cylinder, and therefore gives a higher expansion curve, increasing the area of the indicator diagram probably 20 or 30 per cent at light loads, while the cooling effect of the air injection may cause misfires.

The only reliable method of determining the efficiency or performance is by the simple direct measurement of the brake power developed for a given amount of heat supplied in the fuel consumed, when the air compressor is driven by the engine.

Example 1. An ideal air engine works on the following cycle : air is taken in at atmospheric pressure (15 lb. per sq. in.) and temperature 60° F., and is compressed adiabatically, the pressure at the end of the stroke being

* *Proc. Inst. Mech. E.* (1912), pp. 236–7 and 254–5.

500 lb. per sq. in. Heat is taken in at constant pressure, and expansion afterwards takes place adiabatically, the ratio of expansion being 5. The air is exhausted at the end of the stroke, the heat assumed to be rejected at constant volume. Find the efficiency. Take the specific heats of air $C_p = 0.238$, and $C_v = 0.17$.   (*U.L., B.Sc. (Eng.)*.)

Here $\gamma = \dfrac{.238}{.17} = 1.4 = \dfrac{7}{5}$, and $\dfrac{\gamma - 1}{\gamma}$ $\dfrac{0.4}{1.4}$ $\dfrac{2}{7}$;

$T_1 = 60° + 460 = 520°$ F. (abs.).

In Fig. 95, the ratio of compression,

$$= \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{5}{7}} = \left(\frac{500}{15}\right)^{\frac{5}{7}} = 12.24$$

Also, the ratio of expansion $\dfrac{v_4}{v_3} = 5$, and $r_1 = r_4$

$\therefore$ cut-off $\rho = \dfrac{\text{ratio of compression}}{\text{ratio of expansion}} = \dfrac{v_3}{v_2} = \dfrac{12.24}{5} = 2.448$

In adiabatic compression $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{2}{7}} = \left(\dfrac{500}{15}\right)^{\frac{2}{7}}$

Taking logarithms, $\log T_2 = \log 520 + \dfrac{2}{7}$ (log 100 − log 3)

$= 2.71600 + \dfrac{2}{7}(2 - 0.47712) = 3.15111$

and $T_2 = 1416°$ F. (abs.)

At constant pressure $\dfrac{T_3}{T_2} = \dfrac{v_3}{v_2} = 2.448$,

and $T_3 = 1416 \times 2.448 = 3466°$ F. (abs.)

also $\dfrac{T_4}{T_3} = \left(\dfrac{v_3}{v_4}\right)^{\gamma - 1} = \left(\dfrac{1}{5}\right)^{0.4}$, and $T_4 = 3466\left(\dfrac{1}{5}\right)^{0.4}$

$= 1821°$ F. (abs.)

To check this result,

$T_4 = T_1 \times \rho^\gamma = 520 \times 2.448^{1.4} = 1821°$ F. (abs.)

Now, heat taken in per cycle is

$C_p(T_3 - T_2) = 0.238(3466° - 1416°) = 487.9$ B.Th.U.

and heat rejected per cycle is

$C_v(T_4 - T_1) = 0.17(1821° - 520°)\quad = 221.1\quad$,,

Hence, heat converted into work during cycle $= 266.8$ B.Th.U.

$\therefore$ ideal thermal efficiency $= \dfrac{266.8}{487.9} = 0.547$, or 54.7 per cent.

Alternative method, by expression (2) (p. 274), the ideal air efficiency is

$$1 - \left(\frac{1}{r}\right)^{\gamma-1} \times \left\{\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right\}, \text{ where ratio of compression } r = 12.24.$$

Here $\left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{12.24}\right)^{0.4} = 0.3672$; and $\rho = \frac{v_3}{v_2} = 2.448$,

thus $\rho^{\gamma} = 2.448^{1.4} = 3.5$, and $\gamma(\rho-1) = 1.4 \times 1.448 = 2.0272$

Substituting these values in the above expression, the efficiency is

$$1 - 0.3672 \left\{\frac{3.5-1}{2.0272}\right\} = 1 - 0.3672 \times 1.2334 = 0.547,$$

$$\text{or } \underline{\underline{54.7 \text{ per cent.}}} \qquad Answer.$$

Example 2. An air engine works on an ideal cycle, in which heat is received at constant pressure and rejected at constant volume. The pressure at the end of the suction stroke is 14 lb. per sq. in. (abs.) and temperature 40° C. The ratio of compression is 13, and the ratio of expansion 6·5. If the expansion and compression curves are adiabatic, $pv^{1.4} = $ constant; find the mean pressure and thermal efficiency for the cycle.

$$(U.L., B.Sc. (Eng.).)$$

In Fig. 95, the compression ratio $r = \frac{v_1}{v_2} = 13$, and expansion ratio is $\frac{v_4}{v_3} = 6.5$.

$$\therefore \rho = 13/6.5 = 2$$

since $v_4 = v_1$, $\frac{v_3}{v_2} = 2$, and $v_2 = 1$ Now $\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{1.4}$.

$$\therefore p_2 = 14 \times 13^{1.4} = 507.8 \text{ lb per sq. in.} = p_3$$

Similarly, $\frac{p_4}{p_3} = \left(\frac{v_3}{v_4}\right)^{1.4} = \left(\frac{1}{6.5}\right)^{1.4}$ and $p_4 = 36.95$ lb per sq. in.

Work done during cycle

$$= 144\left(507.8 \times 1 + \frac{507.8 \times 2 - 36.95 \times 13}{0.4}\right.$$

$$\left. - \frac{507.8 \times 1 - 14 \times 13)}{0.4} - \right) \text{ft.-lb.}$$

$$= 144\left(507.8 + \frac{535.2 - 325.7}{0.4}\right) = 144 \times 1031.5 \text{ ft.-lb.}$$

and mean effective pressure

$$\frac{\text{Work done (ft.-lb.)}}{\text{Stroke volume (cub. ft.)}} = \frac{144 \times 1031.5}{144 \times 12} = \underline{\underline{86}} \text{ lb. per sq. in.}$$

$$Answer.$$

Again, $T_1 \cdot v_1^{\gamma-1} = T_2 \cdot v_2^{\gamma-1}$, and $T_2 = T_1\left(\dfrac{v_1}{v_2}\right)^{\gamma-1}$

$$= 313° \times 13^{0.4} = 873.2° \text{ C. (abs.)}.$$

At constant pressure, $\dfrac{T_3}{v_3} = \dfrac{T_2}{v_2}$,

and $T_3 = 873.2 \times 2 = 1746.4°$ C. (abs.)

$$T_4 = 1746.4 \times \left(\frac{1}{6.5}\right)^{0.4} = 826° \text{ C. (abs.)}.$$

During the cycle—

Heat supplied to air

$$= C_p(1746.4° - 873.2°) = 0.238 \times 873.2 = 207.8 \text{ C.H.U}$$

Heat rejected by air

$$= C_v(826° - 313°) = 0.17 \times 513 = 87.2 \quad ,,$$

∴ heat converted into work during the cycle is $= 120.6$ C.H U.

and ∴ ideal thermal efficiency $= \dfrac{120.6}{207.8} = 0.58$, or 58 per cent.

Answer.

Otherwise, by the expression (2) (p. 274), the ideal air engine efficiency is

$$1 - \left(\frac{1}{r}\right)^{\gamma-1} \times \left\{\frac{\rho^\gamma-1}{\gamma(\rho-1)}\right\} = 1 - 0.3584 \left\{\frac{1.639}{1.4}\right\} = 1 - 0.42 = 0.58,$$

or 58 per cent.

The **Brayton Oil Engine,** introduced into England by Messrs. Simon, of Nottingham, in 1878, was the first compression engine using heavy oil, but was not a practical success. The design was modified in 1882, and again in 1890.*

The **Priestman Oil Engine** of 1888 was the first reliable *compression* oil engine to work successfully on " paraffin " or ordinary safe " burning oils." A measured charge was sprayed by compressed air into a vaporizer, where it was heated to about 300° F. by the exhaust gases escaping from the engine at 600° F. The charge of oil vapour was drawn into the power cylinder, together with an additional supply of air, during the suction stroke. The mixture was compressed and then fired by the electric spark. The exact regulation of the proper proportions of air and oil admitted to the spray maker, and controlled by the governor, at different loads, made the oil engine a practical success as a self-contained motor. This engine was built in sizes up to 100 H.P.

* See Drawings in *Engineering*, 15th July, 1892.

Compression Ignition Heavy-oil Engine. In 1890 a new type of oil engine, of simple construction, was introduced, to work with heavy residual oils. Mr. H. Akroyd Stuart was the originator of the method of compressing air alone in the motor cylinder previous to the injection of the fuel oil by means of a pump and spraying nozzle, and of **automatic ignition by compression** of the air in the combustion space at the end of engine cylinder.

Prior to 1890 the compressed combustible mixture in the cylinder of gas and oil engines was ignited by the electric spark, or various devices of flame, heated platinum coil, and tubes kept hot by external burners.

The essential pioneer work which led to the invention of compression ignition of heavy oils in the internal combustion engine was carried out by Mr. H. Akroyd Stuart in his works at Bletchley, Buckinghamshire He began experimental work on oil engines in 1886, and in 1889 he tried to eliminate the possibility of pre-ignition of the mixture of air and oil vapour during compression by taking a lean mixture with pure air alone next the piston. During these attempts he conceived the idea of filling the engine cylinder with air alone on the suction stroke, and experiment proved that the oil spray could be automatically ignited by injection into the hot compressed air in the vaporizer or combustion chamber when the piston was near the end of the compression stroke, and before commencing its second outward stroke. The engine had no cylinder clearance, except the vaporizer, and all the excess air was compressed into this combustion chamber before the oil spray was injected. The combustion chamber was not jacketed ; the walls were corrugated and provided with internal webs, giving a large heating surface. At starting it was heated externally by an oil lamp until the temperature attained was sufficient to ignite a few charges by compression. Then the combustion chamber was maintained at a high enough temperature by the heat retained from the explosions, together with the heat of the compressed air, to ensure regular automatic ignition. In fact, the spring fitted on the air inlet valve at the end of the combustion chamber got overheated, and its position was changed to the side of the cylinder near the exhaust valve. By this method the Akroyd oil engine worked on the four-stroke cycle : Air alone was drawn into the cylinder, compressed into the vaporizer, followed by oil spray injection into the compressed air near the end of the compression stroke, giving perfectly regular automatic ignition and explosion, and then exhaust as stated in the Patent No. 7146 of 8th May, 1890. This engine would work without pre-ignition, *only* when the oil spray was injected near the end of the compression at firing time.

Akroyd Stuart worked out another method of keeping the oil vapour and air apart, and gradually mixing them by compressing **a** portion of the air from the cylinder into the oil vapour in the

vaporizer until an explosive mixture was formed, giving automatic ignition as described in the patent specification No. 15994 of 8th October, 1890. A novel and important feature of this device was a **contracted passage** or **bottle-neck** between the combustion chamber and the motor cylinder, which is still a characteristic of many modern oil engines. The air was kept separate from the oil vapour until the compression stroke."

During his experiments with the bottle-neck vaporizer, he found that the oil spray or jet injection could be timed so as to occur *not only* at firing time, but during any part of the compression stroke, or even of the suction stroke, to allow more time for vaporization, without any sign of pre-ignition. An adjustable cam was used to *time the injection* of the charge of oil, and deliver it quickly into the
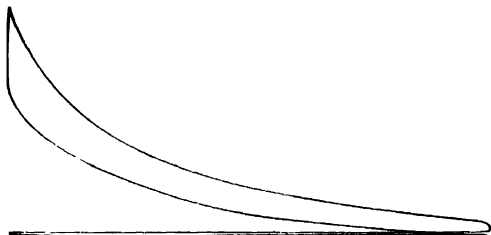
Fig. 97. Indicator Diagram with Small Charge from Akroyd Experimental Engine (1905 Type)

vaporizer at any desired point of the suction stroke or during the compression of the air to ensure the best conditions of working.

The vaporizer, once heated, was maintained at a sufficient temperature by the successive explosions and by the hot residual products, to vaporize the oil spray and ignite the compressed mixture.

Only part of the air drawn into the cylinder was compressed through the narrow neck into the oil vapour in the vaporizer to form an explosive mixture, when automatic ignition and explosion followed in the next out-stroke, and the flame was projected with great turbulence into the surplus air in the motor cylinder.

A clearance space, of a quarter of the cylinder diameter, was left between the piston and the back end of the cylinder, which was filled with relatively pure compressed air to prevent contact of the unburnt or partially burnt products with the piston and cylinder walls ; also to supply " an excess of oxygen to complete the combustion originated in the vaporizer, and thereby burn up any carbon which might otherwise form deposit."

The Akroyd Stuart Patent No. 28045, of December, 1904, was an extension of this *hot bulb* type. Instead of injecting the whole charge of oil fuel into the heated vaporizer, one pump is used to supply two sprays of fuel, one to the hot bulb and the other into the cylinder by means of a nozzle in the contracted neck. Excess

of air drawn into the cylinder between these charges serves the twofold purpose of providing automatic ignition of the charge in the vaporizer, and acting as a cushion to further compress, heat, and fire the charge in the cylinder. In some cases the injection of the fuel charges was delayed until the compression stroke, and, by using a moderate compression and a hot vaporizer for automatic ignition, a higher mean effective pressure was obtained than by the ordinary hot bulb.

This is the principle of dual combustion.

In February, 1891, the Author tested the Akroyd type of compressed air automatic ignition oil engine,* at Bletchley, and working as described in the patent specification No. 7146. The engine used intermediate shale oil of specific gravity 0·854 and flash point 225°F.,
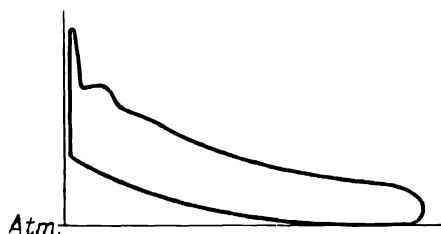
Fig. 97a. Indicator Diagram, Akroyd Engine (1890 Type)

which was drawn from a tank and quickly forced, by a little pump, through a spraying nozzle into the combustion chamber, just at the proper moment for ignition, near the end of the compression stroke. The oil spray was ignited automatically when coming into contact with the compressed air in the combustion chamber or cartridge, and the explosion followed rapidly. The spring of the indicator was not stiff enough to record the explosion pressure quite accurately, but the brake horse-power was carefully measured. The exhaust was perfectly clean, showing complete combustion with excess of compressed air, and the compression ignition was quite regular, as shown in the indicator diagram, Fig. 97a. The quantity of oil injected was regulated to suit the load on the engine, and the speed was controlled by a sensitive centrifugal governor which opened a by-pass valve and allowed the oil delivered by the pump to flow back to the oil tank, instead of passing through the sprayer into the combustion chamber. The cylinder was water-jacketed, the working parts few, of neat design, and remarkable simplicity.

The engine, rated at 6 H.P., worked smoothly and steadily at 216 r.p.m., developing 7·6 B.H.P. The consumption was considerably less than a pint of the heavy oil fuel per brake horse-power hour. In fact, the novel method of ignition and its regularity appeared too

* See *Journal of the Society of Arts*, April, 1881, on " The Uses of Petroleum in Prime Motors."

good to be true, and the Author made trials on another engine driving the shafting in the workshop, with similar result.

This early type of Akroyd oil engine worked on the same principle as the Diesel engine of 1897, inasmuch as excess air was compressed in the cylinder before oil injection, and differed from it in that the compression was not so high and the combustion chamber retained a greater proportion of heat from successive explosions ; while the simple, prompt, and direct airless injection of oil by a pump through a spraying nozzle, at the proper moment, produced explosion at constant volume, and did not necessitate the use of a two or three-stage air compressor with inter-coolers, air receivers, and accessories as in the Diesel oil engine.

Messrs. Hornsby, the licensees, were content to develop the *hot bulb* type of Akroyd engine (Patent No. 15994), although the other type was the first favoured, and is now built with various modifications of design in engines of large output and high efficiency.

The Akroyd water jacket cooled vaporizer (Patent No. 3909, February, 1892) allowed higher compression to be adopted with gradually increased thermal efficiency.

During the years 1905–6, Akroyd Stuart made experiments on a " Hornsby-Akroyd " engine altered to work exactly as described in his original Patent No. 7146, of May, 1890, with timed oil spray injection into compressed air at firing time. By taking advantage of the cooling jacket on the vaporizer, the compression was raised from 90 to 120 lb. per sq. in., with improved economy of fuel.

In practice, low working pressure, combined with simple and substantial design and complete combustion, are important factors for ensuring reliability and durability in self-contained prime motors of comparatively small output for agricultural purposes, in the hands of unskilled attendants.

The **constant volume explosion engine** has the possibility of greater economy of oil fuel and simplicity by airless injection into highly compressed air, without the multi-stage air compressor in the Diesel type, which must compress the air for injection to a temperature much higher than the ignition temperature of the oil fuel, because of the very considerable *cooling effect* of the air from the receiver at 1,000 or 1,200 lb per sq. in., as the air blast passes through the spraying valve and expands down to the pressure of the compressed air charge in the engine cylinder.

One essential of primary importance is the thorough pulverization of the heavy oil fuel into fine mist or fog, of particles so small that each one is surrounded by sufficient air for rapid and complete combustion. Each globule must be completely vaporized and burnt before coming into contact with a cold surface, to which it would adhere and burn slowly, tending to form a deposit of carbon.

By **airless injection** the oil-spray cloud is quickly discharged and cut-off without any dribble of liquid, through the minute orifice of

the nozzle by direct action at very high liquid pressure from a positively driven injection pump plunger, and may be given a combined axial and whirling or rotary motion, in order to produce *turbulence*, which increases the rate of flame propagation throughout the whole mass of the compressed air charge. The term " airless injection " has been adopted to distinguish oil engines of the Akroyd Stuart type from the Diesel engine employing " air injection " of the fuel.
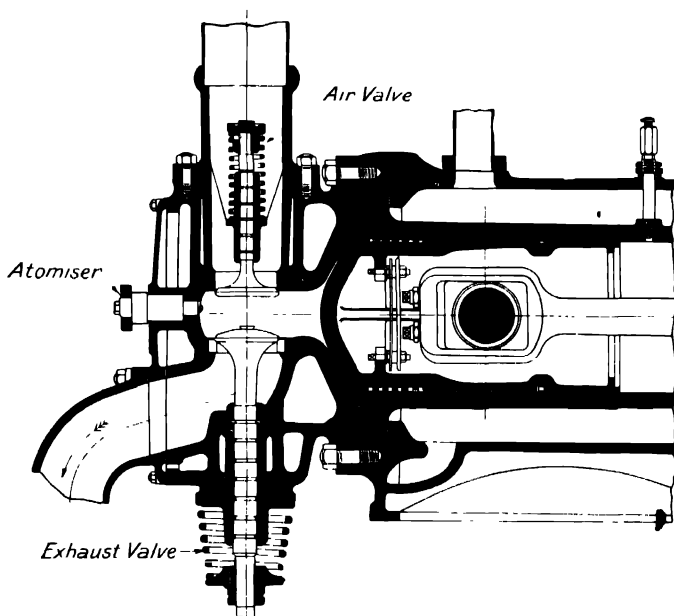
FIG. 98. RUSTON AND HORNSBY HIGH COMPRESSION HEAVY-OIL ENGINE

Modern airless injection, compression ignition heavy-oil engines, working on the four-stroke and two-stroke cycles, have been developed* by British engineers from the original pioneer engine of Akroyd Stuart, and marked progress was made in using efficiently heavy low-grade residual oils during the war. The evolution of the modern Ruston and Hornsby heavy oil engine, Fig. 98, is of special interest. The essential parts for airless injection consist of a spraying nozzle, known as an *atomizer* or *pulverizer*, and a *fuel pump* plunger, driven by a steep cam, which delivers the charge of oil fuel quickly at the correct moment near the end of compression, to the pulverizer, where it is broken up into fine spray cloud and discharged at high velocity into the water jacketed combustion chamber. The pressure of the oil is controlled by a spring loaded needle valve,

---

* See " Some Lincolnshire Oil Engines," *Proc. Inst. Mech. E.*, October, 1920.

and may be 2,000 lb. per sq. in. The synchronism of the pump and pulverizer is automatic ; the positive action of the pump tends to keep the passages in the nozzle clean ; and there is no trouble from leakage or " sticking up " of the fuel valve.

The compression pressure of the air in the cylinder before the oil injection depends upon the quality of oil fuel used, and whether the ignition is wholly due to compression temperature or partially to a hot unjacketed cap. In some engines of the latter type, a pressure of 280 to 360 lb. per sq. in. may be sufficient (see p. 292).

The governor controls the quantity of fuel delivered to the engine to suit the load.

In order to ensure reliable starting from cold by compressed air from a receiver at about 250 lb. per sq. in., with the cylinder head
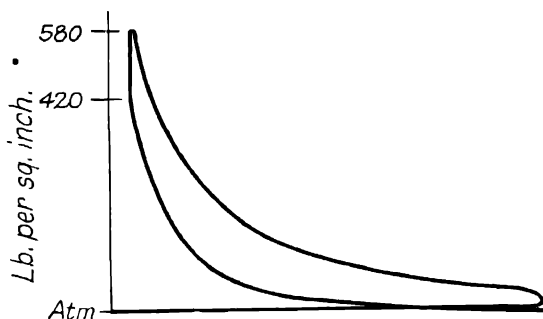
FIG. 99. INDICATOR DIAGRAM FROM RUSTON HEAVY-OIL ENGINE
(*Ruston & Hornsby*)

completely water jacketed, as in Fig. 98, a compression pressure of 380 to about 425 lb. per sq. in. is required to raise the temperature of the air well above the point of spontaneous or self-ignition of the heavier oil fuels, owing to the short period available for the vaporization, ignition, and complete inflammation of the charge of oil fuel.

The more rapidly the heat can be added the greater will be the thermal efficiency, because of the cooling action of cylinder walls. Also the explosion at constant volume gives a vertical line on the indicator diagram, Fig. 99, and attains a maximum pressure of about 560 to 600 lb. per sq. in.

The indicator diagrams of these compression ignition oil engines at full load show that part of the heat is also added by the combustion continuing for a very brief period at nearly constant pressure. When dealing with the heavier fuel oils, the injection generally starts about 15 degrees before the end of the compression stroke.

The light spring diagrams, Fig. 99 (*a*), indicate very low resistance during the exhaust and suction strokes.

The engine, when cold, is started without any external heating by means of compressed air at 250 lb. per sq. in., and after about six revolutions of the crankshaft, the air valves are shut, the oil fuel pumps put into action, and the charge is fired automatically. The air receiver is charged by a small compressor driven by the engine, during about 15 minutes after each start.

In one design the engine develops a continuous working load of 160 B.H.P. and a maximum of 200 B.H.P. per cylinder at 170 r.p.m., giving a brake thermal efficiency of 35 per cent. The consumption of heavy fuel, of calorific value 18,000 B.Th.U. per lb., is even less than 0·4 lb. per brake horse-power hour, under best conditions.

In tests of 6 hours' duration made by the Author on a Ruston high compression engine developing 80 B.H.P. at 210 r.p.m. on Mexican residual oil of specific gravity 0·920 at 60° F., and having
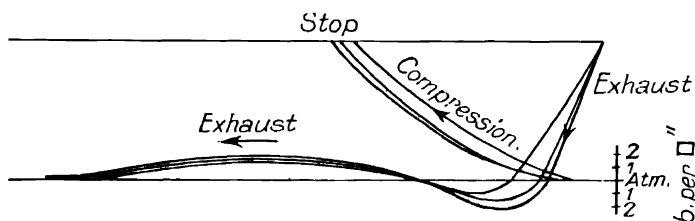
FIG. 99A. LIGHT SPRING DIAGRAMS OF PUMPING STROKES FROM HIGH COMPRESSION HEAVY-OIL ENGINE.

lower calorific value 18,000 B.Th.U. per lb., the rate of fuel consumption was 0·4 lb. per brake horse-power hour, that is, equivalent to 35 per cent brake thermal efficiency. This high efficiency was extremely good for an engine just constructed, and not specially prepared. The engine was started from cold in 30 sec., and throughout the trial the engine ran regularly without any attention, except the supply of fuel and lubricating oil, which was carefully weighed. The water jacket outlet temperature increased from 100° to 120° F. At the end of the trial the engine ran easily at 10 per cent overload— 88 B.H.P. The performance was excellent and the trials in every respect satisfactory.

Indicator diagrams Figs. 99 and 99 (*a*) were taken during the trials.

For cylinders of diameter under 20 in., the exhaust valves and pistons are not water cooled, yet in continuous working at a mean effective pressure of 85 to 90 lb per sq. in., they show no sign of overheating. the exhaust gases being at the comparatively low temperature of about 300° C. (580° F.).

In the cylinder, at the working load, the rate of heat flow is about 34,000 B Th.U. per sq. ft. per hour, and depends largely upon the design of the engine.

Fig. 100 shows the fuel consumption curves from tests* made by the late Captain Riall Sankey on a Ruston and Hornsby twin-cylinder, horizontal, compression ignition oil engine, each cylinder $18\frac{1}{2}$ in. diameter and stroke 30 in., developing 130 B.H.P. at full load, and 175 r.p.m., on a thick, treacly Anglo-American oil, the lower calorific value of which was 18,050 B.Th.U. per lb. The best economy of fuel, 0·405 lb. per B.H.B. hour, was obtained from 75
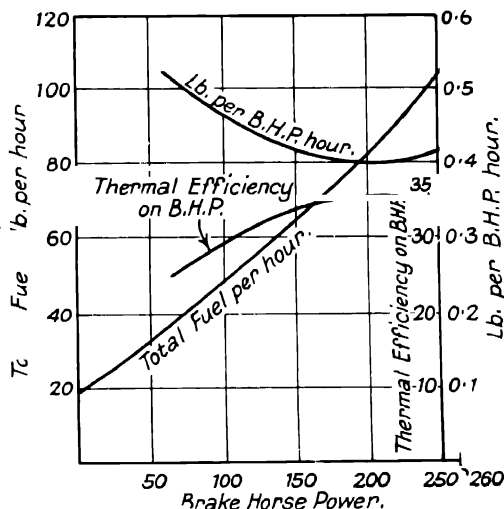
FIG. 100. FUEL CONSUMPTION OF TWIN CYLINDER, RUSTON HIGH COMPRESSION OIL ENGINE

to 85 per cent of full load, and the engine ran perfectly from 8 a.m. until 6 p.m.

Mr. W. A. Tookey found practically the same rate of fuel consumption and thermal efficiency as in Fig. 100 in a series of tests on a Ruston and Hornsby vertical type, cold starting engine, having four cylinders, each 16 in. diameter and 22 in. stroke, developing over 400 B.H.P. at 250 r.p.m., when working on " Diesel " fuel oil of specific gravity 0·895, and (analysis C, 77·3 per cent ; H_2, 12·7 per cent) lower calorific value 18,130 B.Th.U. per lb.

Thick, heavy, low-grade oils of great viscosity and high ignition temperature, like tar oils, have to be warmed to flow through the service pipes and filters to the heater on the exhaust pipe of the engine. A lighter oil may be used at starting, until the engine is heated up and the exhaust hot enough to make the heavy oil fluid. Then the best results, at all loads, are obtained with *pilot ignition*, 6 to 10 per cent of the fuel oil having specific gravity 0·92, by the device shown in Fig. 101. The pilot oil, of low ignition point, is

* *Proc. Inst. Mech. E.*, October, 1920, p. 698.

injected just ahead of the main charge of heavy tar oil. The smaller pilot pump is actuated by the impulse of the oil pressure on the
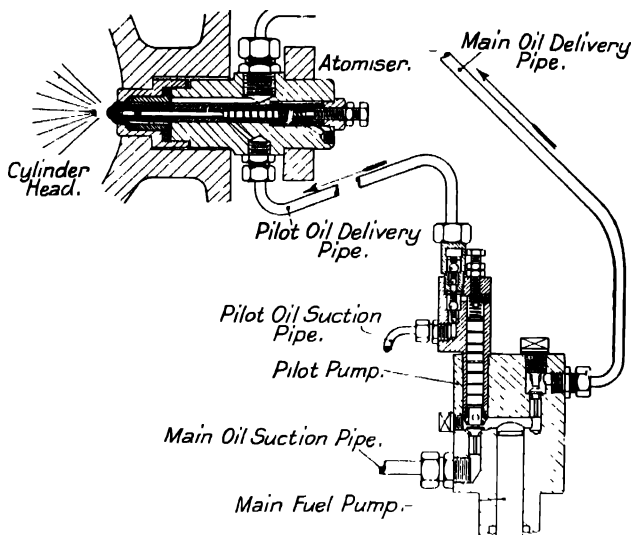
FIG. 101. PILOT OIL PUMP AND SPRAYER FOR TAR OILS
(*Ruston & Hornsby*)

stepped plunger in the main pump. The sprayer contains two spring loaded needle valves, the inner discharges the pilot oil, which is kept constant at all loads, and the outer the main charge of tar oil.
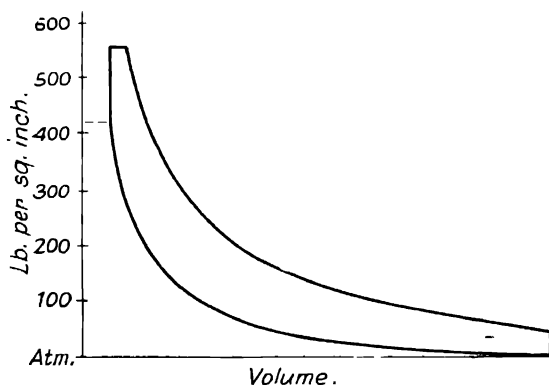
FIG. 102. DUAL COMBUSTION CYCLE

In the ideal **dual-combustion cycle** of internal combustion engines, Fig. 102, part of the fuel is burned at constant volume and the

remainder at nearly constant pressure. This process of combustion is practically obtained in the heavy oil engine at high compression, and is termed " after-burning," defined as the burning which takes place after complete inflammation in a weak gaseous mixture in high speed engines.*

Indicator diagrams clearly show that, in practice, there is dual-combustion in engines working on the constant volume and constant pressure cycles. It is most distinctly marked by the introduction of a pilot charge in advance of, and to ensure the combustion of, the main charge of heavier tar oils which have great viscosity.

The ideal thermal efficiency of this cycle, assuming constant specific heat, is given by the expression

$$1 - \frac{1}{r^{\gamma-1}} \left\{ \frac{a\rho^\gamma - 1}{a - 1 + \gamma a(\rho - 1)} \right\} \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

where ρ is the same ratio as in (2) (p. 274), and a is the ratio of the maximum explosion pressure at constant volume to the pressure at the end of the compression stroke.

The heat added at constant volume raises the efficiency of that at constant pressure, and the advantage of dual combustion is to maintain the thermal efficiency with an increase of mean effective pressure, but this produces a very large increase in the maximum temperature.

In 1891, Mr. Day patented a small vertical two-stroke cycle gas engine of extreme mechanical simplicity, with no valves (except one operated by the governor). The crank-case is enclosed and used as the receiver of the combustible mixture, somewhat like the Robson type. Three ports are arranged in the cylinder wall, and the piston covers and uncovers these, while the engine will run indifferently in either direction. A reduction of speed, with advanced ignition, may cause a sudden reversal of the engine, unless the ignition is set later as the speed falls. Enormous numbers of these little engines have been developed, with modifications, to run on petrol or gasoline, especially in America.

Professor W. Watson, F.R.S., and Mr. R. W. Fenning made experiments on a Day vertical marine petrol engine, in 1910. The cylinder diameter was $3\frac{1}{4}$ in. $\times 3\frac{1}{4}$ in. stroke, rated 2·5 B.H.P. at 900 r.p.m. ; the compression ratio 4·67 to 1 ; the volume swept by piston 27 cub. in., and the total volume from bottom of stroke 34·5 cub. in. The ring of exhaust ports considerably reduced the effective length of stroke. The port area was 15 per cent of the piston area. The proportion of each fresh charge which escaped unburnt through the exhaust port was 36 per cent at 600 r.p.m., reduced to 20 per cent at 1,200 r.p.m ; the volumetric efficiency was only about 40 per cent, and the increase of speed reduced the

* *Proc. Inst. Mech. E.*, December, 1920, p. 1235.

mean effective pressure from 62·5 to about 44 lb. per sq. in. At the
higher speed there was considerable wire drawing between the
carburettor and crank-case. In order to allow more time for
charging, ⅛ in. was cut off the lower edge of the piston opposite the

Fɪɢ. 103. Tʜᴇ Pᴇᴛᴛᴇʀ Tᴡᴏ-ꜱᴛʀᴏᴋᴇ Cʏᴄʟᴇ Oɪʟ Eɴɢɪɴᴇ
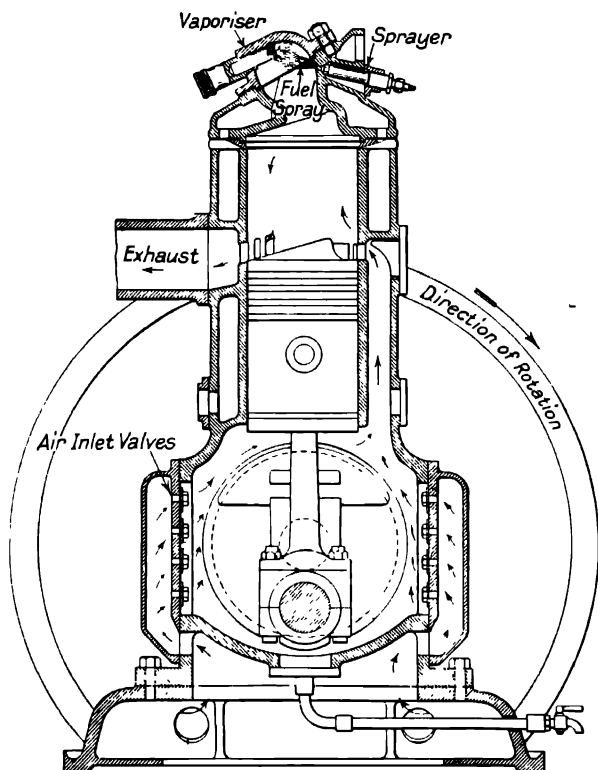
port, with improved results. Increase of speed from 900 to 1,200
r.p.m. reduced the ratio of volume retained to total volume from
33·6 to 32 per cent, while the mean effective pressure was reduced
from 65·5 to 63·5, and the brake horse-power increased from 3·27
to 4·2. Throttling of the mixture to reduce the speed tends to
make the charges weaker and may lead to misfiring, or ignition only
at every alternate out-stroke, with much higher explosion pressures.

The **Petter two-stroke cycle** cold starting **oil engine**, Fig. 103,
embodies an ingenious combination of the valveless engine and the
hot bulb, with air scavenging ; in which the improvements in
design eliminate many of the difficulties inherent in the earlier
engines, and give an impulse every revolution of the crankshaft.

In Fig. 103, the piston is shown at the bottom of its stroke. During the explosion and expansion downstroke the piston uncovers the exhaust and air ports at about four-fifths of its stroke, and the products of combustion are driven out by the air, which has been previously compressed in the crank-case to 3 or 4 lb. per sq. in. above the atmosphere. The exhaust ports are so arranged that the exhaust gases in the cylinder have been reduced to atmospheric pressure before the scavenge air is admitted, and the curved piston head tends to deflect the air flow upwards. During the upstroke the piston covers the air and exhaust ports, and compresses the air in the upper part of the cylinder, while the suction of 3·5 lb. per sq. in. below the atmosphere, produced by the lower end of the piston, draws fresh air into the crank-case.

The air inlet valves, made of special spring steel, are mounted on the front and back of the crank-case. The cylinder barrel and head are well water-jacketed. The oil fuel is injected by a pump into the vaporizer or hot bulb, through the sprayer, near the end of the compression stroke, and intimately mixed with the compressed air so as to ensure automatic ignition. The governor enables the timing of the injection to be varied, as

Fig. 104. Results of Tests of Petter Two-Stroke Cycle Oil Engine

well as the control of the quantity of fuel, by altering the stroke of the pump plunger. As the stroke of the fuel pump is reduced, the governor automatically advances the timing of the injection.

Single cylinder engines up to 18 B.H.P. have a compression relief valve fitted in the cylinder, which allows the engine to be cranked round by one man. With twin cylinder engines a compressed air starter is supplied.

The Petter cold starter consists of a paper tube or cartridge containing a preparation which is slipped into a steel holder and when lighted by a match, is screwed into the vaporizer. This supplies the initial heat to fire the first few charges in the engine until the tube holder, which is flush with the inner wall of the hot bulb, becomes hot enough to carry on the combustion. In this way the engine is started up to normal speed in 30 to 60 sec.

In a series of tests made by Messrs. Burstall and Monkhouse on a 12 B.H.P. engine at 532 r.p.m., the fuel used was Persian gas oil of specific gravity 0·869 at 60° F., and calorific value 19,460 B.Th.U. per lb. Tests were also made using Asiatic Tarakan Fuel oil, of

specific gravity 0·942 at 60° F., and calorific value 18,960 B.Th.U. per lb. The engine was started from cold and ran without load for an hour without any special adjustment ; full load was suddenly put on, the engine ran easily and the exhaust was clean.
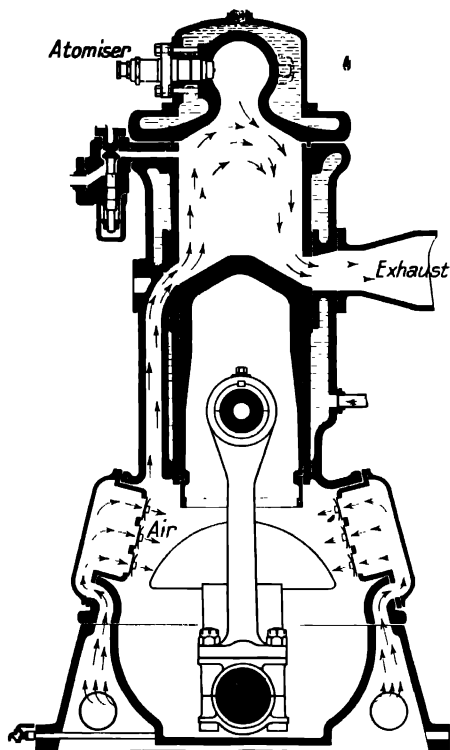
Fig. 105 Vickers-Petters Two-stroke Cycle Heavy Oil Engine

The chart, Fig. 104, has been plotted from the results of tests given in the Report. The oil consumption at full load was at the rate of 0·59 lb. per brake horse-power hour, and the brake thermal efficiency 22·2 per cent ; while the average consumption of lubricating oil was 0·16 pint per hour. At a 26 per cent overload, the actual brake horse-power was 15·18, fuel consumption 0·62 lb. per brake horse-power hour, and brake thermal efficiency 21·1 per cent, calculated on the higher calorific value of the fuel The lower calorific value was about 18,140 B.Th.U. per lb.

The only alteration made to the engine for the heavier oil fuel was to advance the pump eccentric 6 degrees.

Governor tests were made by the full load being thrown off, giving about 3 per cent variation in speed, to which the engine settled in 2 or 3 sec.

The **Vickers-Petters** two-stroke cycle heavy oil engine, Fig. 105, is of great simplicity and works with a moderate compression below 300 lb. per sq. in.

The cylinder head and spherical combustion chamber are completely water-jacketed. The scavenge air is compressed in the crank-case to 4 lb. per sq. in. Oil interceptors and sealing rings are designed to obviate internal pressure on the main bearings, which would lead to excessive consumption of lubricating oil. In starting the engine, initial heat is provided by a special burner or an electric torch, and compressed air from a receiver is admitted through valves operated by cams on the crankshaft. When the

engine is heated up and has attained running speed, the use of the burner is no longer required.

Airless injection is used. A constant supply of oil is delivered at high pressure by a pump to the atomizer, and the centrifugal governor regulates the charge admitted by a valve to suit a varying load, any excess being returned to the oil supply tank.

The automatic ignition of the sprayed oil fuel is effected by the temperature of compression. Owing to the low working pressure adopted, the brake mean effective pressure is about 40 lb. per sq. in., and each cylinder develops not more than 80 B.H.P. at 250 r.p.m. A marine oil engine of output 450 to 500 B.H.P. consists of six cylinders.

High-speed Heavy-oil Engine. An important advance in the development of the oil engine in overcoming the difficulties of using heavy oils efficiently with high compression ratio in high speed engines, having comparatively small bore cylinders, is described* by Mr. Alan E. L. Chorlton.

In order to ensure rapid and complete combustion of heavy residual oils in a multi-cylinder engine running at about 1,000 r.p.m., the fuel injection pump has to measure accurately the exact charge of oil fuel for each cylinder, and to inject the oil under high pressure through the atomizer as exceedingly fine spray into the compressed air in the combustion chamber with a quick start and sharp cut-off to its flow. The necessary conditions are : (1) precise measurement of the fuel charge, (2) injection during a definite period, with (3) instantaneous cut-off, (4) fine division or pulverization of the oil, and (5) avoidance of contact of the oil with the cylinder wall during rapid discharge into an excess of highly compressed air.

The principle of the oil fuel pump devised by Chorlton is shown by the diagram, Fig. 106. The pump plunger actuated by an eccentric gives the oil fuel a reciprocating, to and fro, motion through the passage *A*. Exactly at the right moment this passage is interrupted momentarily by a high speed valve *B*. The effect is to send a vibration through the oil column, between the valve *B* and the atomizer, which lifts the atomizer valve off its seat, and a small quantity of oil is forced past it at high pressure. It is then rapidly drawn to its seating again by the reflex action, giving a quick cut-off, when the valve *B* reopens.

Another feature of this high-speed vertical engine is the use of special cast steel in the construction of the frame, formed as a light square box to each cylinder. The walls of this multi-cellular crank casing are continued to the crankshaft bearings, and carry directly the stress due to the high explosion pressure, and are adequate even when the thickness of the cast steel is only $\frac{3}{8}$ in. The cylinder liners are thin, hard, steel tubes, and the pistons and cylinder covers are

* *Proc. I. Mech. E.*, March, 1926.

of aluminium alloy.   With ample water-jacket cooling, closer piston fits are possible, and in the cylinders of small diameter the piston and rings are found to maintain the tightness with quick revolution engines and high pressure lubrication, under all loads, better than the low speed engine with a large cylinder.   The higher piston speed also gives more rapid firing, desirable on the constant volume cycle.

An 8-cylinder engine, with cylinders $8\frac{1}{4}$ in. bore by 12 in. stroke, normal rating 400 B.H.P. at 750 r.p.m., weighs only $2\frac{1}{4}$ tons.   The average weight/power ratio is 15 to 20 lb. per brake horse-power.

Applied to railway service, the engine, by a special relay governor, runs between 250 and 750 r.p.m., according to the speed of train
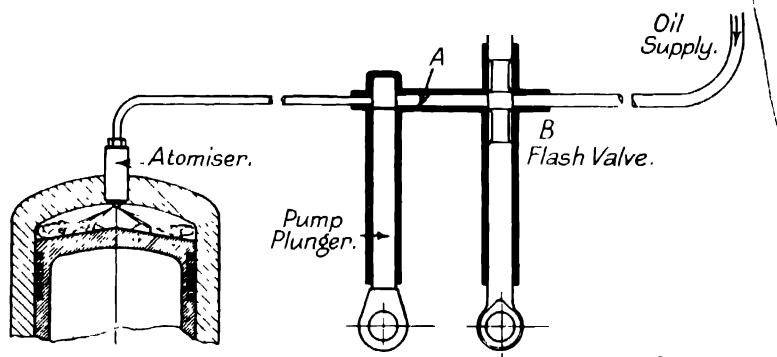
Fig. 106. Oil Fuel Pump (*Chorlton*)

required.   In practice, the cost is stated to be less than that of steam power.   The fuel used is standard Diesel oil.

The results of tests with Shell-Mex Diesel oil fuel are given in the table.   Series *A* was carried out by Professor Mellanby on a standard engine with normal maximum pressure.   Series *B* on a similar engine with improved mechanical efficiency, having been running longer.   In series *C* a high maximum pressure was used.

Test Series	Duration of Test. Hours	Speed. Rev. per minute	Brake Horse-power	Fuel Consumption. Pound per B.H.P. hour	Temperature of Cooling Water.	
					Inlet, °F.	Outlet, °F.
A	$4\frac{1}{4}$	689·3	160·26	0·418	121·5°	127·4°
B	3	700	172	·385	120°	128°
C	3	1007	424	·365	140°	150°

The " Scott-Still" Marine Oil Engine.   This regenerative oil engine, invented by Mr. W. J. Still, consists of a combined internal combustion engine and steam engine, in which heat from the exhaust
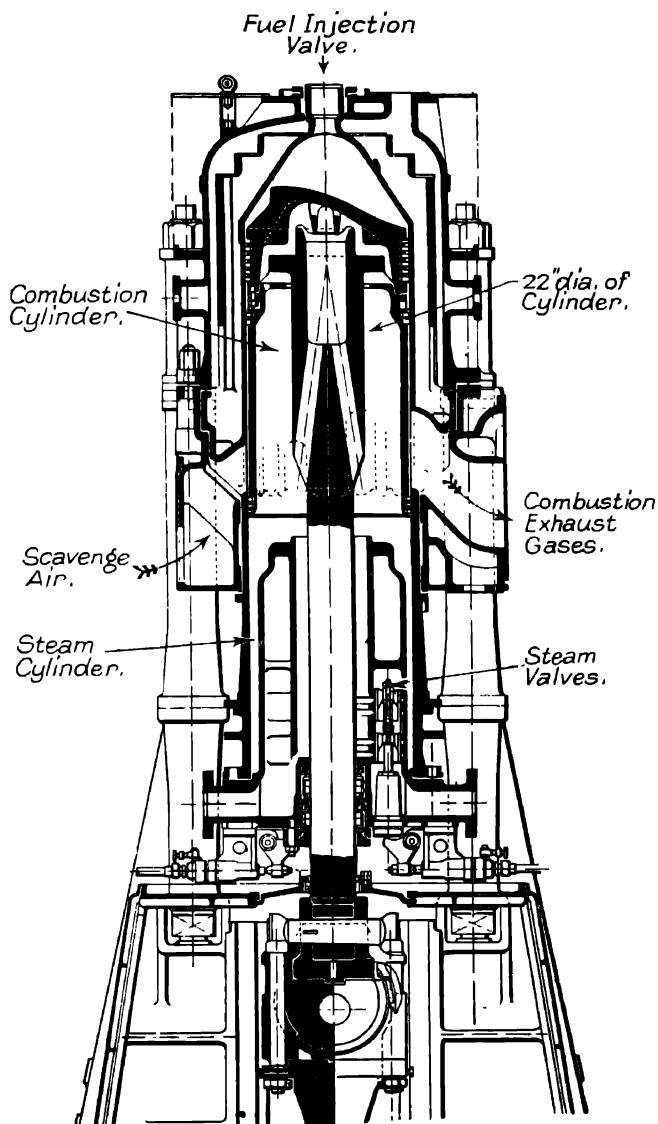
FIG. 107. SCOTT-STILL COMBINED INTERNAL COMBUSTION AND STEAM CYLINDERS (VERTICAL SECTION)

gases and that usually lost from the combustion of the fuel through the walls of the cylinder liner to the jacket cooling water, is recovered and utilized in the generation of steam, which does work in the steam engine cylinders, and the exhaust steam from these drives a low pressure steam turbine.

This particular installation is only one example of the application of the Still principle of waste heat recovery, and is chosen on account of the very complete and independent tests published.* For instance, another ship may be better suited by the Scotch type of steam boiler, and, in the case of a trawler, by a vertical Cochran boiler.

The arrangement of cylinders in the Scott-Still marine oil engine in the motor ship *Dolius* is shown in the vertical section,† Fig. 107.

Still Practice   *Ordinary Practice*

Cast-iron liner, mild steel hoop,   Cast-iron liner and jacket
mild cast steel jacket
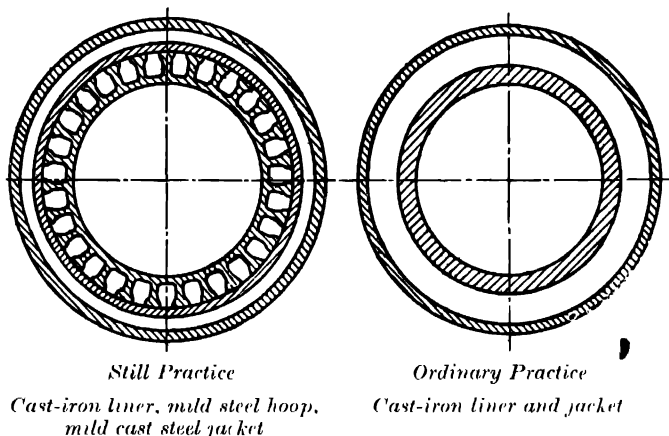
FIG. 108. COMPARISON OF CYLINDER CONSTRUCTION

The combustion cylinder at the top of Fig. 107 has the thin liner ribbed parallel to the axis, Fig. 108, fixed into the steam cylinder below it, and free to expand at its upper end, while the steam cylinder expands downwards.

This liner is reinforced against bursting pressure by steel hoops at the top. This construction, with mantle or shrouds, also aids the special circulation and generation of steam within the water jacket, Figs. 108 and 109; while the small difference of temperature, about 60° F., between the inner and outer skin of the liner reduces the heat stresses.

The oil engine works on the two-stroke cycle, and automatic ignition of the charge is ensured at the comparatively low compression pressure of 360 lb. per sq. in., owing to the high temperature of the jacket and cylinder walls.

* *Proc. I. Mech. E.*, March, 1925.
† *The Engineer*, November, 1923.

The oil fuel is supplied by airless injection through a spring-loaded valve from a pump operated by a cam, which fixes the timing
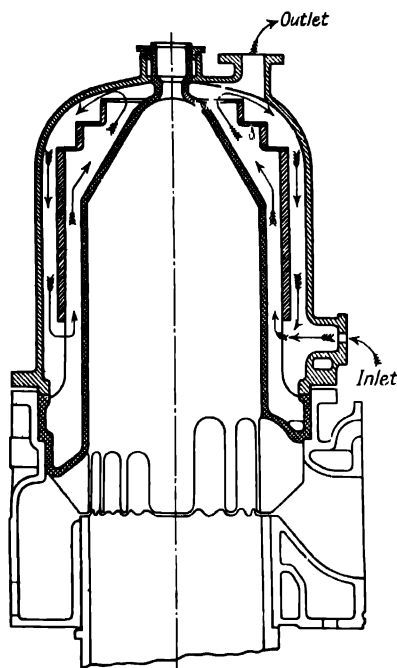
FIG. 109. WATER AND STEAM CIRCULATION IN STILL ENGINE
CYLINDER JACKET

of the fuel spray injection into the cylinder at a pressure of about 3,500 lb. per sq. in. produced by the pump.

Fuel Injection   Scavenging.
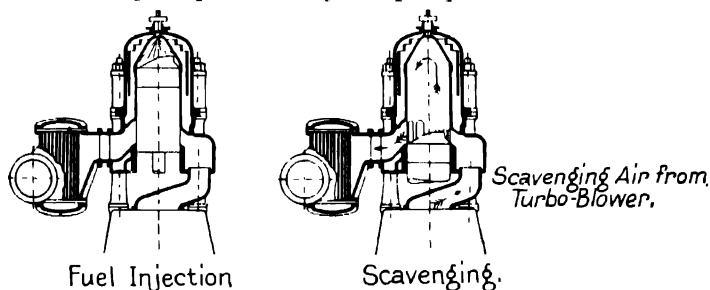
FIG. 110. DIAGRAM POSITIONS OF THE PISTON

The fuel pump mechanism opens a spill-valve at a variable position of the pump stroke, which is controlled by the governor, thus regulating the period of injection and the quantity of fuel

injected.   When the spill valve is opened, the delivery of oil fuel
from the pump to the cylinder is cut off, and the oil flows back to
the supply tank.

The cam rollers are moved sideways by a ram under oil pressure,
to engage with either the " ahead " or " astern " cams for reversal.

The diagram, Fig. 110, shows the positions of the piston for fuel
injection after compression of the air, and for scavenging near the
end of the explosion stroke, when the piston crown uncovers the
scavenger air ports.

The piston head is shaped, Fig. 107, so as to deflect the air upwards.
This scavenge air is delivered at a pressure of about 1·6 lb. per
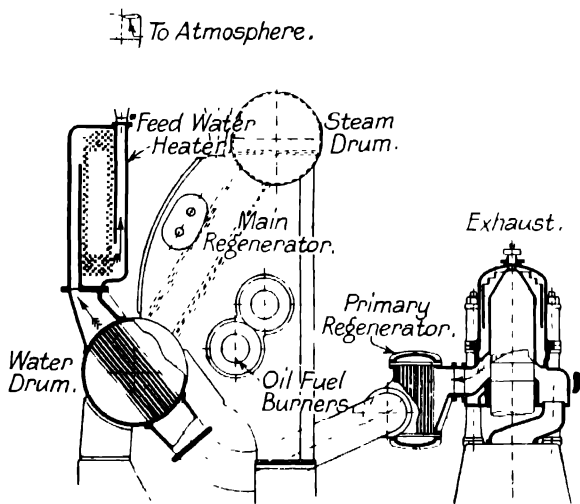
FIG. 111.   REGENERATORS IN SCOTT-STILL MARINE OIL ENGINE

sq. in. (gauge) by a turbo-blower driven by a low pressure steam
turbine, which is supplied with exhaust steam from the low pressure
steam cylinders.

The exhaust gases from the cylinder pass direct through the
small steam generator, or " primary regenerator," Fig. 111, fixed
on the exhaust pipe of each cylinder, wherein part of the heat
carried away in the gases is given to the water as it flows up a
group of vertical tubes on its way to the cylinder jacket.   After
leaving the primary regenerator, the gases from the cylinders enter
a common exhaust manifold, and pass along to the water drum of
the main regenerator.   The water drum contains straight tubes,
through which the gases flow and give up more heat.   The last
quantity of heat is taken from them in passing through a rectangular
counter-current feed water heater, which consists of a nest of water

tubes. The exhaust gases thus reduced in temperature to about 300° F., finally escape up the funnel to the atmosphere.

The water in the cylinder jacket, Fig. 109, is at practically constant temperature, corresponding to the steam saturation pressure, about 120 lb. per sq. in. (gauge) in the boiler and regenerators. The heat from the explosion, transmitted through the liner and head of the cylinder to the water jacket, is absorbed as latent heat of evaporation of the steam generated in the jacket.

The natural circulation of water, from the water drum of the boiler through the primary regenerators and cylinder jackets, and the mixture of water and steam led up a rising main to the steam drum, where the steam is separated, is by thermo-siphon, by the difference of density of the mixture of steam and water.

The steam, separated in the steam drum, passes to the cylinder jacket and through valves, Fig. 107, to the underside of the piston in the high-pressure cylinder In a four-cylinder engine, the aft cylinder of the set is high pressure and exhausts into a receiver which supplies the other three cylinders

The main regenerator, Fig. 111, is a modified Yarrow type boiler, fitted with oil burners, in the combustion chamber under the water tubes connecting the steam and water drums, to augment the steam supply when more power is required, as well as in starting and manœuvring. To start the engine, the steam raised in this oil-fired boiler passes direct to all the single acting steam cylinders, which exhaust through the turbine to condenser.

An outstanding feature is the great flexibility and extremely good manœuvring qualities of the engine, by the movement of the steam valves, through operators and distributors, by oil pressure from a Hele-Shaw multiple cylinder pump at 400 lb per sq. in., and controlled from the hand wheel. The great flexibility is due to the ingenious combination of the steam engine and the internal combustion engine.

High pressure air for injection of oil fuel and in starting, with compressor troubles, are here dispensed with, only the scavenge air blower remains.

The Marine Oil Engine Trials Committee, appointed jointly by the Institutions of Mechanical Engineers and Naval Architects, with representatives from the Admiralty and the Institute of Marine Engineers, to carry out tests of oil engines and oil-engined ships, presented their Second Report in March, 1925, giving the results of tests carried out on the *first* motor ship *Dolius*, having Scott-Still regenerative engines.

The port engine was tested ashore, followed by trial runs at sea at different speeds and loads on the measured mile, and manœuvring trials.

The rating of each main engine is 1,250 B.H.P. at 120 r.p.m., the power being developed in four cylinders, working on the two-stroke

single acting cycle. The dimensions of the port engine are : Stroke, 36 in. ; bore of each combustion, and steam cylinder, 22 in. ; piston-rods, 6·25 in. ; and ratio of compression, 8·56. The fuel was Anglo-American Diesel oil, of specific gravity 0·864 at 59° F. ; flash point, 176° F. ; burning point, 198° F. ; and higher calorific value, 19,500 B.Th.U. per lb. Ultimate analysis : Carbon, 86·09 H_2, 12·95 ; S, 0·4 ; O_2, 0·52 ; and N_2, 0·04 per cent.

In the tests at full rated power, of 4 hours' duration, at average speed 121·9 r.p.m., the indicator diagrams, Fig. 112, were taken.
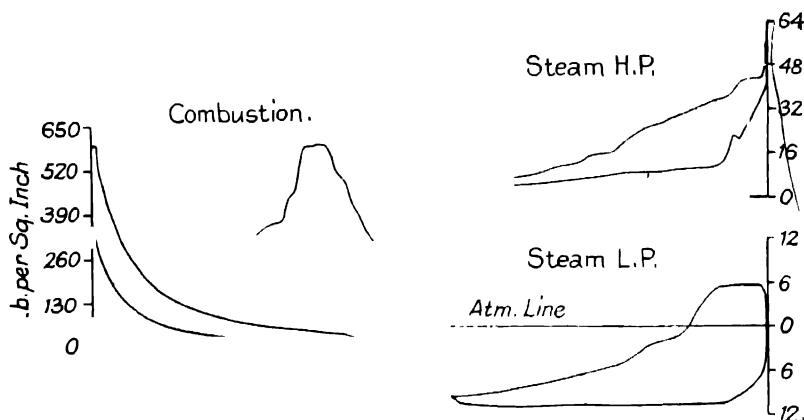
FIG. 112. INDICATOR DIAGRAMS - SCOTT-STILL MARINE OIL ENGINE

The combustion is partly at constant volume and partly at constant pressure, on the dual cycle.

Results obtained from the *combustion cylinders*—

Pressure at end of compression	355 lb. per sq. in.
Pressure at maximum explosion . . .	582 ,, ,,
Pressure at end of expansion . . .	39 3 ,, ,,
Mean indicated pressure, average of all cylinders .	76·6 ,, ,,
Indicated horse-power (combustion only) .	1290

Thermal equivalent of indicated horse-power for 1 min. 54,700 B.Th.U.

On the *steam side*—

Temperature at exit from water-drum . −	344° F.
Pressure in steam drum . . .	115 8 lb. per sq. in
Pressure at admission, high pressure cylinder .	47·5 ,, ,,
Pressure at admission, low pressure cylinders .	4 8 ,, ,,
Pressure at exhaust (below atmosphere) .	10·6 ,, ,,
Average, mean indicated pressure of all cylinders	6·9 ,, ,,
Indicated horse-power (steam only) . .	116
Thermal equivalent of indicated horse-power for 1 min.	4920 B.Th.U.
Total indicated horse-power, combustion and steam .	1406

Work done by exhaust steam from one engine on turbo-blower (intended for both sets of engines)—

Heat drop in turbine per pound of steam . . 88 B.Th.U.

Effective brake horse-power by Heenan & Froude
absorption dynamometer 1271

Brake mean effective pressure . . . . 75·4 lb. per sq. in.

Mechanical efficiency $= \dfrac{\text{Brake horse-power}}{\text{Total indicated horse-power}} = \dfrac{1271}{1406}$

$= 90\cdot4$ per cent

Oil fuel consumption per brake horse-power hour $= 0\cdot352$ lb.

Oil fuel consumption per indicated horse-power $= 0\cdot322$ lb.

a remarkably good result.
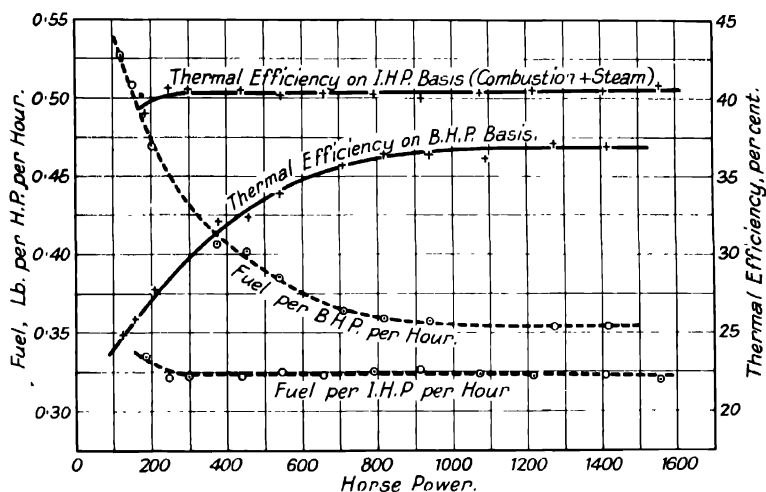
Fig. 113. Fuel Consumption and Thermal Efficiency Curves
(Scott-Still Regenerative Oil Engine)

In order to enable comparisons to be made with the performance of other engines, and to subdivide the thermal equivalents of the work done in the Thermal Balance Sheets, the combustion side of the engine is debited by the Committee with all the frictional resistances, and so may be said to give at the brake a quantity called the "combustion brake horse-power." Then the effect of the assistance of the steam is to add all its power (indicated horse-power steam) to the combustion brake horse-power to make up the observed effective brake horse-power.

Thus, Friction H.P. $= 1406 - 1271 = 135$ H.P.

and Combustion B.H.P. $= 1290 - 135 = 1155$ H.P.

Effective B.H.P. $= 1155 +$ Steam I.H.P. $116 = 1271$

Thermal equivalent of effective B.H.P. for 1 min. is 53,910 B.Th U.

Thermal efficiency on total I.H.P. (not including the L.P. turbine)
$$= 40 \cdot 5 \text{ per cent}$$

Thermal efficiency on effective B.H.P. $= 37 \cdot 1$ per cent.

$$\text{Efficiency ratio} = \frac{\text{Thermal efficiency on total I.H.P.}}{\text{Efficiency of ideal engine (50 per cent)}} = 0 \cdot 81$$

Thermal Balance Sheet, from Test No. 10 of the Marine Oil Engine Trials Committee (Second Report)—

Fuel supplied—0·352 lb. per brake horse-power hour		100 per cent
Distribution of Heat—	Per cent	
Effective brake horse-power, 1,271 at 121·9 r.p.m.		
(Combustion B.H.P. 1155 + steam I.H.P. 116)	37·12	
Power for scavenger blower from L.P. turbine	2·37	
Friction	3 93	
Total indicated horse-power . .		43·42 per cent
Heat Losses –		
Finally carried away in exhaust gases .	23·07	
Rejected in condenser cooling water .	25·82	
Radiation	7·69	56 58 per cent
Heat in fuel.	100·00	
The indicated power is made up as follows—		
Combustion indicated horse-power		37·67 per cent
Regenerated steam indicated horse-power . .		3·38 ,,
Regenerated steam in L.P. turbine (heat drop) . .		2·37 ,,
Total indicated horse-power		43·42 per cent

The power developed by the low pressure steam turbine is obtained from the fuel burned in the combustion cylinder, and is here added to the indicated power of the engine, as part of that from the steam generated by the waste heat. This makes the total indicated horse-power 1,457, and the rate of fuel consumption 0·31 lb. per indicated horse-power hour, increasing the thermal efficiency to 42·1 per cent, instead of 40·5 per cent, and the curves (Fig. 113).

The mechanical efficiency becomes 87·2 per cent.

The results mark a distinct advance in heat engine practice. Details in design will be modified and improvements made by experience.

Already, in actual service, the motor ship *Dolius*, fitted with these Scott-Still engines, on the third outward voyage to the Far East, during a run of over 10,000 miles at average speed 11·3 knots, and engines at 121 r.p.m., gave an excellent performance. The average fuel consumption per brake horse-power hour, for all purposes, was 0·393 lb. of Borneo (Tarakan) fuel oil of specific gravity 0·94 at 60° F., based on the overall daily consumption of 8·46 tons.

The First Report of the Marine Oil Engine Trials Committee gave the results of tests on the Diesel type of oil engine of the M.V. *Sycamore*, with air-blast injection. The Anglo-Persian Diesel oil fuel used had specific gravity 0·875, and higher calorific value 19,320 B.Th.U. per lb. The lowest rate of fuel consumption was 0·425 lb. per brake horse-power hour, or a brake thermal efficiency of 31 per cent, and mechanical efficiency 79·8 per cent.

Petrol Engines. The development of the automobile, propelled by the petrol engine, is largely due to the invention of Gottlieb Daimler.

The small, high-speed, inverted vertical engine (Patent No. 4315 of 1885) had splash lubrication in the enclosed crank-case. The automatic inlet valve was above the exhaust valve, the latter being operated on the four-stroke cycle by a rod from a double cam groove in the enclosed flywheel.

Daimler was the first to use a *handle starter*, and first applied his motor to a bicycle in 1885, and to a motor-car in 1887, giving a fresh start to road locomotion, which was delayed in England until the free use of the roads was obtained by the Act of November, 1896.

In 1889, Messrs. Panhard and Levassor acquired the Daimler patent rights in France, and adapted this motor to their carriages. Daimler assisted them in the construction of the first automobile. The typical Daimler engine had two vertical cylinders cast together, with hot tube ignition, and governed by keeping the exhaust valve closed when the engine ran above the normal speed of 700 r.p.m. The engine used the lighter fractions of petroleum, known as petrol, such as petroleum spirit, gasoline, and benzoline, which evaporate freely in the air at the ordinary temperature, and a current of air through this highly volatile spirit becomes saturated with hydrocarbon vapour and called carburetted air.

The **carburettor** supplies the mixture of petrol vapour and air in suitable proportions, usually about 2 per cent of petrol vapour, and in varying amount of charge, according to the speed or output of the engine.

The early form of *surface carburettor*, designed by Daimler in 1885, consists of a petrol tank, containing a float, forming an inverted conical basin at the centre. Air drawn down a central tube, by the suction of the engine, bubbles up through the petrol in the conical basin, passes baffle plates and through a wire gauze screen, to stop any liquid spray, which falls back into the basin. The supply of carburetted air is thus prepared and ready to be mixed with air, just like gas. Such a process of fractional evaporation is not without its drawbacks. The quantity of petrol in each charge is not accurately measured, and varies with the suction or speed of the piston. When the charge of carburetted air is drawn into the cylinder, the sudden inrush of air through the petrol causes rapid evaporation and cools the carburettor. The more volatile

portions of the petroleum spirit pass off first, and the remaining liquid becomes heavier, cooler, and more difficult to volatilize. The richness of the charge is gradually reduced by the fall in temperature and moisture in the air, especially when the petrol dregs become " stale " in cold weather ; and the heavier hydrocarbon vapours are apt to condense when conveyed through cold pipes or subjected to pressure. Hence the carburettor must be close to the engine cylinder, and the air warmed to supply the latent heat of vaporization of the petrol and ensure the complete evaporation of the heavier fractions.

In the more recent *Lanchester surface carburettor* these difficulties were successfully overcome. The petrol in the float was kept apart
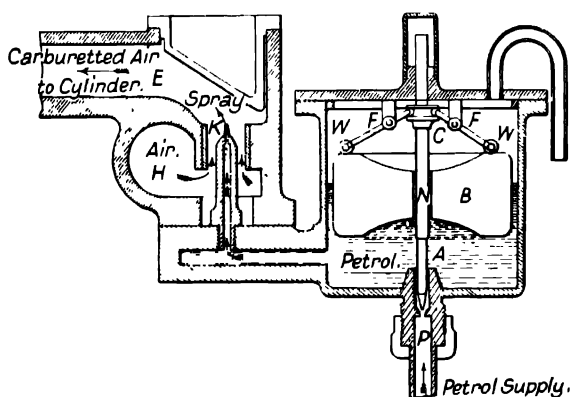
FIG. 114. DAIMLER-MAYBACH FLOAT FEED JET CARBURETTOR

from the free surface of the supply in the tank, and wicks were arranged in the float feed, with the upper loose ends against a gauze screen. Hot air drawn by the piston suction through the upper loose part of wicks was charged with petrol by capillary action, and passed through the gauze to a mixing chamber. An auxiliary supply of fresh air was mixed with the carburetted air on its way to the engine cylinder.

The *jet spray carburettors*, fitted with automatic float fed regulator and now in common use on the petrol motor, are modifications of that patented by William Maybach in 1893, although anticipated by the invention of Butler in 1889. The petrol supply from a tank above the float-feed chamber A, Fig. 114, or by slight air pressure in the tank, passes through a filter of very fine gauze, and enters by the needle valve P, which maintains the desired constant level in the small float chamber, and of the petrol a little below the top of the orifice of the spray nozzle K at slow speed, or low rate of piston suction. The hollow and perfectly oil-tight brass float B is free to move up and down on the needle valve spindle N, as the

oil level rises and falls in the feed chamber. Two small levers, turning about *FF*, have their weighted ends *WW* resting on the top of the float, and their other ends are fitted in the grooved collar *C* on the valve spindle. When the level of the petrol in the float chamber *A* falls slightly, the float sinks with the ends *W*, causing the levers to lift the collar *C* and the needle *N*, allowing petrol to flow into the chamber until the float is raised to the *constant level*, when the levers press the needle valve downwards on its seat to shut off the petrol supply. The *choke tube* around the jet is reduced in area so as to increase the velocity of the air and thereby reduce the pressure over the jet during the suction stroke of the motor piston. The inrush of the charge of air carries the petrol spray and vapour along the induction pipe *E* into the cylinder. The air inlet is adjusted to form an explosive mixture. The area of the jet orifice is designed or *set* so that the velocity of air, when the engine is running· at normal speed, is just sufficient to give the correct proportion of petrol to air, about 1 to 15 by weight in the charge. At higher speeds, the momentum or inertia of the flow of petrol, combined with the increased resistance of the air, tend to make the charge too rich in petrol, and various devices have been adopted to introduce *auxiliary air* and obtain a mixture of constant proportion.

The latent heat of evaporation of the petrol controls the suction temperature and therefore the weight of charge in the cylinder. Heat is added to the fuel and air mixture by contact with the hot valves and cylinder walls ; also by admixture with the hot residual products left in the clearance space of the cylinder.

In 1898, at Coventry, the Author carried out a brake test on a Daimler motor having two vertical cylinders, each 90 m.m. diameter by 120 m.m. stroke, which developed 6 to 6·5 B.H.P. as the speed was raised from 700 to 750 r.p.m. The consumption of petrol, of specific gravity 0·680, was 0·8 lb. per brake horse-power hour. When the motor was running at full load, the cooling water entered the cylinder jacket at the temperature of the room and left the outlet as steam, yet a thin layer of frost was formed on the induction pipe between the Maybach carburettor and the cylinder, owing to the latent heat absorbed in the rapid vaporization of the liquid petrol. Hence the necessity for pre-heating by the induction pipe to reduce or prevent condensation of the heavier fractions of the oil vapour in the pipe before entering the cylinder.

Fig. 115 is a transverse section through a four-cylinder petrol engine of the motor-car type, and diagrammatically shows the simple construction.

The head is detachable, the cylinder and top half of the crank-case are cast together. The cylinders are cast separately and often in pairs *en bloc*. The exhaust valve seating is formed in the cylinder casting. The valve rod guide is well water-cooled. For high speeds, the valve stem is bored out hollow, the metal on the large head

being reduced to make it as light as possible, with large bearing surface on the conical seating. There is also ample water-jacket space in the cylinder head and around the liner.

The circulation of the cooling water is usually by *thermo-siphon.* The water, when heated in the engine, flows upwards through the outlet from the cylinders into a large pipe with very gradual slope into the top of the radiator placed in front of the engine. This
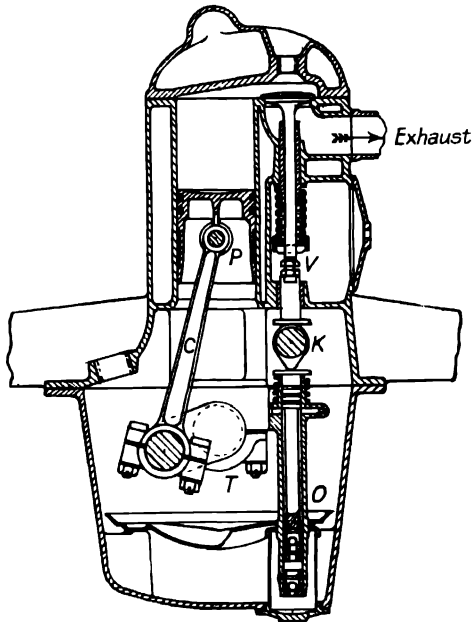
FIG. 115. PETROL ENGINE (VERTICAL SECTION)

cooler consists of a small reservoir at the top and bottom, connected by a nest of small tubes or pipes exposing a large surface, over which fresh air is drawn by a fan or the motion of the car, and cools the circulating water as it flows downwards to the cylinders. In small aeroplane and other engines the cylinders have cooling fins on the outer surface, and are simply air-cooled.

Continuous and correct lubrication of all bearing surfaces is of primary importance. Suitable oil is regularly poured into the filler through a fine gauze filter and strainer. Distribution is by a cam-driven pump. O, of the plunger type, with suction and delivery ball valves. A scoop on the big end of the connecting rod C dips into the oil trough T, and splashes the lubricating oil in the enclosed crank-chamber on the cylinder walls, piston, cams, and rollers ; while the oil to the bearings is under pressure from the oil pump.

The trunk piston, P, is light in weight, and made of cast iron,

pressed mild steel, or aluminium alloys for high speeds, with flat, slightly concave, or arched top, and has two or three narrow piston-rings, split diagonally, and with ample clearance at the joints to allow for expansion without butting together. These rings suffice to keep the cylinder gas-tight, and conduct part of the heat from the piston to the cylinder liner. One good design of aluminium alloy, ribless piston, for normal engine speed of 2,500 r.p.m., has a flat crown, four deep narrow piston-rings, and a long skirt split into sectors with springs inside, ensuring uniform contact over a large bearing surface on the liner. The piston acts like a cross-head and receives the side thrust from the connecting rod.

Grinding and polishing of the piston and combustion chamber retards deposit of carbon, and absorbs less heat than a rough surface from the burning gases, while a slightly concave piston crown aids the circulation and turbulence of the charge, giving more rapid combustion. The stroke/bore ratio varies from 1 to about 1·5.

Ignition is by electric spark from a magneto, or an induction coil and battery for starting. The sparking plug is above the exhaust valve, its best position is at the top of the cylinder head, equidistant from the side walls. The order of firing in the four cylinders is usually 1, 3, 4, 2 in petrol car engines working on the four-stroke cycle, and so giving two impulses during each revolution of the crankshaft, with fairly uniform torque and balance, but not so good as six-cylinder engines for larger power output.

Carburation. In the steady flow of liquid petrol from a jet spray carburettor at the nozzle the velocity varies as the square root of the pressure difference or head, between the float chamber and the nozzle, according to the parabolic law $V = C \sqrt{2gh}$; and the discharge becomes $W = 60·2CA \sqrt{\rho h}$, where W is weight in pounds of the liquid fuel discharged per minute, ρ the density of the liquid, h the head or pressure drop in inches of water column, A the area of passage in square inches, and C the coefficient of discharge, including losses due to skin friction of the pipes and fluid friction or viscosity, and loss due to contraction of the stream. (See p. 149, *et seq.*)

Viscosity is the internal friction or resistance offered by the particles or interfaces of a fluid to steady flow over each other, or to change of shape of the mass, and which renders its flow sluggish. The relative viscosity of liquids at different temperatures is readily measured by the Redwood standard viscometer. The kinematical viscosity of a fluid is the viscosity divided by density (p 156). When the velocity of a viscous fluid, as air, exceeds a critical value found by Osborne Reynolds and

$$v \text{ ft./sec.} = \frac{1000 \times \text{viscosity}}{\text{Density} \times \text{radius of pipe}},$$

the motion becomes turbulent, and the resistance varies as the

square of the velocity.   Besides, the viscosity of air increases with
its temperature, whilst that of liquid petrol diminishes.   The rate
of flow or fluidity of a liquid is the reciprocal of its viscosity.   The
relation between fluidity and temperature is shown in Fig. 116 from
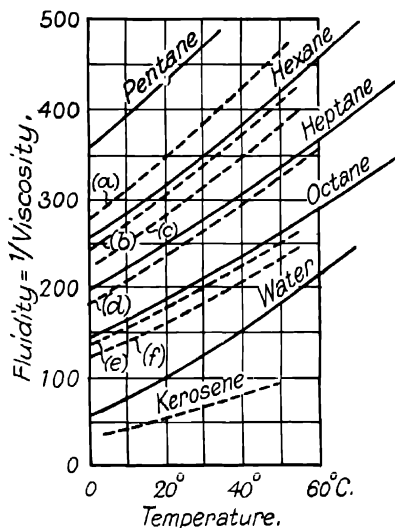
FIG. 116. RELATION BETWEEN FLUIDITY AND TEMPERATURE OF
PETROL

experiments* by W. H. Herschel, on gasoline and various aviation
petrols -

Sample	a	b	c	d	e	f	Kerosene
Specific Gravity	·680	·694	·702	·722	·757	·748	·813

The curves,† Fig. 117, are for the discharge of petrol flowing at
different temperatures through the same submerged orifice with
chamfered ends, length of passage 0·407 in. and diameter 0·0344 in.,
in which the effect is greater than with the smaller values of the
ratio length/diameter found in carburettor practice.   Whereas for
very short square-edged passages, length 0·005 in. by diameters
0·042 in. and 0·02 in , discharging into air, the variation is inappre-
ciable, both with respect to change of head, and temperature from
24·5° to 4° C.

* Bureau of Standards Technologic, U.S.A., Paper No. 125.
† Report issued by the National Advisory Committee of Aeronautics,
Washington, U S.A.

Professor Watson measured the petrol that issued from the spray nozzle or jet, and the quantity of air that passed through a
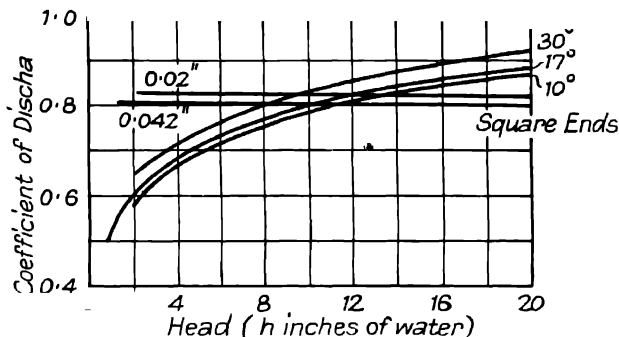
FIG. 117. EFFECT OF TEMPERATURE ON FLOW OF PETROL

carburettor, at *constant suction*. In Fig. 118. the curve *OBA* shows the relation found between the quantity of air that passed through the carburettor and the suction ; while *CBD* gives the air which, when mixed with the petrol found to be discharged from the nozzle,
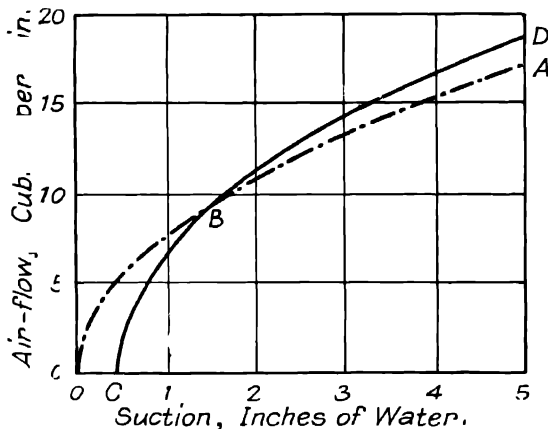
FIG. 118. FLOW THROUGH CARBURETTOR. (*Watson*)

would form a mixture of constant proportion air to petrol.   Until the suction was more than ½ in. head of water, no petrol issued from the nozzle, and above 1·5 in. head of suction the mixture became too rich in petrol.

Mr. H. E. Wimperis takes the energy equations* in Perry's *Applied Mechanics* (pp. 533 and 537), but with the assumptions that (1) the flow of air through the carburettor is steady and without

* *The Internal Combustion Engine*, pp. 266–7.

friction, and (2) viscosity may be neglected. For steady motion of a fluid, the stream line law of constant energy is

$$\frac{v^2}{2g} + h + \int \frac{dp}{w} = \text{constant.}$$

The total energy per pound of fluid is the sum of the kinetic, potential, and pressure energy, and " remains constant, except in so far as friction may diminish it, ' and this must be determined by experiment in each case.

Apply this energy equation to the adiabatic flow of air from rest to a velocity v_a through the carburettor in a horizontal direction; and take p the pressure on the surface of the petrol in the float chamber, and p_a at the top of the petrol jet, then the small difference

$$\delta p = p - p_a = w_a \frac{v_a^2}{2g}$$

where v_a is the velocity of air and w_a its density.

Again, for the liquid petrol, w, the weight per cubic foot, is constant, and the energy equation becomes

$$\frac{v_2}{2g} + h + \frac{p}{w} = \text{constant,}$$

here h ft. is the height of petrol in the jet above the free surface of the petrol at rest in the float chamber, where the stream line begins. Equating the total energy here and at the jet, we have

$$0 + 0 + \frac{\delta p}{w} = \frac{v^2}{2g} + h + 0,$$

where v is the velocity of flow of petrol at the jet, or, substituting the above value of δp,

$$\frac{v^2}{2g} = \frac{\delta p}{w} - h = \frac{w_a}{w} \cdot \frac{v_a^2}{2g} - h$$

hence $\qquad v^2 = v_a^2 \cdot \frac{w_a}{w} - 2gh \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$

when $v = 0$, there is no flow of petrol at the jet, and the velocity of the air flow is

$$v_a = \sqrt{2gh \cdot \frac{w}{w_a}} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

the velocity of the air and suction must be greater than this before the petrol begins to flow.

Take, for example, petrol (p. 234) of specific gravity 0·72 at 60° F., of which 1 cub. ft. weighs $0·72 \times 62·4 = 45$ lb. (nearly) $= w$; and 1 lb. of air at atmospheric pressure and 60° F. occupies 13·07 cub. ft., or 1 cub. ft. of air weighs 0·0765 lb. $= w_a$.

Substituting these values in equation (2), gives the critical velocity of air for no flow of petrol—

$$v_a = \sqrt{64\cdot4 \times \frac{45}{\cdot0765} \times h} = 194\cdot64\sqrt{h}.$$

If h is $\frac{1}{2}$ in. $= 0\cdot042$ ft., $\sqrt{h} = 0\cdot205$,

then $v_a = 194\cdot64 \times 0\cdot205 = 39\cdot9$ ft. per sec.,

nearly 40 ft. per sec., or 2,400 ft. per min. When h is $\frac{3}{8}$ in., the critical velocity is $34\cdot4$ ft. per sec.

Equation (1) becomes $v^2 = 0\cdot0017v_a{}^2 - 2gh$, or

$$\frac{v}{v_a} = \sqrt{0\cdot0017 - \frac{2gh}{v_a{}^2}} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

from which it is obvious that with increase of the air flow, v_a, the ratio of petrol to air increases, and the mixture becomes too rich, as in Fig. 118.

The rapid pulsation of flow due to the varying suction of the piston at very high speeds will give much greater momentum to the petrol than to the air, which, being lighter, responds more readily to the pulsations, and a surging action may take place, owing to the rapid changes of suction and the inertia of the jet tending to precipitate the finely divided petrol held in suspension only by the velocity of the air. Short induction pipes of large diameter are used to reduce wire-drawing or throttling effect.

The effect of friction is to reduce the velocity of flow and the weight of charge. Bends in the inlet pipe system also increase the frictional resistance and cause eddies, the effect is equivalent to loss of head ; and for a given diameter of pipe the frictional loss of heat varies inversely as the radius of the bend.

In some carburettors the friction of the orifices is made so large that inertia effects are almost damped out. High friction is one of the means adopted to counteract the inertia of the petrol, and this may be obtained by making the orifices small and increasing their number. Another advantage of this multi-jet device is to expose a greater surface of petrol spray to the air, and produce a more intimate homogeneous mixture with air to assist the evaporation of petrol. It is usual to warm or pre-heat the main air supply by taking it from around the exhaust pipe or other hot surface.

In order to supply the latent heat of evaporation of the petrol, and to prevent condensation of the vapour on the inside of the induction pipe between the carburettor and engine cylinder, it is absolutely necessary to use some heating device. The best is hot water from the cylinder jackets when there is pump circulation, but when this is by thermo-syphon, the induction pipe may be heated by the exhaust gases or clean hot air ; and the carburettor is kept near the cylinder, shielded from cold winds in the winter.

The quantity of the mixture of air and petrol vapour entering the engine cylinder is controlled by the *throttle valve*, which also varies the suction. When the throttle is nearly closed, as at starting, only a small charge is drawn into the cylinder, and the compression is reduced, so that a richer mixture is needed to ensure ignition, while the rate of flame propagation is reduced. A *choke tube* in carburettors reduces the area of the induction pipe around the jet, and greatly increases the velocity of the inrushing air, and the suction or depression at that section.
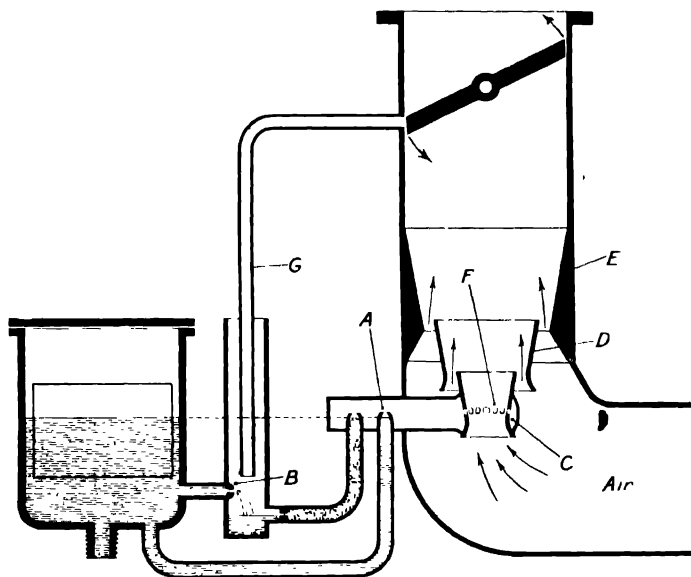
FIG. 119. DIAGRAM OF THE ZENITH CARBURETTOR

The **Zenith Carburettor** has three choke tubes or diffusers, *C*, *D*, and *E* in the diagram, Fig. 119, and a compound jet consisting of the ordinary main jet *A*, giving too weak a mixture at low piston speed and which grows richer as the engine speed increases. In order to correct this, a compensating jet is fed from the well by the orifice *B*, which is at atmospheric pressure, and is not appreciably affected by the suction of the engine. The flow of petrol from the compensating jet cannot increase with the engine speed, but the air passing through the carburettor does, therefore the mixture given by this jet becomes gradually weaker in petrol as the engine speed increases. By combining these two separate petrol feeds a correct proportion of petrol to air in the mixture is maintained at all speeds.

The petrol, correctly measured by the main and compensating jets, is sucked through the ring of diffuser holes *F* by only a little

air passing through the small choke tube *C* at a very high velocity. When the mixture of sprayed petrol and air passes out of the small choke tube *C*, it is sucked up by another stream of air through the concentric choke tube *D*, at a lower velocity, forming a mist of petrol and air. This mist is taken up by a further supply of air, through the large main choke tube *E*. The quantity of air added is necessary to maintain the correct quality of the mixture. The three streams of air flowing, at different velocities, set up turbulence and tend to produce a homogeneous mixture or charge, which, when heated in the induction pipe, and in the engine cylinder by admixture with the residual gases, is all converted into vapour before the inlet valve closes.

The starting and slow running tube *G* dips into the well *B* that feeds the compensating jet, at atmospheric pressure, and provides
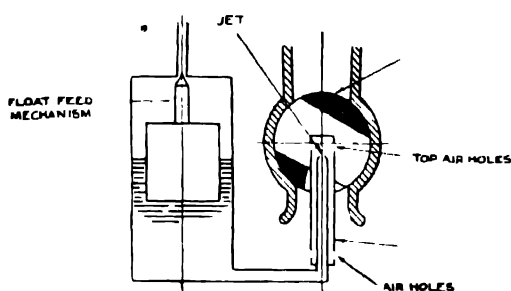
FIG. 120. SLOW RUNNING POSITION   FIG. 121. FULL THROTTLE POSITION
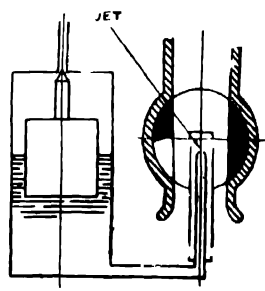
a suitable rich mixture for starting. When the throttle is just slightly open for starting, there is little suction on the main and compensating jets, and the velocity of the air past the top end of the tube *G* draws the petrol up the tube, and through a small outlet at the edge of the throttle valve into the induction pipe. The mixture strength can be adjusted to a nicety to suit the particular engine by means of a spring-controlled screw, which opens or partially shuts a small air passage and so increases or reduces the richness of the mixture for slow running. After the engine has been running for a little time at slow speed, petrol rises in the well, which, added to that from the main jets, enriches the mixture so that rapid acceleration is possible when the throttle, shown at the top of Fig. 119, is suddenly opened.

On this type of Zenith carburettor there may be fitted a *mixture control*, connected to a small indicator on the dashboard of the car, and the mixture strength changed by moving the control button to one of three positions for "starting," "normal," and "weak mixtures." There are two valves, operated by a lever from the

button—one controls the air supply to the slow-running tube, and the other admits air to the passage between the jets and diffuser.

Claudel-Hobson Carburettors may be divided into two classes. First, the air injector type, fitted with a single jet tube, surrounded by a concentric tube or sleeve, drilled with rings of small air holes near the top and bottom, into the annular air space between the tubes.

The top of the tubes above the nozzle and upper ring of air holes is covered by a cap, and projects into the mixing chamber of the horizontal revolving barrel throttle, as shown in the diagrams, Figs. 120 and 121.

In the lower edge of the throttle radial air slots are cut, Fig. 122, to regulate the correct proportions of air to petrol entering the mixing chamber at closed and intermediate throttle positions.

FIG. 122. CLAUDEL-HOBSON THROTTLE

When the throttle is closed as far as possible, Fig. 120, the engine does not stop, but most of the air enters the lower ring of holes and, passing the jet, carries the petrol spray through the upper ring into the throttle chamber to mingle with the extra air admitted by the slots, and give a rich mixture for starting and running light. The area of the holes in the air tube is adjusted by the makers.

The diffuser type, Fig 123, is designed for multicylinder engines running at a very wide range of speed on less volatile fuels, such as various mixtures of petrol, benzole, and alcohol, which require more thorough pulverization than gasoline.

In both types any grit or deposit in the petrol settles in the space below the diffuser jet and may readily be removed by unscrewing the cap. From the float chamber the petrol enters the main submerged orifice, K, Figs. 123 and 124, and flows into the three concentric tubes, B, N, and M, when the engine is developing average power. The outer tube, P, contains only air, and has a ring of holes at H, Fig. 123, to the annular space Y and the air intake passage I, Fig. 124.

For starting and slow running, the central tube B, Figs. 123 and 124, has the petrol orifice X, the air supply for spraying drawn through the holes DD above X, and the ring of delivery holes CC.

The main orifice K admits more petrol than is required for slow

running, and the level of petrol rises in the diffuser tube N up to about $\frac{1}{8}$ in. to $\frac{1}{4}$ in. below the top of the guard tube M, so that the air holes FF are successively submerged.

As the engine is accelerated and the ignition slightly advanced, the petrol level in the diffuser tubes falls, and automatically diverts part of the engine suction to the atmosphere for the main air supply. Part of the air is drawn through any holes FF which are uncovered and in communication with the atmosphere through the tube M, open at the top to the outer air tube P. Then the petrol is also sprayed through the ring of holes G. The main air enters below and around the top of the diffuser tubes, Fig. 124.

When the throttle is in the closed position for slow running, the delivery may be increased and the mixture enriched
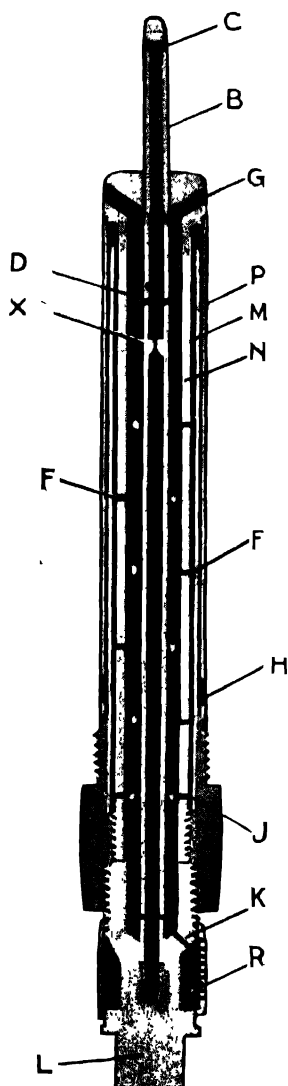
FIG. 123. SECTION OF
CLAUDEL-HOBSON DIFFUSER
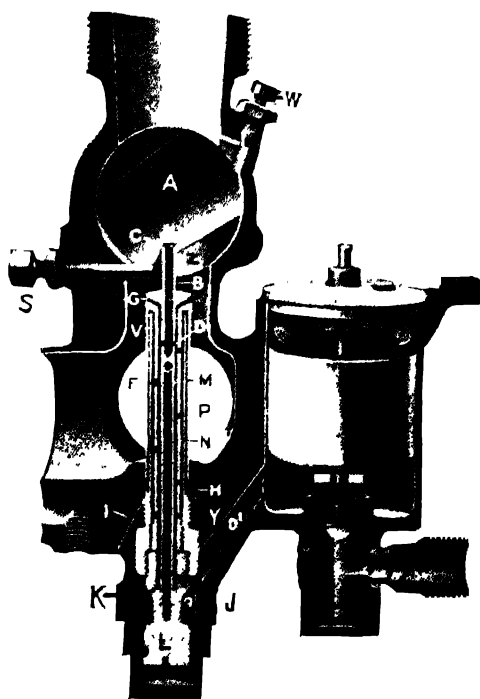
FIG. 124. CLAUDEL-HOBSON DIFFUSER
CARBURETTOR

by further screwing in the air screw S, Fig. 124. A by-pass screw W gives a fine adjustment on the quantity of mixture passing to the inlet or induction pipe. If the mixture is too strong, the engine keeps going irregularly, fast and slow, instead of running smoothly and slowly; but with too weak a mixture, and the throttle closed as far as possible, the engine may stop.

In *making adjustments* the size of jets may be selected, but not reamered or tampered with. The main jet orifice K should discharge just sufficient petrol for medium acceleration, and is kept small for economy of fuel through the throttle range. The power jet, in the petrol passage between the float chamber and diffuser, provides extra supply of fuel at full throttle for maximum power and acceleration. The correct size of these two jets, for any particular car, is found by testing the maximum power, under load, at full throttle on top gear, both on the level and up a hill steep enough to reduce the engine speed. In case the speed is reduced too much in climbing the hill, it is better to increase the size of the power jet rather than the main orifice K, if the acceleration at half throttle is satisfactory. Next, adjust the throttle-stop for slow running. If the air screw is right in and the mixture is still too weak, open the by-pass slightly, and reset the throttle-stop so that the throttle can close a little more. This enriches the mixture by allowing the slot in the throttle to close farther around the slow running tube, increasing the velocity of the air and the suction at the jet. For engines with a stroke more than $1\frac{1}{2}$ times the bore of cylinder, or running at higher speeds than 2,000 r p m., a larger size than the standard carburettor may be required.

Successful working has been obtained by many aero and other petrol engines fitted with Claudel-Hobson carburettors, and the principles of working have given both economy of fuel and reliability under widely varying conditions.

Various arrangements of other multiple jet carburettors have been employed. Adjustment to keep the mixture strength constant at different speeds of suction is usually obtained by varying the size or number of jets. The number of jets, or area of opening uncovered and exposed to the suction is proportional to that of the throttle, and for full throttle a main jet with extra air supply comes into action. A pilot or primer jet is generally used for starting and supplies a very rich mixture.

Starting from Cold. In addition to the pilot jet in the carburettor, a very rich mixture may be obtained by " flooding " the carburettor, or by " doping " the cylinder with a little of the liquid petrol, which spreads over the internal walls of the induction pipe or cylinder, and a sufficient quantity of the more volatile fractions evaporate from the large surface, thus exposed, to form a combustible mixture with the air. When turning the starting handle by hand, the velocity through the induction pipe is low.

The readiness of a fuel to start an engine from cold is indicated by the vapour pressure and the latent heat of evaporation, as well as the ratio of air to fuel required to give complete combustion. These values for the more volatile fractions of petrol in comparison with alcohol and ether, from experiments by Mr. Ricardo, are as follow—

	Specific Gravity at 15° C.	Boiling Point, °C.	Vapour Pressure at 0° C.; m.m. Mercury	Latent Heat, B.Th.U. per lb.	Ratio Air/Fuel by Weight.
Hexane . .	0·670	69	45·0	156	15·2
Cyclohexane	·780	81	27·5	155	14·7
Benzene	·884	80	26·0	172	13·2
Ethyl Alcohol	·794	78	12·7	397	8·95
Ether . .	·719	35	185 0	158	11·14

The addition to alcohol of a small proportion of ether, with its high vapour pressure, renders starting easy in a suitably designed engine and carburettor.

Exhaust Gases from Petrol Engines. Professor B. Hopkinson and Mr. L. G. E. Morse made experiments on a four-cylinder, 16 to 20 H.P. Daimler engine, compression ratio 3·85, to determine the conditions under which the poisonous gas CO is formed in an internal combustion motor, and the relation between the composition of the exhaust gases, the strength of the petrol and air mixture, the power developed by the engine, and the thermal efficiency.

Separate controlling valves were fitted on the air stream over the carburettor jet, and on the extra air added to the mixture before entering the inlet pipe of the engine, and by slightly varying the throttle the petrol consumption could be adjusted, *at constant speed and with constant amount of air taken into the engine per stroke.* The main air inlet was kept open to the atmosphere with but little throttling, so that the suction in the inlet pipe of the engine near the air inlet valves, measured by mercury gauge, was always within $\frac{1}{2}$ lb. per sq in. of the atmosphere. When running at constant speed, and with constant suction, the mechanical and pumping losses in the engine were also constant and amounted to 2 H.P. at 725 r.p.m.

In making a test, the air passages were first set to give the desired petrol consumption. The timing of the ignition spark was then adjusted until the brake horse-power was a maximum. When all the conditions had become steady, the petrol used during 1.000 revolutions was observed. Samples of the exhaust gases, taken over mercury, were analysed by the ordinary volumetric methods, the CO_2 being absorbed by potash, the oxygen by pyrogallol, the CO by an acid solution of cuprous chloride, and the hydrogen by palladianized asbestos. The measured amounts were reckoned as

percentages of the volume of dry exhaust gases. The residual gas, after all these were absorbed, was nitrogen, mixed possibly with traces of some unburnt hydrocarbon vapours, and with water vapour for saturation, were taken as about 2 per cent. There was some soot always formed, and probably acetylene, which tend to reduce the burnt carbon as compared with the burnt hydrogen.
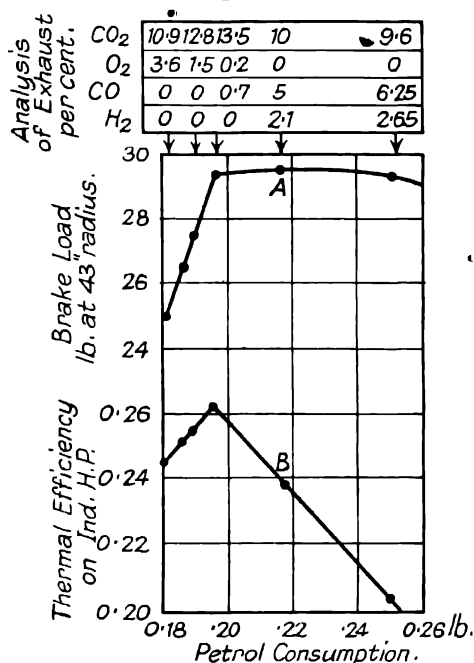
FIG. 125. PETROL CONSUMPTION AND EXHAUST GASES

The nitrogen found by difference, multiplied by 0·266, gives the total amount of oxygen in the air used. Deduct from the total oxygen that which went to form the absorbed gas (its volume is equal to the sum of the CO_2, the O_2, and half the CO), the remainder is the oxygen which combined with hydrogen to form steam, and the volume of steam is thus calculated.

The petrol used was Pratt's motor spirit, of density 0·715 to 0·720, and its lower calorific value 18,900 B.Th.U. per lb. Its composition, determined by combustion, and weighing the CO_2 and water produced, was H_2, 14·86, and C, 84·66 per cent ; from which the volume of steam formed should be 1·05 times the combined volume of CO_2 and CO, in case the petrol is all burned. The ratio of these two volumes actually found is always considerably greater than this, the smallest volume for 0·197 lb. of petrol being 1·18.

The analyses show that when the petrol consumption is about 0·2 lb. per 1,000 revolutions, the available *oxygen* is completely burned to CO_2 and steam. However, the petrol is not completely burned, since there is always some soot and hydrocarbons in the exhaust.

In later experiments, after absorbing the CO_2, O_2, CO, and H_2, Hopkinson exploded the *residual gas* with some additional oxygen and electrolytic gas. He invariably obtained a further yield of CO_2, which, with a mixture giving no CO in the exhaust, amounted to about 5 per cent of the total amount of CO_2 produced by the combustion. He also observed that the residual gas was soluble in water, probably indicating acetylene, which would account for a good deal of the missing carbon.

The available oxygen is about sufficient to burn 0·19 lb. petrol. If the supply of petrol exceeds 0·2 lb., oxygen disappears from the exhaust, but CO and hydrogen are present in increasing quantities ; if it be less than 0·197, there is excess of oxygen and no CO.

Curve *A*, Fig. 125, giving the relation between brake load and petrol consumption, has a very flat maximum from 0·196 to 0·25 lb. per 1,000 revolutions, and the quantity of the poisonous gas CO rises to 6·25 per cent, while the *power is nearly a constant maximum* over this range. The thermal efficiency, curve *B*, on the indicated power, based on the lower calorific value of the fuel, reaches a very sharp maximum, 26·1 per cent at 0·197 lb. per 1,000 revolutions, with an air to petrol ratio about 14, the CO and O_2 at a minimum, and falls off for either less or more fuel, due to incomplete combustion. The weaker mixtures diluted by the residual products of the previous explosion burn slowly.

If the carburettor be set in the usual way for maximum power, and the consumption of petrol reduced to the lowest amount consistent with nearly maximum power, the formation of CO may be prevented. This result may be attained at a sacrifice of 1 or 2 per cent of power.

Professor Watson* measured directly the weights of air and petrol supplied to four-cylinder petrol engines, and made **analyses of the exhaust gases.** He plotted the results of a large number of tests and analyses for various mixtures by weight, shown in the diagram, Fig. 126. In these experiments the maximum thermal efficiency was obtained with the ratio of air to petrol, about 17, which is too weak a mixture for general use, while about 14 lb. of air to 1 lb. of petrol gave complete combustion of petrol of density 0·720. For richer mixtures no oxygen is present, while for weaker mixtures no CO is found in the exhaust. The practical working range of mixture strength is from 11 to 17 lb. of air to 1 lb. of this petrol. Dr. Watson also gave curves† of exhaust gas analyses, showing the results for alcohol-benzol mixtures.

* *Proc. Inst. Auto. Engrs.*, 1909. † *Ibid.*, 1914–15

Free oxygen and CO do not occur together in the exhaust in appreciable quantity. Dr. Watson suggested that the diagram, Fig. 126, enables the proportion of air to petrol in the mixture to be determined for ordinary purposes from analyses of the exhaust gases, and should be the same as that obtained from direct measurement, when the combustion is complete.

Both in these results from analysis, as well as in those obtained by Hopkinson, the air/petrol ratios do not agree with the ratios found by direct measurement. Also for mixtures within the common working range the usual method of analysis and calculation gives too little carbon in the exhaust gases.
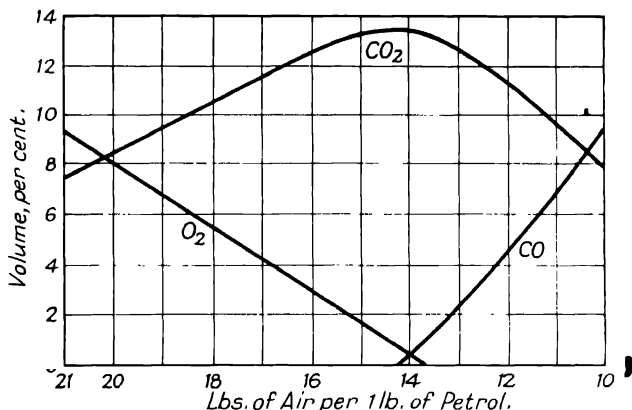
Fig. 126. Analysis of Exhaust Gas (*Watson*)

Professor Bone found a considerable disappearance of both hydrogen and carbon from the combustion of heavy hydrocarbons in a Diesel engine. This discrepancy he explained by the presence of aldehydes found in solution in the water condensed in the exhaust pipes of both petrol and heavy oil engines. Hence it is evident that the fuel is not completely burned.

Tests made by the Royal Automobile Club on the exhaust from various motor-car engines also proved that the poisonous gas, CO, was present in all cases, and it was decided that the standard mixture for tests of carburettors should be that giving exhaust gases containing about 1 per cent of free oxygen. From Watson's experiments, Fig. 126, this corresponds to no CO, and to the air/petrol ratio 14·5 ; about the average between the ratio giving maximum power 11 to 14, and that of 17 for maximum efficiency.

Recent research indicates that for any volatile liquid fuel the highest thermal efficiency is always obtained with about 15 per cent excess of air in the charge.

Valve Setting.   Effective carburation is greatly assisted by the correct design and timing of cams, valves, and piping.   For ordinary high speed touring car engines, early opening of the exhaust valve about 35° to 45° before the bottom centre, during the explosion stroke, prevents back pressure during the exhaust stroke, and tends to keep the engine cool, although the pressure of gases in the cylinder should be reduced as much as possible by doing work on the piston before release, as well as to reduce the noise of the exhaust.   Then, in order to clear out the burnt products, especially CO_2, which retards combustion, advantage is taken of the kinetic
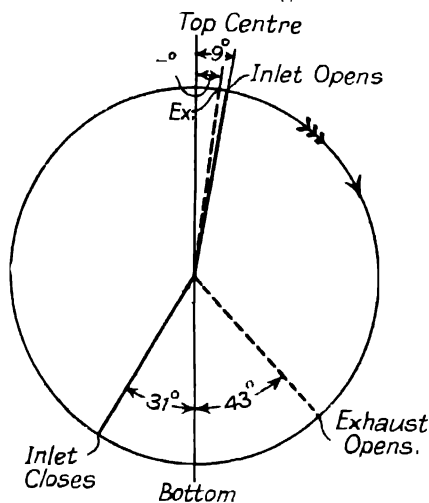
FIG. 127. VALVE SETTING

energy of the column of escaping gases, and the exhaust valve is left open long enough after the piston has reached the top centre, at the end of the exhaust stroke, until the pressure of the gases in the cylinder is below that in the inlet pipe near the valve.   It is found in practice that, at high speeds, free exhaust increases the power of the engine and effects a saving of petrol.

For racing cars, slight overlapping is sometimes used, the inlet valve commences to open before the exhaust valve is completely closed, so that the induction of the fresh charge tends to sweep out the residual products.   At this point in the cycle of a four-cylinder engine, the piston of the cylinder which fires next is at the bottom of its stroke ; its exhaust gases are flowing down the exhaust pipe with great velocity, if its exhaust valve was opened early.   Thus the scavenging action in the first cylinder is assisted by the kinetic energy of the gases from the succeeding cylinder in firing order. This overlap may be produced automatically above certain speeds by the lag of the exhaust in closing owing to the rollers " jumping "

the cams. But by this scavenging action a considerable portion of the incoming charge will pass unburnt down the exhaust pipe, especially when the inlet and exhaust valves are close to one another.

During induction with pulsations the relative inertia of the light air and heavier vapour tends to cause lag of the petrol spray behind the air, but when the periods synchronize fairly closely, and the flow of the air and petrol coincide, the petrol is well pulverized. The point for closing the inlet valve is usually about 20° after the bottom centre, under full throttle, when the pressure inside the cylinder is atmospheric. Any longer opening will cause pulsations by inlet back flow, and any less will stop the flow before the cylinder is full of the fresh charge. There cannot be one standard setting for all engines, since the correct setting will depend upon the diameter and lift of the valve, strength of springs, the suitable angle of inlet and exhaust passages, and can only be arrived at by experiment.

A typical example of poppet valve setting is that adopted in the four-cylinder engine of the Wolsely Company, Fig. 127, which shows the positions of the crank at the opening and closing of the inlet and exhaust valves.

The valve timing in the Daimler sleeve-valve engine, with 25° overlap, indicates different conditions of working —

ENGINE	VALVE SETTING		
		Opens	Closes
Wolseley . .	Inlet	9° late	31° late
	Exhaust	43° early	7° late
Daimler . . .	Inlet	5° early	54° late
	Exhaust	64° early	20° late

Interesting experiments* were carried out on a four-cylinder engine, bore and stroke $4\frac{1}{2}$ in. by 5 in., having valves $1\frac{5}{8}$ in. diameter and $\frac{7}{16}$ in. lift, both of these being too sma for the size of the engine.

Different sets of cams, varying in profile, were fitted, giving the various valve settings in the table, and the results of brake tests of the engine are shown by the curves, Fig 128.

In test *A* the inlet valve was not open long enough to admit a full charge, and the late opening of the exhaust would produce high back pressure during exhaust, and the gases would not be cleared out of the cylinder.

In *B* the increased duration of admission allowed a greater weight of charge to enter the cylinder, and the gas had time to escape

* *Automotor Journal,* June 12th, 1909.

due to the early opening of the exhaust valve, while its late closing with 3° overlap increased the power and speed.

The setting in test *C*, without overlap, gave the best curve of the series, and sustained increase of power at high speeds.

With an engine of the same cylinder dimensions, but with connecting rods 10 in. long instead of the previous 12 in., and valves 2 in. diameter having $\frac{5}{16}$ in. lift, timed as in test *C*, except that
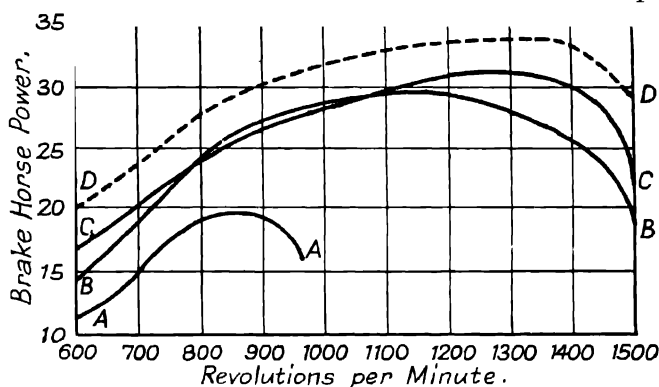
Fig. 128. Curves Showing Change in Power Due to Various Cams

the exhaust valve opened at 37° early, test *D* showed the possibility of increased power, due chiefly to the larger valves giving freer admission and exhaust.

Tests		Valve Setting		Maximum B.H.P. Obtained	Speed, Revolutions per Minute
		Inlet	Exhaust		
A	Opened	8° late	14° early	19·3	865–877
	Closed	0° late	0° late		
B	Opened	9° late	47° early	29·2	1150
	Closed	17° late	12° late		
C	Opened	13° late	39° early	31·2	1300
	Closed	22° late	10° late		
D	Opened	13° late	37° early	33·7	1365
	Closed	22° late	10° late		

Volumetric Efficiency may be defined as the ratio of the actual volume of combustible mixture or charge drawn into the engine cylinder per cycle at standard temperature and pressure, to the volume swept by the piston in one stroke. In practice, the actual volumetric efficiency depends upon several factors, including chiefly (*a*) the temperature of the charge as it enters the cylinder, (*b*) the

resistance of the inlet valves and pipes, and (c) the back pressure of the residual products left in the clearance space.

The curves, Fig. 129, give the average volumetric efficiency at standard pressure and temperature, obtained by Mr. H. R. Ricardo from a large number of direct tests, and from many indicator diagrams. With valves in the cylinder head, the volumetric efficiency *A* is higher than *B* when they are in the side pockets. Experience shows that, with a gas velocity of 100 ft. per sec. through the inlet valves in the head, the average volumetric efficiency is 77 per cent, and the highest recorded is 79 per cent.

It is necessary to heat the incoming charge in order to supply the latent heat of evaporation of the petrol and to prevent condensation in the induction pipe. After passing through the inlet valve, the fresh charge will receive heat from the combustion chamber, cylinder
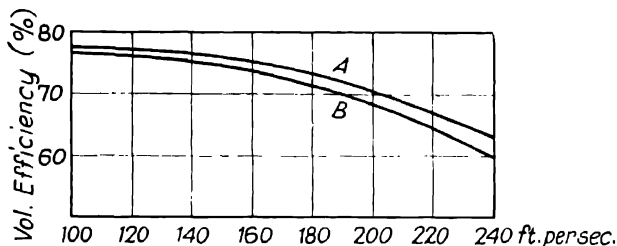
FIG. 129. RELATION BETWEEN VOLUMETRIC EFFICIENCY AND VELOCITY OF FLOW THROUGH VALVES (*Ricardo*)

walls, and valves. Before mixing with the residual products in the cylinder, the temperature of the charge will be about 60° C., or 333° C. (absolute), and its density is reduced to $\dfrac{273}{333} = 0{\cdot}82$, or 82 per cent of that at standard temperature and pressure, provided the residual gases are at atmospheric pressure and the inlet valves and pipes offer no resistance. The higher the velocity of the gas through the inlet valve the greater will be the turbulence of the charge and the rate of propagation of flame after ignition, but the greater also the loss by throttling and the lower the volumetric efficiency. Experience tends to show that, for turbulence and rapid combustion, the velocity of the fresh charge at entry must be at least 100 to 130 or 150 ft. per sec., and, neglecting friction, the difference of pressure between the outside atmosphere around the carburettor and that in the cylinder necessary to produce this velocity may be calculated by $\dfrac{wv^2}{2g}$ where *w* is the density of the charge, i.e. 0·085 lb. per cub. ft. at 0° C. This difference of pressure is $\dfrac{{\cdot}085 \times 100 \times 100}{64{\cdot}4 \times 144} = 0{\cdot}0917$ lb. per sq. in

When a petrol engine, with the inlet and exhaust pipes removed to reduce friction and pulsations, was motored round at 1,450 revolutions per minute, such that the gas velocity through the inlet valve was exactly 100 ft. per sec., the light spring diagrams indicated a pressure drop of 0·3 lb. per sq. in. The final pressure is thus 14·4 lb. per sq. in., and the volumetric efficiency reduced to

$$\frac{14\cdot4}{14\cdot7} \times 82 = 80 \text{ per cent,}$$

the maximum attainable under these conditions.

In actual practice at high speeds it is difficult to keep the inlet velocity down to about 100 ft. per sec., because of the resistance in the carburettor and induction pipe. The velocity through a suitable choke tube may be 250 ft. per sec. to pulverize the petrol spray, with a mean " head " of 0·6 lb. per sq. in. during the suction stroke, while at the end the velocity falls and part of the kinetic energy is converted into pressure. By utilizing the kinetic energy of the gases in the exhaust pipe, the residual gases in the cylinder may be brought to atmospheric pressure. Taking the volumetric efficiency, 80 per cent, the indicated mean effective pressure in the cylinder will be reduced in proportion to 80/100, or 0·8.

Again, the temperature of the residual exhaust products after expansion and subsequent drop to atmospheric pressure can be determined from light spring indicator diagrams.

Thus, for a compression ratio of 4, the final temperature is ·1100° C. (abs.), and the density of the exhaust gas $\frac{273}{1100} = 0\cdot248$.

The clearance volume is one-third of the swept volume, and the cylinder is full of gases at atmospheric pressure at the end of the suction stroke. Then there is, in the mixture, 25 per cent of residual gas at 1100° C. and 75 per cent of fresh charge at 333° C. and density 0·82.

The proportion by weight is $25 \times 0\cdot248 = 6\cdot2$ per cent of burnt products and $75 \times 0\cdot82 = 61\cdot5$ per cent of fresh charge. Therefore the mean temperature is

$$\frac{6\cdot2 \times 1100° + 61\cdot5 \times 333°}{6\cdot2 + 61\cdot5} = 403° \text{ C. (abs.), or } 130° \text{ C.}$$

This result is in fair agreement with the suction temperature obtained by direct measurement.

There is a uniform decrease of volumetric efficiency produced by increase of the temperature in the water jacket of the cylinder. The volumetric efficiency also falls as the compression ratio is increased at constant speed for the mixture strength giving complete combustion, while the jacket temperature is kept constant. Experiment shows a slight increase in volumetric efficiency when the mixture is either richer or weaker than that giving complete

combustion ; in the latter case because of the lower combustion temperature of weak mixtures, and in the former owing to the larger proportion of liquid fuel and the fall in suction temperature by increase of the latent heat of evaporation.

The effect of throttling an engine is to reduce the weight of the charge that can be drawn into the cylinder, giving low volumetric efficiency and compression of the charge and low mean effective pressure, so that the power is reduced to suit the load on the engine. The ratio of compression and expansion is not changed, and, provided the ratio of fuel to air in the *mixture* is *kept constant,* and the *ignition advanced correctly* to allow for the slower rate of burning due to the larger proportion of inert gases in the cylinder, the thermal efficiency, that is, the fuel consumption per indicated horse-power hour, remains nearly the same, to less than half-load torque. Below that, the proportion of residual exhaust products in the cylinder becomes so large as to cause delayed and incomplete combustion. The mechanical losses remain practically the same at constant speed, and as the indicated power is reduced, so is the mechanical efficiency.

The **temperature of the mixture** in the gas engine cylinder is calculated from the value of the gas constant $R - pv/T$, just before compression begins, from the indicator diagrams, when the composition of the fresh charge is known, and allowance made for the residual burnt products left in the clearance space after exhaust, from light spring diagrams, on the assumption that the temperature of the gases remaining in the combustion chamber can be estimated. The temperature during compression is then obtained from measurements of p and v along the compression curve.

Also, on the expansion curve after explosion, allowance is made for the chemical change in volume on combustion. The constant R is independent of the change in the specific heat of the gas, and is equal to the difference of the specific heats at constant pressure and constant volume.

The difficulty in the early attempts at the *direct measurement* of temperature within the engine cylinder, by means of a platinum wire fine enough and of so small thermal capacity as to follow the rapid variation of temperature without appreciable lag, was that the high explosion temperature at once melted the fine platinum wire.

Professors Callendar and Dalby have overcome this difficulty by the ingenious device* of fitting the most delicate platinum thermometer in a jack-in-the-box valve, which exposes the fine wire to the mixture in the cylinder during compression, withdraws and screens the wire from the action of the gas momentarily at the highest temperature of explosion, and exposes it again during each suction and compression stroke.

* " On the Measurement of Temperatures in the Cylinder of a Gas Engine," *Proc. Roy. Soc.,* Series A, Vol. 80 (1907), p. 57.

The ordinary admission valve AB, Fig. 130, has an axial hole drilled through the stem and head, in which a tube moves to and fro, carrying the insulated leads from B to the platinum thermometer loop at P. This tube forms the spindle of the thermometer valve T, which fits into the little conical seating on A,
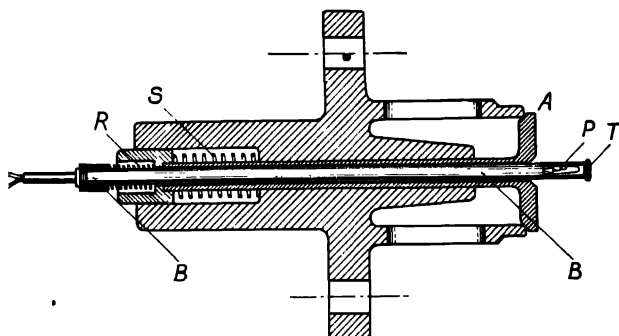
FIG. 130. ADMISSION AND THERMOMETER VALVES (*Callendar & Dalby*)

and two thin ribs join the head to the spindle so as to allow the gas in the cylinder to act on the fine wire loop P, when the valve T is pushed out as shown in Fig. 130. When this valve is momentarily closed it shields the thermometer wire from the highest explosion
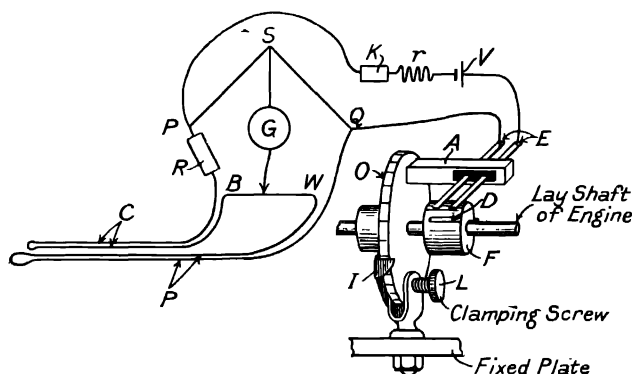
FIG. 131. CONTACT MAKER AND THERMOMETER BRIDGE
(*Callendar & Dalby*)

temperatures. The thermometer valve is pushed out and closed at the correct times by a separate cam and mechanism.

The electrical resistance or temperature of the platinum thermometer is measured by the *bridge* and *contact-maker* shown in the diagram, Fig. 131. P is the pair of leads to the fine platinum loop, and C equal and similar leads to compensate the resistance of the other pair alongside of them in the tube, while the short piece of

fine platinum wire across their ends compensates the end effect or
cooling action by conduction from the thermometer wire to the
thick leads. Thus the difference of resistance will be that of the
hottest part of the thermometer wire exposed to the gas within
the cylinder. The bridge wire *BW* is carefully calibrated.

The *contact-maker* consists of a pair of springs fitted with platinum
contacts and carried on the arm *A* of a graduated and adjustable
disk *O*, which is secured by clamping screw *L* for any desired point
I of the cycle. A pair of stepped cams *D* rotate with the lay shaft,
and allow the platinum point of one spring to make electric contact
with the other for an interval of about 10° of the crank angle,
depending upon the distance between the steps

Readings are taken at several points in the compression stroke
when the index *I* is set for the springs to make contact, and the
electric balance is adjusted on the bridge wire by the indication of
the galvanometer *G*.

Obviously, with rapidly varying temperatures, there will be time
lag between the true temperature of the gas at any moment and
that of the finest wire. On this account, Professors Callendar and
Dalby made measurements on the compression curve where the rate
of variation of temperature is a minimum, rather than on the
expansion curve after explosion when the change of temperature
may be 1000° C. in 1/10 sec., that is, at the rate of 10,000° C. per sec.
The platinum thermometer wire used was 1/1000 inch in diameter.

At engine speed 130 r.p.m., the lag of the thermometer, for points
on the compression curve, was not more than 10° of crank angle,
with a temperature variation of 200° C. in half a revolution, which
corresponds to a lag of about $\dfrac{10°}{360°} \times \dfrac{130}{60} = 0.06$ sec. To determine
the exact correction for this lag effect, the engine was motored round
with the platinum thermometer exposed and readings noted when
only air was compressed and expanded, while indicator diagrams
were taken, from which the temperatures were calculated. Com-
parison of these temperatures gave the lag effect correction.

By this method it was found that the suction or charge tem-
perature varied from 95° C. at light load to 125° C. at full load, the
atmospheric temperature being 20° C. and water jacket 27° C.
The following are the results of two tests—

	Test I	Test II
Revolutions per minute . . .	130	114
Ratio air to gas	7·1	5·8
Atmospheric temperature . . .	20° C.	21° C.
Jacket temperature . . .	27° C.	27° C.
Thermometer reading at 360° crank angle	122° C.	—.
Thermometer reading at 26° crank angle	111° C.	130° C.
Chemical contraction on combustion .	4·3%	5·1%
Maximum explosion temperature . .	2250° C.	2500° C.

The pressure, measured from the expansion curves of the indicator diagrams taken during these two trials, was practically the same, although the temperature during expansion was much higher with the richer mixture, which used more gas, without developing more power.   This may be accounted for by the greater loss of heat to the walls of the combustion chamber from the rich mixture during combustion, since the heat loss from the flame or incandescent gases and from the black piston would then be proportional to the fourth power of the absolute temperature,* but there was very little radiation from the hot gas.   The values of the maximum temperatures were probably rather high owing to the indicator available in 1907. Professor Dalby has improved the optical indicator (p. 20) ; and by this method accurate measurements of the gas constant may be obtained which will give a higher degree of accuracy.   During this research, temperatures were measured accurately, not only during suction and compression, but also at points along the expansion curve, after the maximum explosion pressure.

ı   Professor E. G. Coker and Mr. W. A. Scoble measured the **cyclical changes**† **of temperature in the cylinder** of a " National " gas engine having cylinder 7 in. diameter by 15 in. stroke.   The swept volume was 577 cub. in., clearance 120 cub. in.   The engine was built to develop 12 I.H.P. at 240 r.p.m.   During the investigation the engine indicated about 10 H.P. at 200 r.p.m., when the air and gas inlets were adjusted to ensure an explosion every two revolutions.

The thermo-couples used were 10 per cent alloys of platinum-rhodium, and platinum-iridium, of thickness 0·0005 to 0·0008 in. and projecting $\frac{1}{2}$ in. into the cylinder, were found to withstand for some time the highest temperature and shock of the explosions from mixtures 6·7 to 7·35 of air to 1 gas, of which the lower calorific value varied from 280 to 254 C.H.U. per cub. ft.   The electromotive force of this couple is 7 to 8 millivolts at the highest temperatures attained. The melting point of platinum is taken approximately 1750° ; for rhodium about 1920° ; and for iridium 2300° C.   Sir William Crookes has shown that these metals sublime at high temperatures ; and from tests‡ at 1300° C., it is estimated that the fall in electromotive force of the thermo-couples is 5 per cent after an 8 hour run, while for an ordinary trial it is less than $2\frac{1}{2}$ per cent.

Experiments by Dr. Harker, at the National Physical Laboratory,

* Stefan's law that the rate of radiation from a black body like the piston, to a black enclosure, expressed in B.Th.U. per square foot per hour, is $16 \times 10^{-10} (T_1{}^4 - T_2{}^4)$, where T_1 and T_2 are the absolute temperatures in degrees Fahr. of the two surfaces.   See " Heat Transmission," by Professor Dalby, *Proc. I. Mech. E.* (1909), p. 924 ; also Report No. 9, by Engineering Committee of the Food Investigation Board, including results of experiments on the transfer of heat by convection of air between parallel vertical walls. October, 1922.

† *Proc. Inst. C.E.*, Vol. CXCVI, pp. 1–74.

‡ *The Measurement of High Temperatures*, G. K. Burgess and H. Le Chatelier (London, 1912).

show that the melting point of these 10 per cent alloys is about 100° C. higher than that of platinum, or 1850° C.

The couple was in a two-way fire-clay tube, secured in a steel sheath by plaster of Paris, and the sheath formed a conical valve fitted on the cylinder above the combustion chamber. Indicator diagrams were taken with a 200 lb. spring, and another with the phase altered to draw out the explosion part. In this way the
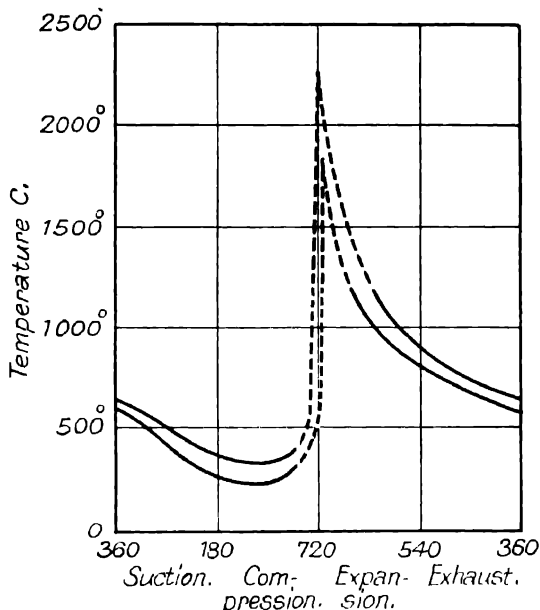
Fig. 132. Temperature Cycle of Gas in Cylinder

values of the constant pv/T for a number of points on the compression and expansion curves were obtained, the crank angle being set by a contact-maker similar to that used by Professors Callendar and Dalby. In one series of tests the constant R was 22 for compression and 21·45 selected for the expansion curve in order to avoid the correction for the molecular contraction of the gases during combustion, and to use the comparatively high temperatures and pressures measured on the expansion stroke.

The average temperatures obtained for normal running with moderately rich mixtures are given by the lower curve, Fig. 132, and the upper curve with the ratio of air to gas 5·66 to 1, the strongest the engine would take, to compare with 5·8 to 1 (p. 328). In this case the maximum pressure was 433 lb. per sq. in. and maximum temperature reckoned at 2250° C.

The weakest mixture, 7·35 of air to 1 of gas, gave a maximum temperature 1836° C. The measured values are shown in Fig. 133,

and the higher temperatures for the peak of the explosion were calculated from indicator diagrams and the gas constant, pv/T. The calculation of the suction temperature based on the mixing of the fresh charge with the residual gases gave a lower value than that observed, 200° C., or nearly double the value found above (p. 328) which is usually taken. In this engine the air-inlet seating was not jacketed and the heat was conducted along the inlet pipes; also considerable back pressure was indicated by the diagrams,
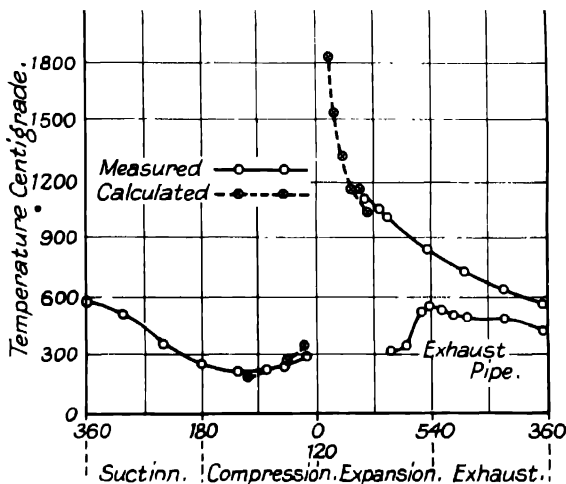
Fig. 133. Temperature in Gas Engine Cylinder

which increased the weight of the residual gas. The incoming air was heated by these, as well as by the piston and the hot walls of the combustion chamber. Thus the charge was admitted to the cylinder at a comparatively high temperature, and the power developed was reduced accordingly. The maximum pressure and temperature occurred earlier as the strength of the charge increased, and at normal charge strength the highest explosion temperatures ranged between 1830° and 1950° C.

The average heat, 800 C.H.U. per min., was supplied to the engine, of which 240 C.H.U., or 30 per cent, was converted into indicated work and 275 C.H.U. passed to the cooling water. The total surface available for conduction of this heat to the water jacket, including the piston area, was 3·5 sq. ft., and the surface area of the combustion chamber about 1 sq. ft., so that the heat flow could not be greater than 400 C.H.U. per sq. ft. per min. The heat flow is proportional to the temperature gradient, so that the mean temperature of the inner surface of the cylinder liner was less than 40° C. above that of the jacket outlet with a very small cyclical change in the metal.

Thermo-couples, consisting of cast-iron plugs with wrought-iron wires were also used to measure directly the temperatures at or near the surface of the cylinder, and of the valves and piston. The highest temperature of the inlet valve was 310° C., with a cyclical change of 10° to 20° C. ; that of the exhaust valve was 400° C., with a variation of 10° to 15° C. ; and the centre of the piston 340°, changing not more than 80° C. The maximum surface temperature of the combustion chamber was 250°, with a cyclical change of about 20° C. ; and that of the cylinder liner was not more than 40° above the jacket outlet, with an inappreciable cyclical change.

Transference of Heat. The heat generated by the explosion of the charge in the engine cylinder is transmitted to the cylinder walls, piston, and jacket water by radiation, conduction, and convection.

Radiation. Heat, like light, is radiated, with the velocity of light, from the sun ; both are transmitted as *radiant energy* in waves through ether, and radiant heat has the longer wave-length. Radiant energy and light have precisely the same laws of propagation, reflection, and refraction. Radiant energy does not warm the medium through which it is propagated from the sun to the earth's atmosphere until the waves reach the receiver and are absorbed by a body, raising its temperature. Dark radiation, from a heated ball in an enclosure, may be reflected by a mirror or " burning " lens.

Stefan's Law. Stefan suggested that the rate of radiation from an incandescent black body into space is proportional to the fourth power of the absolute temperature. This law agreed with the results of experiments by Dulong and Petit, Tyndall, and others. *The maximum amount of heat in B.Th.U. radiated from a black incandescent body per square foot per hour is* $16 \times 10^{-10}T^4$, *where T is the absolute temperature (Fahr.).* A black body at 600° C. radiates roughly 160 B.Th.U. per sq. ft. per min.

If the flame temperature of the burning gases is T_1°C. (abs.), and that of the enclosure T_2°C. (abs.), since the flame of incandescent particles fills the vessel, the area of the radiating surface of the mass of gas is the same as the absorbing surface, the greatest quantity of heat energy radiated by the gas, considered as a black body, per square foot of its boundary surface, is

$$9 \cdot 33 \times 10^{-9} (T_1{}^4 - T_2{}^4) \text{ C.H.U. per hour.}$$

This formula does not apply when the gases are not incandescent, because burnt gas (even at flame temperature) radiates comparatively little heat.

Professor Callendar showed[*] that the heat radiated from explosions in enclosed vessels and in the internal combustion engine is

of the same order as radiation from a large non-luminous Bunsen flame and that of the Mêker burner, which radiate about 15 per cent of the heat of combustion. With very large flames the radiation tends to become proportional to the enclosing surface. He found with a mixture of gas and air in the proportion for complete combustion, the average quantity of heat radiated during explosion was 12·5 per cent of the heat of combustion, and with rich mixtures the maximum of 20 per cent was reached.

Although it is impossible to estimate, separately, the exact amount of radiation to the cylinder walls, it is probable that the loss by radiation is proportional to the area of the wall surface of the enclosure, and is practically independent of the time, because the duration of flame is very short in the rapid explosions of the most efficient mixtures. The assumption is that, as a rough approximation, two similar masses of flame in which the temperatures at corresponding points are the same, and in which the densities are inversely proportional to the volumes, will radiate heat equally.

The total heat energy radiated in the engine cylinder from the mass of burning gas is proportional to the area of the combustion chamber, and therefore the radiation received per square foot of the boundary may be calculated.

Sir Dugald Clerk reckoned the heat loss by radiation during explosion at ordinary compression in a gas engine, using the most economical mixture and a maximum temperature of 1800° C., to be about 3 per cent when the time of explosion is 1/30 sec.

Professor Hopkinson showed* from explosion vessel and other experiments that the heat loss by radiation is $K\sqrt{t}$, where t is the time of explosion and K a constant. In the cooling of the flame the heat loss is $2T \cdot \sqrt{\dfrac{K \cdot c \cdot t}{\pi}}$, where T is the flame temperature, K the thermal conductivity, and c the thermal capacity per unit volume.

He also determined the relation between heat loss and the conditions of the surfaces in the combustion chamber. It is well known that lampblack is a good absorber and radiator of heat, but does not reflect ; on the other hand, silver, brass, tinfoil, or metal when polished, reflect both heat and light, but are not good absorbers.

Mixtures of coal gas and air in the same proportions were exploded in one case in a cast-iron vessel coated inside with lampblack, and in another lined with tinfoil or silvered. With lampblack it was estimated that about one-third of the total heat loss to the walls was due to radiation up to 1/10th sec. after maximum pressure was attained, when the temperature was 2100° C. and fell to 1200° C. In cooling the burnt products to atmospheric temperature, at least one-fifth of the total heat given to the black walls was due to

* *Proc. Roy. Soc. A.*, Vol. 79 and Vol. 84 (1910).

radiation. In the vessel with polished inner surface the loss by radiation to the walls was reduced, the maximum pressure increased about 3 per cent, and the rate of cooling was considerably less than in that with the lampblack surface.

In actual practice, petrol engines, having ground and polished combustion chambers, not only give increase of mean effective pressure and thermal efficiency, but also have less tendency to deposit carbon, which absorbs heat rapidly. A layer of carbon deposit 1/10th in. thick, of thermal conductivity about 1/50th that of iron, in the combustion chamber of a petrol engine, for the ordinary heat flow, gives a mean temperature of 700° C., and tends to set up pre-ignition of the charge during compression, while any such hot spot gets still hotter because it is a centre of ignition (pp. 345 and 350).

Conduction and Convection. The quantity of heat, Q, transferred by conduction in a steady flow through a clean plate is directly proportional to the constant difference of temperature, $T_1 - T_2$, between the two surfaces of the metal plate, and inversely as the thickness l in. of the plate.

If the temperature is Fahrenheit, in time t min., the quantity of heat transferred in B.Th.U. is

$$Q = k \frac{(T_1 - T_2)}{l} At,$$

where A is the area of the surface of the plate in square feet, and k, the thermal conductivity, varies with the material in the plate, and its mean temperature. The value of k for cast iron is 4·3 about 140° F. in these units (see p. 339) by this formula.

There is a thin film of gas clinging to the inner walls of the combustion chamber, which is not apparently affected by convection currents or turbulence of the gases, and which offers great resistance to the passage of heat, other than radiant energy, by conduction.

On the outer side of the cylinder liner there is a similar film of water, only slightly affected by the circulating water, which offers smaller resistance to the transfer of heat from the metal to the water. There must be great differences of temperature in these very thin films or layers of air and water close to the interface of their contact with the metal, especially when there is a large flow of heat by conduction from the gas through the metal to the cooling water.

In addition, there may be carbon or sooty deposit and a film of oil on the gas side of the cylinder walls, and a layer of lime or sediment on the water jacket side, which also offer resistance to the heat flow.

As an illustration of the relative temperature gradients due to the thermal resistances of the air film, the iron, and the water film

in the transmission of heat through them by conduction from the gas to the water, suppose a clean iron plate 1 in. thick coated with a film of air on one side and a film of water on the other side, each film being $\frac{1}{200}$ in. thick. Lord Kelvin points out* that the thermal conductivity of iron is 80 times that of water and 3,500 times that of air. Since thermal resistance is inversely proportional to thermal conductivity, taking the thermal resistance of the iron plate unity, the resistance of the air film will be $\frac{3500}{200} = 17\cdot5$, and that of the water film $\frac{80}{200} = 0\cdot4$. The total difference of temperature or head necessary for a given flow of heat will be proportional to these thermal resistances : $17\cdot5 + 1 + 0\cdot4 = 18\cdot9$; that is, $92\cdot6$ per cent of the whole drop in the temperature gradients is required for the heat flow to overcome the resistance of the gas film, $5\cdot3$ per cent the iron plate, and $2\cdot1$ per cent the water film.

A gas engine developing 40 B.H.P. on coal gas has the cylinder kept cool by the water jacket. In order to reach the water from the inner surface, the heat has to flow by conduction through the metal of the walls. The cylinder liner is 1 in. thick and the average rate of flow of heat through this metal to the water jacket is about 600 B.Th.U. per sq. ft. per min. The temperature gradient necessary to sustain this flow is about 60° C. per inch thickness of metal liner ; according to the experiments† of Callendar and Nicolson on the thermal conductivity of cast iron, which showed that k is about 10 per cent less at 180° C. than at 40° C. Therefore the inner surface of the cylinder liner is only 60° C. hotter than the jacket water.

Considered in more detail, the transference of heat from the hot core at the centre of the combustion chamber to the jacket water is illustrated by the diagram, Fig. 134.

Professor Dalby clearly explains in the similar case of the hot gases from the furnace of the steam boiler to the water, supposing the heat flow steady and constant, by the analogy of a steady direct electric current flowing through parts of a circuit consisting of various resistances, in which the differences of potential at the various points correspond to the differences of temperature necessary to produce the flow of heat through the various thermal resistances.

The temperature Aa of the hot central core of gas in the engine cylinder falls gradually to Bb at the gas film, as the heat is removed to the boundary walls by convection currents and turbulence, and very slightly by conduction, gas having a low thermal conductivity.

A great difference of temperature, $Bb - Cc$, is needed for the transfer of heat through the exceedingly great resistance of the gas

* " Heat," in *Ency. Britt.*, 9th Edition, p. 579.
† *Proc. Inst. C.E.*, Vol. CXXXI, p. 159.

film. The temperature falls to Dd through the oil film. A drop of temperature, dd', is needed to transfer the heat flow across the surface, where the gas or oil film is in contact with the metal, like the potential difference at a joint in an electric circuit. A similar fall of temperature, ef, is needed across the surface of contact between the metal and water film.

The temperature drop, $Dd' - Ee$, through the metal is comparatively small, and $Ef - Gg$ for the water film to that of the
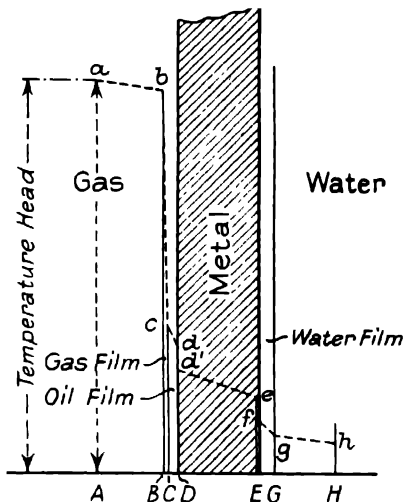
FIG. 134. TEMPERATURE GRADIENTS IN THE ENGINE CYLINDER
(*Dalby*)

water close to the metal, which finally falls to Hh, the mean temperature of the jacket water which removes the heat.

The heat given to the piston-face has to be conducted radially to the edge, and then across the film of oil between the piston and cylinder liner, and through the liner to the jacket water.

From experiments* on the steam boiler it is inferred that 98 per cent of the total available temperature is required to force the heat from the gas into the metal, and the remaining 2 per cent transfers the heat from the metal to the water in the boiler. Also, the gas side of the plate of the heating surface is only $36°$ hotter than the water side. Sir John Durston found similar results, and with a clean plate and the water at $212°$ F., the temperature on the gas side of the plate was $68°$ F. higher than that of the water. When 5 per cent of mineral oil was added to the feed water, the difference of temperature increased from $68°$ to $98°$ F. ; 2·5 per cent of paraffin in the water increased the difference to $118°$; and with a greasy

* * *The Engineer*, Vol. LXX, p. 523 (Hudson).

deposit 1/10 in. thick, the temperature rose to 338° F. above that of the water.

Experiment shows that the rate of transfer of heat by conduction from the hot gas to the cylinder walls in the internal combustion engine depends on the nature and condition of the surface and is found to increase with the density or compression, and with the velocity, eddying motion, or *turbulence* of the gas, which, by scrubbing or scouring the inert gas film, brings fresh particles of gas into contact with the surface to which they impart their heat, tends to increase the rate of heat flow from the gas to the metal, according to the law of heat transmission first enunciated* in 1874 by Professor Osborne Reynolds : $h = (A + B\rho v) (t_1 - t_2)$, where ρ is the density, v velocity, and t_1 the temperature of the gas, and t_2 the temperature of the metal surface ; A and B are constants depending on the nature of the working fluid, and h the heat transmitted per unit area of surface in unit time (see p. 339).

Professor J. T. Nicolson applied this effect to the design of steam boilers for utilizing the waste heat in the exhaust gases from internal combustion engines.

Professor B. Hopkinson measured the **metal temperature** and **heat flow**† in a Crossley gas engine working on the ordinary four-stroke cycle with " hit-and-miss " governing, and rated to give 40 B.H.P. at 180 r p.m. Diameter of cylinder $11\frac{1}{2}$ in. by 21 in. stroke, compression space 407 cub. in., compression ratio 6·37, and compression 160 lb. per sq. in. (gauge).

The thermo-couples consisted of nickel wire 0·015 in. diameter, passed through a hole drilled in a wrought iron bolt, brazed into the iron at the end and insulated by a glass tube set in plaster of Paris ; the second connection being made with wrought iron wire. Each thermo-couple was calibrated up to 700° C. in an electric furnace against a Callendar pyrometer. The same leads, resistances, and galvanometer were used in the calibration as in the engine ; the bolt containing the couple being simply changed from its place in the engine to the furnace. In all cases the absolute measurements were correct to within 15° C., and the difference of temperatures between two points to within 5° C.

The gas was measured by the fall of a standard gas-holder, of 10 cub. ft. capacity, with the main supply cut off ; and its lower calorific value measured in a Boys or a Junker calorimeter. An exhaust calorimeter of the spray type was fitted, and the total heat given to the engine was occasionally measured as a check upon the calorific value measurements ; and either the brake or indicated work could be predicted correctly to within 2 per cent.‡

* See *The Steam Engine*, by Prof. Perry (1899). pp. 586–8 ; also by Dr. T. E. Stanton, *Trans. Roy. Soc.*, Vol. CXCIX (1897), p. 67.

† *Proc. Inst. C.E.*, Vol. CLXXVI (1909), p. 210.

‡ *Proc. Inst. Mech. E.*, 1908, p. 417.

The fluctuations of temperature at the metal surface to a depth of 3/16 in. was determined by means of a rotating contact-maker, and the electric potential of the thermo-couple at definite points in the cycle, measured by a potentiometer. The fluctuation during a cycle was not more than 5° C. above and below the mean.

The cyclical variations in the metal diminish very rapidly with the depth, and those registered by the couple will be less than at the surface. However, the *mean* temperature, correctly registered by the thermo-couples, remained the same, within 3° or 4° C., at the surface and at a point $\frac{1}{8}$ in. in the metal.

In the investigation, comparison is made between the temperatures of the various parts and the total heat received by the engine and removed by the jacket-water, also by radiation and conduction. The excess of the temperature of any portion of the engine over the mean temperature of the water jacket, multiplied by the average thermal conductivity of the metal along the line of heat flow from that point to the jacket-water, is taken as proportional to the heat carried away. Experiment proved that the capacity for heat of the engine was equivalent to 1,000 lb. of water ; that is, an increase of 1° C. in the mean temperature of the jacket-water during the tests, or of 2° C. at exit, meant an absorption of heat into the engine of about 2,000 B.Th.U.

The surface of the compression-chamber, including the piston-face, exposed to the hot gases at the in-centre, was about 2·5 sq. ft. ; at the other end of the out-stroke the total surface was about 8 sq. ft. At full load the quantity of heat carried away by the jacket-water varied from 1,400 to 2,000 B.Th.U. per minute, or an average of about 1,600 B.Th.U. per minute.

The heat removed by conduction and radiation was estimated by comparison of pairs of trials in which the conditions were exactly the same, except that the temperature of the jacket water was varied by changing the rate of flow. It was found that, when the mean temperature of the jacket exceeded that of the air by 28° C., the heat removed by the water was 100 B.Th.U. less than when the difference was 9° C. Assuming the rate of loss proportional to the temperature difference, then the total heat lost by radiation and conduction in the former case is 150 B.Th.U., or, roughly, 10 per cent of the total heat passing into the walls of the engine. In most of the tests the temperature of the jacket water at exit was 70° C., and the mean jacket temperature 42° C., or 28° C. in excess of the air temperature. Under these conditions, the loss by radiation for all the tests would be practically 150 B.Th.U. per min.

The total heat Q removed by the jacket water and by conduction and radiation at full load varied from 29 per cent of the whole heat supply, with a charge containing 8·5 per cent of coal gas, up to 34 per cent with 11 per cent of coal gas. In a series of tests, Q was varied from 1,300 to 2,000 B.Th.U. per min., and in each the ratio

θ/Q remained constant, where θ is the excess of piston temperature over the mean jacket-temperature (42° C.), proving that within these limits the proportion of heat received by the *piston* is practically unaffected by the strength of the mixture. For instance, with the mean total loss, Q, of 1,600 B Th.U. per min., the excess temperature θ at the centre of the piston was 330° C., or

$$\frac{\theta}{Q} = \frac{330}{1600} = 0.206, \bullet$$

when the temperature at the piston centre was 330° + 42°C. = 372°C. Under normal working conditions, the temperature at the centre of the piston may be taken as 340° C., and at the periphery 170° C., above the mean jacket-temperature. With the richest charges this temperature rises at the centre to 400° C., and at the edge 220° C. above the mean jacket temperature.

From the temperature gradient in the piston, a rough estimate can be formed of the rate at which it is receiving heat. Considered as a thin circular disk receiving heat, $4kt$, at a uniform rate over the surface, and that the heat is removed equally all round the edge, then the difference of temperature, T, between the centre of the piston and a point in the face, at radius r in., can be shown to be very nearly $\dfrac{hr^2}{4kt}$, provided the distance of the point from the edge is not less than the uniform thickness, t in., of the disk or crown, where h is the rate of heat reception in B Th.U. per square inch per minute ; k the average conductivity of the metal, which decreases as the temperature rises.* The experiments of Callendar and Nicolson showed that k is about 10 per cent less at 180° C. than at 40° C., and so $k = 0.056$ at 300° C. by extrapolation. In this engine piston $t = 1.5$ in.

Under normal working conditions, when the total loss by heat flow is $Q = 1,600$ B.Th.U. per min., the difference of temperature, T, between the centre of the piston and a point in the piston face at radius $r = 3.62$ in. was found by experiment to be 72° C., then the rate of heat flow into the disk of radius 3.62 in. is

$$h = \frac{T \times 4kt}{r^2} = \frac{72° \times 4 \times 0.056 \times 1.5}{(3.62)^2} = 1.845 \text{ B.Th.U. per}$$
$$\text{sq. in. per min.}$$

This is the difference between the heat taken in at the piston face and that lost by the metal by convection and radiation from

---

* See *Proc. Inst. C.E.*, Vol. CLXXVI (July 1909), Appendix II, p. 246. The difference of temperature between the centre and edge of a thick disk heated uniformly over one face lies between $\dfrac{hr^2}{4kt}$ and $\dfrac{h(r^2 + 2t^2)}{4kt}$. When the disk is very thin these expressions become equal to that given in the text. A mean between the two, namely, $\dfrac{h(r^2 + t^2)}{4kt}$ is probably a very close approximation.

the back of the piston exposed to the air. The latter loss was found by lagging the piston with magnesia and running under identical conditions. The temperature at the centre of the piston was not more than 10° C. higher with the lagging than without it. It follows that only 2 or 3 per cent of the whole heat received by the piston face is lost from the back.

The area of the piston face is 104 sq. in., and the total heat received by it is $104 \times 1 \cdot 845 = 192$ B.Th.U. per min., or only 12 per cent of the whole loss Q by heat-flow into the walls.

At the in-centre, when the density and temperature of the gas is highest, and the heat flow is therefore most rapid, the piston area is 30 per cent of the whole exposed surface, and at the out-centre it is 11 per cent. If all the gas were at the same temperature, the piston would be expected to receive one-fifth of the total heat Q lost in the engine. Part of this loss occurs during the rush of the hot gases past the exhaust valve at the moment of release, and practically the whole of this goes to the valves and passages, and none to the piston. Probably the gases in contact with the piston after combustion is complete are cooler than anywhere else in the cylinder. The mixture there must be rather weaker than the average, because the piston takes with it on the suction stroke a large part of the gases left in the clearance space, which will dilute the mixture near the piston more than in other parts of the cylinder. Moreover, the gas near the piston is the last to be ignited, the flame starting from the ignition at the opposite side of the compression space, and, consequently, the gas near the piston after ignition will be much cooler than that near the point of ignition (see p. 350). The trials show that the piston only receives about one-eighth of the total heat lost in the engine.

Professor Hopkinson compared the above result with direct determinations of the heat given to the water-cooled piston by Professor Burstall in the Third Report* of the Gas Engine Research Committee. The compression ratio was 6·79. At the in-centre the area of the piston was about 0·3 of the exposed surface, and at the out-centre one-ninth, and the piston took in 16 per cent of the heat carried away by the cooling water. If to the heat removed by the water were added that taken away by radiation and conduction, the proportion would be reduced to about 15 per cent of the total. In this case the water-jacketed piston was 200° C. the cooler.

The heat given to the piston face has to be conducted radially to the edge and part down the piston skirt. It has to cross the film of lubricating oil between the piston and liner, and through it to the water jacket. Part of the heat goes through the piston-rings and part by the skirt. The temperature will increase as the square of the diameter if the thickness of the piston crown remains constant and the metal surfaces clean. A deposit of carbon, tar, or other

bad thermal conductor may receive heat nearly as fast as clean metal, but, in order to get the heat away, a higher temperature is reached and a greater temperature gradient is necessary.

The internal stress in a disk of uniform thickness, not quite like the piston head, in which the net expansion between heating at the centre and cooling of the edge is 0·0038 in , corresponds to a tensile stress in the cast-iron piston of $7\frac{1}{2}$ tons per sq. in. ; and with normal charges the tension will amount to $5\frac{1}{2}$ tons per sq. in. The clearance between piston and liner in this engine when cold is about 45 thousandths of an inch.

With an average gas charge, and the loss Q, 1,600 B.Th.U. per min., the temperature at the centre of exhaust valve was 400° C., and of the inlet valve 250° C.

The tests showed a decrease of 10 to 15 per cent in the ratio θ/Q for the exhaust valve when the strength of the mixture is increased, that is, the proportion of total heat loss received by the exhaust valve is less when the charges are rich than when they are weak. This noteworthy feature may be explained by the fact that the heat loss which occurs during the exhaust, just after release, goes mainly to the valve. This view is confirmed by experiment when the total heat loss Q is reduced by diminishing the load on the engine, so that it misses ignitions ; the charge of gas remaining the same, the value of θ/Q for the exhaust valve is unaltered, showing that the proportion of heat received by the exhaust valve is unchanged.

The temperature of the metal at any point in the cylinder depends chiefly upon two factors : the *heat flow*, or rate at which heat passes into the surface per unit of area at that point, and the *distance* the heat has to travel to the place where it is removed. The heat flow is conditioned by the *strength of the mixture*, the *degree of compression*, and the *time of ignition* ; and the distance mainly by the size and design of the metal in relation to the cooling water. Thus, in a geometrically similar design, if the thickness of the piston crown were halved, the fall of temperature from centre to edge would be doubled, tending to produce pre-ignition with strong charges.

Strength of Mixture. The following table gives the temperatures obtained in experiments with very weak and very rich mixtures, the engine running at 180 r.p.m. and firing every time, with jacket water 75° C. at outlet. The lower calorific value of the gas was 580 B.Th.U. per cub. ft. at atmospheric temperature and pressure—

	WEAK	RICH
Gas Charge, cubic foot per suction . . .	0·1	0·13
Percentage of gas in cylinder contents . . .	8·5	11·0
Total heat-loss, B.Th.U. per minute . . .	1510	2300
Total heat-loss as percentage of total heat supply .	29	34
Temperature of piston at 3·62 m. from centre .	300° C.	430° C
Temperature of exhaust valve . . .	400° C.	540° C
Temperature of inlet valve	240° C.	355° C

Thus the total heat loss with the richer charge and more rapid combustion increases over 50 per cent when the gas charge is increased 30 per cent. With the weaker charge, the volume increases during combustion, thus the temperature is lower, and less heat is lost, but the share of heat received by the exhaust valve is greater.

Compression. The compression was reduced by keeping the half-compression (starting) cam in action, which holds the exhaust valve open during the first part of the compression stroke. The valve closes when the volume enclosed by the piston is 41 per cent of the total volume at out-centre and the pressure 10 lb. per sq. in. above atmosphere. The gas taken into the cylinder per suction was the same as the normal, but only 57·5 per cent of the whole charge is retained and the other part expelled unburnt, so that only the density of the charge is varied. Trials were made in pairs on the same day ; one with the engine working normally, the other with the half-compression cam in action. With diminished pressure the temperature in the cylinder was very much lower, and the total heat loss greatly reduced, from 1,900 to 1,100 B.Th.U., nearly, but not quite, in proportion to the density of the charge.

In each case the piston friction 100 B.Th.U. per min. (2·5 H.P.) is included in the total heat loss, and should be deducted to obtain the heat actually given out by the gas. Then the losses become, with reduced compression, 32·2 per cent, and, with normal compression, 30·2 per cent.

When the compression is increased by reducing the clearance, without other alteration, if the clearance is halved, the density during and soon after explosion will be doubled, and the heat loss per unit area of exposed surface will be nearly doubled. The *temperature of the exposed metal* inside the engine cylinder is raised, roughly, in proportion to the compression ratio, except that of the exhaust valve, where the exhaust gases at release will be cooler when the compression or expansion is greater. This increase of temperature sets a practical limit to the compression-ratio allowable for higher efficiency.

At the in-centre of the compression stroke until shortly after explosion, experiment shows that the heat flow is three or four times that obtained for the mean of the whole expansion stroke, depending upon the density of the mixture.

Time of Ignition. The ignition in this engine commences 5° before the in-centre, and is completed 10° to 15° of the crank angle after the centre, according to the richness of the mixture. *Retarding the ignition* reduces the total heat loss and alters its distribution, so that the exhaust valve receives a larger share of the heat, while the piston and inlet valve receive less.

The results shown at top of next page were obtained with mean jacket-temperature 40° C., as usual.

	Ignition Retarded 20° of Crank angle	Normal Ignition
Gas charge, cubic foot per suction . . .	0·1224	0·1218
Total heat-loss (Q) B.Th.U. per minute . .	1619	1805
Total heat-loss as percentage of total heat supply .	28 1	31·3
Piston temperature, at 3·62 in. from centre .	258° C.	317° C.
θ/Q for piston	0·132	0·151
Inlet valve temperature	207°	255°
θ/Q for inlet valve	0·10	0·118
Temperature at release (exhaust) . . .	1200° C.	1000° C.
Thermal efficiency, per cent . . .	28·5	34·5

When the ignition is retarded the engine is cooler, except the exhaust valve, and there is loss of thermal efficiency.

Advancing the ignition by 10° of crank angle causes an increase of temperature in the engine and greater heat loss. The piston and inlet valve get a larger share of the heat and their temperatures are raised 50° C.

Missing Ignitions. Several pairs of trials were made, in the first of which the engine was run on the ordinary load firing every time, while in the second trial the load was reduced so that ignitions were missed every other cycle, and the gas cock adjusted to make the charge of gas taken per suction the same in the two cases. In one pair the gas per suction was 0·128 cub. ft., and, the temperature of the engine being precisely the same in the two cases, the total heat removed by the jacket and radiation was 2,030 B.Th. U. per min. at full load, and 1,103 B.Th.U. per min. at half load ; or 25·2 B.Th.U. per explosion at full load and 23·7 B.Th.U. in the half load trial. Deducting 1 B.Th.U. from each for piston friction, leaves 24·2 B.Th.U. and 22·7 B.Th.U. per explosion as heat received from the gas and removed by the jacket water and radiation.

When this engine is partially loaded and *scavenging*, less heat is given to the jackets per explosion, because the charge is diluted with a greater weight of air left in the clearance space, and about 4 per cent of the gas taken into the cylinder is then discharged unburnt.

In another experiment the engine was run at full load until the inner surfaces were thoroughly heated. The load was suddenly thrown off and the engine motored round with the gas supply cut off, so that it took in, compressed, and expelled a charge of air every cycle. The rise in temperature of the air passing through the engine was observed by a thermometer inserted in the exhaust pipe. It was found that the air in the exhaust pipe was 45° C. hotter than in the inlet pipe. About 30° C. of this rise was due to the work done upon the air, leaving 15° C. to represent the heat taken by 1 cub. ft. of air per suction from the metal surfaces, and the heat required to warm this air through 15° C. at constant

pressure is nearly 1/2 B.Th.U. This is the quantity of heat taken up by the air passing through the engine in a scavenging stroke, and is not more than 5 per cent of the heat received from the hot gases during an explosion stroke. The temperatures of the piston, exhaust valve, and inlet valve were observed and found to change very little during the short time taken to get the temperature of the air in the exhaust pipe.

Pre-ignition. When any part of the surfaces exposed to the gas in the engine cylinder exceeds a certain temperature, *spontaneous ignition* of the charge takes place. In order to produce pre-ignition, an iron bolt, about 4 in. long, was screwed into the exhaust valve cover and projected into the centre of the combustion chamber or compression space. The bolt carried at the inner end a thermo-electric junction, formed by passing a nickel wire down through a hole drilled along the axis of the bolt and brazing it to the iron at the projecting end. It was found that the bolt could be raised to 700° C. and kept continuously at that temperature without any apparent effect on the working of the engine. The metal temperature indicated by the thermo-junction would be a little lower than the actual gas temperature which caused ignition.

Increase of density or compression is found to lower the temperature of ignition.

With a gas charge of 0·1265 cub. ft. per suction the temperature at the tip of the bolt rose rapidly at first and in a few minutes attained a steady value about 670° C. without any effect. When the gas charge was increased to 0·1282 cub. ft. per suction, about 1 per cent, the temperature of the bolt rose to 690° C., at which ignitons from it occurred occasionally. These ignitions tended to become more frequent and earlier, each raised the bolt temperature a few degrees, until, at 725° C., they were continuous ; the charge was fired as soon as it entered the cylinder and the engine pulled up.

Another bolt was tried, 6 in. long by $\frac{3}{4}$ in. diameter, with a screw thread cut on it to give greater surface, and the thermo-couple was in the centre of the flat end. In this case there was no ignition below 730° C., but always occurred at 750° C. The inference is that a clean metal surface will ignite the gas mixture when the surface temperature is a little above 700° C., and will not ignite it below that temperature. The true ignition temperature of the gas mixture is above 740° C. A similar *copper* bolt caused ignition at the same temperature measured by a constantan wire. Possibly carbon deposit tends to lower the ignition temperature.

The diagrams, Fig. 135, taken with the Hopkinson optical indicator, show the normal diagram, also pre-ignitions at *C*. The loss of heat was. about 12 B.Th.U. per explosion along *CED*, after explosion while the flame is being compressed, so that the mean temperature of the gas at *ED* was practically constant, although

work was being done on it by compression at the expense of the energy stored in the flywheel.

Professor Hopkinson pointed out by experiments on explosion in a closed cylindrical vessel (p. 350) that when a mixture of coal gas
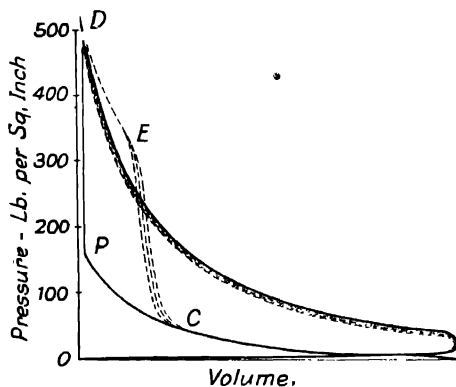
Fig. 135. Diagram showing Pre-ignitions

and air is ignited by electric spark from one point, and the flame has spread completely throughout the vessel, the temperature of the portion first ignited is raised by compression, and even in a non-conducting vessel is very much higher than the temperature of
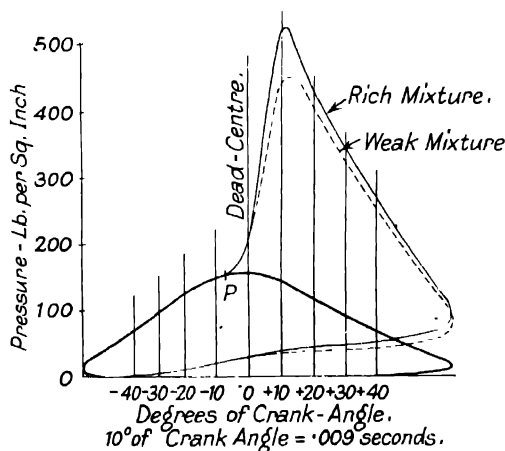
Fig. 136. Explosion of Rich and Weak Mixtures

the gas at a distance. Thus the origin of ignition being at the bolt increases the temperature of the gas around it, and raises the temperature of the pre-igniting point of the bolt itself with great rapidity during successive explosions.

Some designs of sparking plug, for petrol engines, without sufficient metal or water cooling to conduct away the heat received from explosions, may become overheated and cause persistent pre-ignitions in the same way as the overheated bolt, exhaust valve, piston head, or patch of carbon deposit.

The indicator diagrams, Fig. 136, were taken under normal conditions of working, the phase altered through a right angle, and show the rise of pressure from the point of ignition P, during explosion on a time basis.

The *time of explosion* is the time taken after ignition from the moment visible rise of pressure begins until maximum pressure is
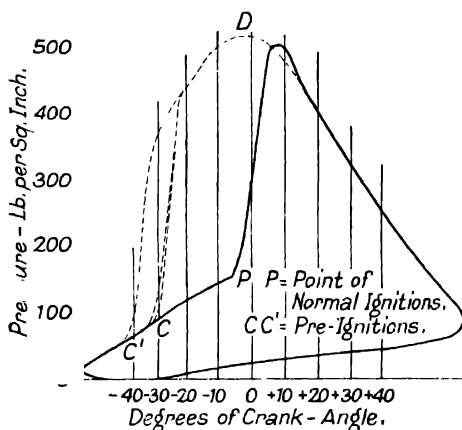
FIG. 137. NORMAL AND PRE-IGNITION INDICATOR DIAGRAMS

attained, and measures the explosiveness of the mixture. By the diagram, maximum pressure was reached from 0·014 to 0·02 sec. after ignition at P, according to the strength of the mixture.

The altered phase indicator diagrams, Fig. 137, show normal ignition at P, before the bolt was hot enough to fire the charge, and the pre-ignitions at CC'' by the bolt, indicate the rapid advance of ignition as the bolt gets more heated. The shutter of the camera was opened for one or two explosions and then closed for five explosions, until the ignition had advanced to C'.

Pre-ignitions of considerable violence were produced by dropping oil on the top of the exhaust valve, when the temperature of the valve was only 450° C. The discharge of a good deal of carbon into the exhaust indicated that the oil was " cracked," or decomposed, by contact with the hot metal surface. A mixture of oil with air is readily vaporized in contact with the surface of the exhaust valve, and if the temperature is high enough the oil is cracked. Then the rapid rate of burning of the hydrogen and hydrocarbons from the

decomposition of the oil, and their high flame temperature, under the comparatively high compression pressure in the gas engine, would give rise to *detonation* and eventually to *pre-ignition*. The familiar noise of the metallic hammer-like blow of detonation heard in oil and petrol engines is well known as " pinking," ever since the days of the Priestman oil engine The noise of the rapid explosion in the engine when it was well heated and working at full load, especially on *paraffin oil* fuel, was at once reduced by a few drops of water taken into the combustion chamber during the suction stroke. In this way the pre-ignitions in the gas engine, due to the oil added, were completely stopped by the injection of a small quantity of water.

The tendency of rich gas and light paraffin petrols to detonation and pre-ignition in homogeneous charges is the principal factor controlling the compression ratio, and both the power output and efficiency of any gas or petrol engine, working on the ordinary explosion cycle.

Professor Hopkinson carried out important experiments* on the **explosion of mixtures of coal gas and air in a closed vessel,** and a continuous photographic record was taken of the change of temperature at different points of the vessel by fine platinum wire thermometers, and on the same revolving drum the pressure change was recorded by his optical indicator (p. 21). The spread of the flame through the mixture was traced by the sharp rise of temperature at the various points, due to the conversion of the chemical energy of the fuel into heat energy. The internal energy of the mixture is equal to the heat of combustion of the gas used, less the heat lost to the walls of the vessel. The maximum pressure is attained at or about the time when the gas is completely burned to CO_2, steam, and inert gas.

At the outset careful experiments were made to determine the relation between the resistance of the loop of about 5 cm. of pure platinum wire 0·001 in. diameter and the temperature of the gases. The necessary corrections are due to the *time lag* of temperature of the wire behind that of the gases ; also the end effects of conduction to the stout copper leads, convection currents of gas, and heat radiated by the platinum wire.

The cylindrical explosion vessel, Fig. 138, was 28·75 in. in length by 23·4 in. diameter, and capacity 6·2 cub. ft. *A* is the sparking point near the centre, and three thermometers : *B* near the spark, *C* 4 in. from one end, and *D* 0·4 in. (1 cm.) from the other end wall. Thermometers were also placed at *B'* and other parts of the vessel in many different trials.

Experiments were made with one mixture 9 of air to 1 of gas, and another 12 of air to 1 of gas. The Cambridge coal gas had a higher calorific value of 630 to 680 B.Th.U. per cub. ft. at 0° C. and

* *Proc. R.S.,* Vol. 77A (1906), p. 387 ; also *Engineering,* 15th June, 1906.

760 mm. mercury, and a lower value between 570 and 620 B.Th.U. per cub. ft.

AVERAGE COMPOSITION OF CAMBRIDGE COAL GAS (*Prof. Hopkinson*)

	Per Cent by Volume	Oxygen required for Combustion	Steam Produced	CO_2 Produced
		Volume	Volume	Volume
Hydrogen (H)	47·2	23·6	47·2	—
Marsh Gas (CH_4) . . .	35·2	70·4	70·4	35·2
Heavy hydrocarbons . .	4 8	22·6	16·0	14·4
Carbonic oxide (CO) . .	7 15	3 6	—	7·15
Nitrogen (N) . . .	5·4	— ·		· ·
Other gases . . .	0·25	·	—	· ·
	100·00	120·2	133·6	56·75
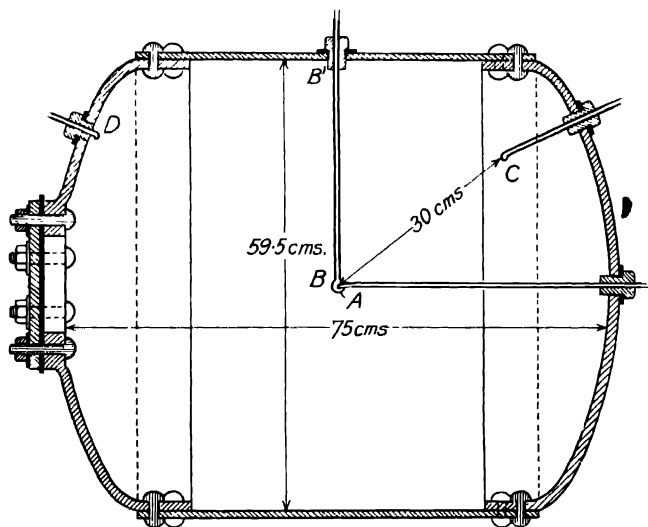
FIG. 138. EXPLOSION VESSEL (*Hopkinson*)

By experiment, 100 volumes of gas require 576 volumes of air for complete combustion ; and 100 volumes of gas burned in 900 volumes of air give about 133 volumes steam, 57 of CO_2, and 780 inert gases ; total products, 970 volumes. That is, a contraction of 3 per cent during combustion.

In making an experiment, steam was blown into the explosion vessel to saturate the air with moisture. The vessel was exhausted to 9/10ths atmospheric pressure, and coal gas admitted to increase the pressure up to atmospheric, giving a mixture of 1 gas to 9 air.

Then 4 to 6 hours was allowed for gaseous diffusion of the mixture containing 10 per cent of gas at 20° C.

Fig. 139 shows the result of one set of photographic records. There were two thermometer wires, one giving the temperature on curve B at the centre near the spark, and the other the temperature curve at D, Fig. 138, as far from the spark as possible. Curve A, Fig. 139, is the indicated pressure during explosion and subsequent cooling of the burnt products.

Fired at atmospheric pressure, the flame spreads from the spark at the centre at a velocity of 150 cm., or 59 in. per sec., and the maximum pressure from 76 to 82 lb. per sq. in. above atmospheric
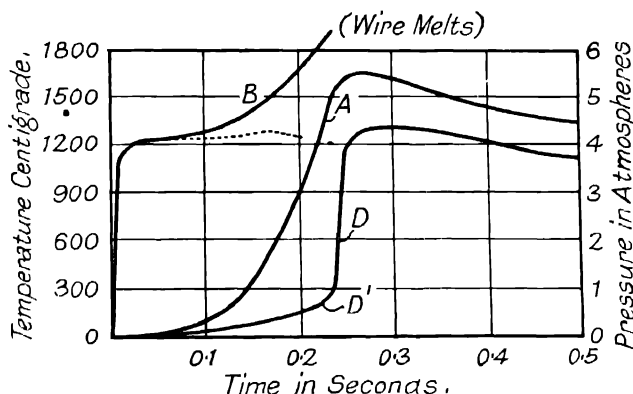
FIG. 139.   EXPLOSION AND COOLING CURVES   (*Hopkinson*)

is attained in 0·25 sec. after firing.   At B, Fig. 138, the temperature rises almost instantaneously from 20° C. to 1200° C. in less than 1/40th sec., and the combustion at the centre takes place at nearly constant pressure and is complete before the pressure, on Fig. 139, has risen 2 lb. per sq. in. above atmospheric.   The temperature at B remains nearly constant during the earlier part of the spread of the flame, after which it rises, due to increase of pressure in the explosion vessel shown by curve A.   If deduction be made for this adiabatic compression, the temperature of the gas had it burnt at constant pressure would have reached 1250° C., as seen by the dotted curve.

The point B' at the side wall nearest the spark is first reached by the flame.   The gas here is ignited when the pressure is about 2 atmospheres, its temperature rises instantly to 1300° C., and at once begins to fall, and here there is much loss of heat.

The temperature by thermometer at D, Fig. 138, the most distant point from the spark, was observed to rise slowly at first, due to nearly adiabatic compression of the gas in its vicinity by the

advancing flame. At the point D', Fig. 139, the pressure by curve A has risen to 6 atmospheres (absolute), as in a gas engine, and the temperature, due to this compression of the unburnt gas at D, is 200° C. The flame now reaches this wire and there is a sharp rise in temperature to 1300° C., without any further rise of temperature, the pressure being at its maximum, and there is little or no adiabatic compression of the gas at this point after ignition.

The explosion vessel is completely filled with flame and is losing heat to every part of it when the pressure reaches 70 lb. per sq. in., and the maximum pressure 82 lb. per sq. in., is attained in less than 1/30 sec. after complete inflammation.

At a later stage of the explosion, adiabatic compression of the burnt gas at the centre of the vessel raises the temperature, and the platinum wire at B is melted when the pressure is 6 atmospheres (absolute). At this temperature the platinum wire radiates heat rapidly, and it must have been receiving heat at a similar rate from the gas, which is considerably hotter than the wire. From experiment on wires of different sizes, Professor Hopkinson calculated the temperature of the gases, about 1900° C.

Since the gas can lose no heat by radiation after the flame has once passed the wire, any after-combustion would be evident in a rise of temperature greater than that to be expected from the compression. The experiment, therefore, proves that, in a mixture of this strength, combustion at any point is complete, within 4 per cent, 1/40 sec. after it is started, and that the process may be taken as complete, for practical purposes, in 1/30 sec. In other words, "after-burning" cannot affect the phenomena of the gas engine unless the speed is abnormally high.

Professor Hopkinson concludes that the combustion is everywhere complete at the time of maximum pressure ; also, taken together, the two temperature records show that 1/30 sec. after maximum pressure, the gas is a mixture of CO_2, steam, and inert gas, in chemical equilibrium in the vessel, except perhaps very close to the walls. At the moment of maximum pressure the *distribution of temperature* is, roughly, as follows—

Mean temperature (inferred from pressure)	1600° C.
Temperature at centre of vessel near the spark, thermometer B	1900° C.
Temperature at 10 cm. (4 in.) within the wall, thermometer C.	1700° C.
Temperature 1 cm. (0·4 in.) from end wall, thermometer D	.1100° to 1300°
Temperature 1 cm (0·4 in.) from the wall at the side . .	850° C.

Half a second after maximum pressure the *distribution of temperature*, due to the motion of the gas set up by the explosion and convection currents, is—

Mean temperature (calculated from the pressure) . .	1100° C.
Mean temperature, exclusive of layer 1 cm. thick at walls.	1160° C.
Temperature at centre of vessel	1100° to 1200° C.

If the vessel were impervious to heat, the portion of the mixture at the centre would be hotter than the outer portions by 500° C. when the combustion of the whole mixture was complete.

The mass of the gas during cooling may be described as a hot centre core in which the temperature is fairly uniform, surrounded by a thin layer wherein the temperature falls to that of the walls. If this layer is $\frac{1}{2}$ cm. thick, and the drop of temperature uniform, the mean temperature inferred from the pressure would be 60° C. below the hot core—the value actually observed.

Specific Heat. Assuming loss by radiation from the flame about 15 per cent of the heat of combustion, as observed by Professor Callendar for a Bunsen flame, then, in compressing the products of combustion at the centre of the vessel from 1 atmosphere to 6·5, the temperature rises from 1200° C. to 1900° C., so that their average capacity for heat is 1·3 times that of air. Since, in adiabatic compression, the absolute temperature varies as $p^{\frac{\gamma-1}{\gamma}}$ it follows that the average value of $\frac{\gamma-1}{\gamma}$ is about 0·2 for these gases, and (the ratio of the specific heats) is 1·25 for temperatures between 1200° and 1900° C. If the ordinary gas law holds for the mixture, the mean specific heat at constant volume, between 1200° and 1900° C., is 30 ft.-lb. per cub ft., or 1·5 times as great as at ordinary temperatures. There is uncertainty as to the loss of heat by radiation from the outer surface of the advancing flame. In order to eliminate most of the radiation, this gas mixture was burned inwards and the average specific heat at constant pressure of the burnt products between the ordinary temperature and 1300° C. would appear to be about 33·1 ft.-lb. per cub. ft., or 25·4 ft.-lb. at constant volume.

Before explosion, the gas and air were saturated with moisture at 20° C. and 14·7 lb. per sq. in., and the products of the combustion of 1 cub. ft. of gas and 9 of air calculated to consist of—

	Cub. ft.		Per cent
Carbonic acid (CO_2) . .	0·56	—	5·78
Steam (H_2O) . . .	1·53	=	15·79
Nitrogen and Oxygen (N and O)	7·60	—	78·43
Total . .	9·69		100·00

Assuming that the CO_2 and H_2O occupy their molecular volumes.

In the *explosion of the weak mixture*, with 12 volumes of air to 1 of gas, containing 7·7 per cent of coal gas, the maximum pressure 50 lb. per sq. in. above atmospheric is attained about 2·5 sec. after the spark. The temperature of a wire close to the spark rises in 0·07 sec. to 1000° C., and then remains steady for some time. About 1 sec. after ignition, and while the pressure is less than 10 lb. above atmospheric, the upper half of the vessel is filled with burnt gas,

which is in contact with and losing heat to the upper walls. In the lower parts of the vessel the gas is still unburnt. A wire placed 15 cm. (5·9 in.) vertically below the spark, shows a gradual rise of temperature to 210° C. by adiabatic compression during more than 2 sec., followed by a sudden rise in 1/10 sec. to 1300° C., due to ignition.

In all the experiments, complete inflammation of the gas, even in the weakest mixtures, is nearly simultaneous with the attainment of maximum pressure, and combustion, when once started, at any point is almost instantaneously complete. With the 12 of air to 1 of gas mixture, it is safe to assume that ¦ sec. after maximum pressure, when loss of pressure by cooling is still less than 5 per cent, there is present in the cylinder a mixture of CO_2, steam, and inert gas in chemical equilibrium. In the mixture 9 of air to 1 of gas this state is reached very much sooner, since the flame travels in this about 10 times as fast as in the 12 to 1 of gas mixture. With the latter the effect of *convection currents* is important from the outset.

The heat produced by burning a cubic foot of the gas in a calorimeter and cooling the products to 20° C. is 620 B.Th.U. The heat produced by condensation of the steam formed is 60 B.Th.U. If the cooling were stopped at 55° C., the net heat obtained is 550 B.Th.U., about 10 B.Th.U. being due to cooling the gases from 55° to 20° C. Deduct 15 per cent loss by radiation from the surface of the flame and there remain about 470 B.Th.U., as the heat evolved in cooling the products of the explosion of 1 cub. ft. of gas and 9 of air at constant pressure from 1230° to 55° C. The same volume of air cooled through the same range would evolve 370 B.Th.U. Thus the average volumetric specific heat of the products is, roughly, 1·3 times that of air.

In a research by Professor Watson on **combustion in the petrol engine cylinder,** the engine used had compression pressure 60 lb. per sq. in. and ran at speeds from 700 to 900 r.p.m.

The progress of combustion was observed through a quartz glass *window in the combustion head.* In this way the *colour of the explosion flame,* as seen directly, appeared to change with the mixture strength, as in a Bunsen flame. At any point in the explosion stroke a rotating disk, with a perforated sector slit, was made to uncover the window and expose it to the collimator of a spectroscope, and gave a line spectrum of the explosion flame.

It was noticed that, in order for maximum pressure with normal mixtures, as shown by the indicator diagram, to occur at or very shortly after the dead centre end of the compression stroke, the ignition spark must pass at a crank angle of 40° before the dead centre ; and the actual period of combustion was 0·01 to 0·018 sec. after firing. The point of maximum intensity of combustion was nearly always either just before that of maximum pressure or

coincident with it, the difference being less than 10° crank angle or 0·002 sec.

The period of combustion for very weak mixtures was 0·025 sec., but burning continued to the end of the exhaust stroke, and when the inlet opened the fresh charge was ignited in the induction pipe causing " popping back " in the carburettor, well-known to motorists as a sign of too weak a mixture. In this case the indicator diagram, Fig. 140, becomes quite flat.

Throttling the charge also reduces the compression pressure and the rate of combustion.

Obviously, the rate at which flame spreads in a homogeneous combustible mixture depends on the compression pressure before ignition, as well as on the nature of the fuel and the richness or proportion of fuel to air in the mixture. The rate of burning is
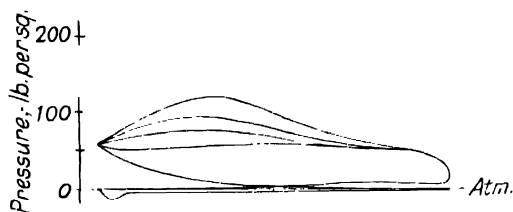
Fig. 140. Indicator Diagrams with Weak Petrol Mixture
(*Watson*)

retarded by increasing the proportion of cool residual products or inert diluent added to the mixture. The inert gases lower the flame temperature during combustion, and therefore reduce (1) the loss of heat to the cylinder walls, and (2) the specific heat of the gases.

It is found, by explosions in closed vessels, that the maximum flame velocity occurs when there is an excess of hydrogen, coal gas or petrol vapour above that required for the true explosive mixture with air or oxygen to give complete combustion.

Also, in actual practice, maximum power is obtained with fuels such as petrol and benzol when the mixture strength in the engine cylinder is 20 per cent greater than that required for complete combustion.

Effect of Turbulence. The turbulence of an explosive mixture, in a closed vessel, produced by a little fan driven at high speed, before ignition, causes the flame to spread more quickly than when the mixture is ignited at rest, and the maximum pressure attained is also increased. In cooling, the hot gas, when kept in circulation by the fan, the rate of transfer of heat to the cold walls is two or three times greater than when the gas is stagnant.

Sir Dugald Clerk made some experiments* on turbulence in a gas engine. The indicator diagram *AB*, Fig. 141, was taken from the engine working as usual, giving the time of explosion 0·037 sec. By an arrangement, the next explosive charge was not fired after the first compression stroke, but the valves were closed and the turbulence allowed to subside before the ignition spark was passed after the third successive compression stroke, giving slow and prolonged combustion during 0·092 sec., as indicated by the diagram *A'B'*.

Indicator diagrams, from gas and petrol engines, show that the rate of flame propagation in the cylinder varies from about 35 to 100 ft. per sec., due to turbulence of the incoming charge. In a gas engine running at 120 r.p.m. on the four-stroke cycle, the suction stroke   kes 0·25 sec , and during this time the average velocity of the gases through the inlet valve is 100 ft. per sec., and the time of explosion about 1/30 sec. In an ordinary petrol engine running at 1,200 r.p.m., the average velocity of the charge through the inlet valves, 100 to 130 ft. per sec., or even higher, produces great turbulence within the cylinder, which continues during the compression stroke and tends to spread the flame uniformly and rapidly throughout the charge   luring the time of explosion, about 1/120 sec.

FIG. 141. INDICATOR DIAGRAMS SHOWING EFFECT OF STILLING TURBULENCE (*Clerk*)

Further, turbulence appears to be one of the principal factors rendering possible the very high speeds over 2,400 r.p.m. in the modern small petrol engine, so that the highest thermal efficiency is obtained with small cylinders. Moreover, experiments by Professor Hopkinson show that the rate of heat loss through the cylinder walls increases with the higher speed of the gases, by the scrubbing or scouring effect, sweeping away the stagnant layer of gas adhering to the cylinder walls, and the increased convection produces greater heat flow, which counterbalances the shorter time intervals of contact, so that a larger proportion of the mixture is burnt quickly and efficiently, and, by speeding up the propagation of the flame, advantage is taken of heat developed early in the stroke by greater expansion, rather than by slow and incomplete burning of weak mixtures, which continues until the exhaust valve opens. He found, in the gas engine at 120 r.p.m., the heat loss through the cylinder walls was, roughly, 30 per cent of the total heat of the gas. When the speed of the engine was doubled, the burning gases remained in contact with the cylinder walls only

* *Report of Gaseous Explosions Committee*, Britt. Assoc., 1912.

half the time, and yet the heat loss to the walls was about the same as at half the speed.

Piston Temperature. Professor A. H. Gibson measured the piston temperatures* in various high-speed petrol engines under full load conditions by an ingenious device for taking observations, and deduced the heat flow by the method of cooling curves.

In each case the engine was kept running during 5 hours before making a test with a new piston or after cleaning an old piston. Experiment showed that the pistons of small engines attained a steady temperature after running 5 min. on full load, so that measurements were made 15 min. after starting the engine. Professor Gibson could stop the engine suddenly, turn the flywheel until the piston was on the top dead centre, and through a hole in the sparking plug boss, quickly insert the point of the thermo-couple in dimple holes prepared in the piston, and take the observation within 10 sec. after stopping the engine. Fine lines were marked on the stem of the thermo-couple, and when any one of these was flush with the outside of the sparking plug boss, the point of the thermo-couple was in the hole at the correct radius in the piston crown.

It was found that an experiment could be repeated under similar conditions, and the fluctuation of temperature was not more than $\pm 5°$ C. If at the instant of measuring a temperature of 220° C. was indicated, that point in the piston was never hotter than 225° C. or cooler than 215° C. throughout the cycle. In aluminium pistons of 4 or 5 in. diameter, the drop of temperature through the piston head was about 20° C., so that the mean metal temperature was not more than 10° C. lower than the surface temperature at any particular point, since the aluminium alloys are good conductors of heat. The pistons were made of various aluminium alloys. Only three are given here, and their measured conductivities in the chill cast state at 200° C. are as follows—

| Alloy | Percentage Composition (approx.) | | | | Conductivity |
	Al.	Copper	Tin		C.G.S. Units
B 4	91	7	1	Zinc 1	0·40
B 24	93·5	4	---	Iron 2, Mg. 0·5	0·38
B 18	89·5	8	1·5	Mn. 1	0·27
Cast iron					0·11

One series of tests was made on an air-cooled aluminium cylinder, 100 mm. (3·937 in.) bore by 140 mm. (5·51 in.) stroke, in order to determine the effects of a variation in : (1) Piston clearance ; (2) material and design of the piston ; (3) bearing area ; (4) the

* *Proc. Inst. Mech. E.*, January, 1926 ; see also Advisory Committee for Aeronautics, L.A.S.C. Report No. 13, (May, 1918).

working mixture ; (5) compression ratio ; and (6) the spark advance These tests were carried out under full throttle, full load, and, except (4), the petrol flow was adjusted to the weakest mixture capable of giving full load at 1,800 r.p.m.

Piston temperatures were measured at the centre and at radii of 0·5, 1·0, and 1·5 in. along a line from the centre to the spare sparking plug position at the back of the cylinder, on the side remote from the cooling blast, and consequently at the hottest side of the cylinder. On this account, the hottest point of the piston is not at the centre, as with cylinder walls of uniform temperature, but at a point about 0·4 in. from the centre nearer the hottest wall. The temperature of the cylinder wall was measured at four points down the back of the cylinder.

Effect of Piston Clearance. Piston *A*, 99·5 mm. (3·9175 in.) skirt diameter, was first tested with a clearance (cold) of 0·024 in., it was then turned down to a smaller diameter, giving a clearance 0·045 in. and again tested. The results are given in tests (1) and (2) of Table I ; the effect of the increased clearance is to increase the drop in temperature between the edge of the piston crown and the cylinder wall by 29° C.—from 27° C. to 56° C. Both the piston and cylinder wall are slightly hotter. The results indicate that the skirt clearance should be reduced to the safe limit found by experience in order to obviate the " slap " of the piston. On the other hand, if the clearance is made too small there is the risk of the piston tearing and seizing. In aero engines of diameters up to 8 in., the skirt clearances were about 0·0025 in. per inch of diameter. It was stated that one firm obtained satisfactory working by allowing a skirt clearance of 0·014 in an aluminium piston of 17 in. diameter having three piston-rings, with floating gudgeon-pin to prevent distortion of the piston and assist uniform lubrication all round it. The design of piston-rings is important. It is essential to allow sufficient space at the joints to avoid butting together and eventually seizure. On a 12-in. aluminium piston the ring is not more than $\frac{3}{8}$ in. wide, and very little wider for larger diameters. An aluminium piston of $14\frac{1}{2}$ in. diameter gave satisfaction at speeds up to 380 r.p.m. under arduous conditions at sea for several years.

Three *piston designs*, *A*, *G* and *D* of the same alloy *B*4, were compared, of thickness at the crown 4·0, 6·5, and 6·5 mm. respectively, and mean diameter at top lands 3·9085, 3·9075, and 3·9125 in. It appears from tests 1, 6 and 4 that the ribless, hollow crown piston *A*, with three rings, is about 30° C. cooler than the lighter piston *G*, gave good contact with the rings, and was the best piston. The ribless design is also readily adapted to variation of thickness of metal according to the amount of heat flow.

The *bearing area* of piston *B* was reduced 4 sq. in. to *B*₁ by drilling 30 holes in the skirt, the normal piston having the total

TABLE I

RESULTS OF TESTS ON AIR-COOLED PETROL ENGINES (*Prof. Gibson*)

Test No.	Piston	Type	Alloy	Petrol. Pints per B.H.P. Hour	Brake H.P.	Temperature of Piston °C. at					Mean Back Wall Temperature °C.	Difference of Temperature between Piston and Wall
						Centre	0·5 in. radius	1·0 in. radius	1·5 in. radius	Edge		
1	A Clearance 0·024 in.	Ribless	B 4	0·594	17·85	194°	206°	184°	175°	173°	146°	27°
2	A' Clearance 0·045 in.	Ribless	B 4	0·605	17·90	234°	248°	226°	213°	210°	154°	56°
3	B	Ribless	B 24	0·598	18·00	197°	207°	184°	176°	174°	148°	26°
4	D	Heavily ribbed	B 4	0·595	17·85	210°	223°	198°	182°	177°	152°	25°
5	E	Heavily ribbed	B 18	0·598	17·70	236°	246°	215°	194°	186°	160°	26°
6	G	Lightly ribbed	B 4	0·597	18·10	215°	238°	218°	197°	184°	156°	28°
7	K	Cast iron	B 24	0·644	17·05	410°	430°	387°	310°	220°	156°	64°
8	B'	As B with holes in skirt	B 24	0·620	17·65	206°	218°	198°	188°	185°	146°	39°
9	Magneto Advance 35°	Heavily ribbed	B 4	0·605	17·25	244°	250°	230°	205°	187°	155°	32°
10	48°			0·604	17·50	254°	259°	238°	210°	189°	156°	33°
11	18°			0·827	13·00	259°	266°	249°	213°	189°	166°	33°

peripheral contact area 33·4 sq. in. The result, shown by tests **3** and **8**, was that the temperature of the piston was increased ; and the temperature drop between the piston crown and cylinder was increased from 26° to 39° C. The conclusion is that it does not pay to save weight by reducing the bearing area of the skirt by which there is a considerable transfer of heat to the cylinder wall.

Variation in the Material of the Piston. The pistons *A* and *B* were cast from the same patterns, and *D, E* were also identical in design. The results of tests 1, 3, 4 and 5 show little difference in the temperatures, except that of the piston *E*, which is about 20° C. hotter than the others at the centre. This higher temperature is caused by the lower conductivity of the alloy in *E*. The ratio of the conductivities of the alloys *B*4 and *B* 18, in the pistons *D* and *E* is 1·34, but the ratio of the temperature gradients is 1·30. In similar pistons the temperature gradient along a radius should be inversely proportional to the conductivity of the alloy. The heat flow through the ribbed metal evidently alters the temperature gradient, since the heat flows by conduction through the ribs and skirt to the bottom, which is cooled by oil splash.

The comparison of cast iron and aluminium alloy pistons, *G* and *K*, is shown in tests 6 and 7 (Table I). The pistons were of the same general type, but not of identical design. The thickness of the crown in *G* was 6·5 mm., and in *K* 3·0 mm., and their weights were 1·262 lb. and 1·776 lb. respectively. The maximum temperature in the cast iron is nearly 200° C. higher than that of the aluminium piston, and the difference between the hottest point and the edge is 210° C. in cast iron and only 54° C. in the alloy. The cast iron clearance at the mean working temperatures is only 0·0003 in. greater, with the same wall temperature, so that the drop between the piston edge and wall is 64° C. with the cast iron, and 28° C. with the aluminium piston. The alloy piston gives 6 per cent more power on 8 per cent less petrol per horse-power hour. On the other hand, a cast-iron piston in a motor-car engine, running on partly closed throttle, would be more efficient than an aluminium piston, because the hotter piston would raise the temperature of the charge in the cylinder during the compression stroke.

In experiments on a small high-speed engine of about 4 in. diameter, with cooling water and aniline in the jackets over a range of jacket temperatures from 40° C. to 230° C., the best results were obtained at about 130° C.

By increasing the *compression ratio* with a maximum load mixture at constant speed, less heat appears to be transferred to the piston from the gases in tests *A, B, C,* and *D* (Table II).

Tests *B* and *C* with aluminium piston, also *G* and *H* with cast-iron piston, give clearly the cooling effect of a very strong petrol mixture on the temperature of the piston. The other tests in

Table II show that the *weakest mixture capable of giving maximum power output also gives the highest piston temperature.*

In a series of tests with the weakest mixture for maximum load, *increase of the compression ratio* from 4·6 to 5·8, so as not to reach detonation compression, reduces the mean temperature of cylinder wall by 26° C. at the top and 20° C. at the bottom of the barrel; while the petrol consumption falls from 0·586 to 0·525 pints per brake horse-power hour, and the brake mean effective pressure increases from 116·2 to 125 lb. Evidently a larger proportion of the heat of combustion is converted into useful work, leaving less heat to be lost to the cylinder walls and piston.

TABLE II

RESULTS OF TESTS ON THE SAME ALUMINIUM AIR-COOLED CYLINDER
(*Professor Gibson*)

Piston	Test	Compression Ratio	Speed r p.m	Brake M.E.P. lb /sq in	Brake H.P	Petrol, Pints per B.H P Hour	Temperature Centre of Piston Deg C.	Mean Temperature of back wall of Cylinder Deg. C.
Aluminium	A	4 6	1800	113	17·4	0 615	214	156
	B	5 1	1800	119	18·2	0·620	213	150
	C	5·1	1800	116	17 8	0·860	196	141
	D	5 3	1800	117	17 9	0·605	195	151
	E	5 3	2000	111	18·9	0·595	202	159
Cast iron	F	4 6	1800	109	16 5	0 643	356	155
	G	4 6	1800	110	16 6	0·675	380	155
	H	4 6	1800	109	16 5	0 795	336	141

In ordinary practice, as the *spark* is *advanced* in a petrol engine to a certain point, so as to avoid detonation and pre-ignition, the power output and efficiency of the engine are increased; the temperature of the exhaust valve is reduced, while more of the heat developed in the cylinder head is converted into work, and this, with the cooler exhaust valve, appears to keep down the temperature of the piston and walls. The results of tests 9 to 11, in Table I, show the marked increase in power and economy of fuel by the spark advance from 18° to 48°, and, at 35° advance, the lowest temperature is attained on an aluminium piston having a thin crown, the engine speed being 1,800 r.p.m. The *highest piston temperature is obtained with the minimum* spark advance 18°.

Professor Gibson also carried out tests to determine the *distribution of temperature down the skirt of the aluminium piston* in a 5 in. by 5 in. air-cooled cylinder, with a compression ratio of 5, at a speed of 1,700 r.p.m.

The clearance of the piston (cold) varied from 0·045 in. over the top lands to 0·035 in. over the bottom lands, and 0·025 in. over the piston skirt. The mean temperature of the cylinder wall was 197° C., and at the edge of the piston head about 250° C., so that the drop across the oil gap was 53° C. At this point, the mean temperature of the skirt was 222° C., and the difference between the

mean wall temperature and that of the skirt was 25° C. at this part of the wall. The temperature of the piston skirt at 3·5 in. from the top of the piston was 168° C.

Tests* on Aluminium Pistons in a Water-Cooled V Aero-Engine having eight cylinders of 120 mm. (4·72 in.) bore by 180 mm. (7·09 in.) stroke and compression ratio 5·06. The water-jacket casing was of aluminium, into which a steel liner was screwed. After a short run there was often bad contact between the upper end of the liner and its aluminium backing. That was made worse by the sparking plugs being fitted into ferrules screwed through the aluminium casing and then into the steel liner. In the case of some cylinders which gave trouble from burnt pistons, when afterwards cut open, the screwing in of the ferrules had forced the liner out of contact with the casing, which soon caused the liner to become abnormally heated.

Two of the cylinders of the engine tested had previously burnt pistons.

Professor Gibson measured the temperatures of the cylinder liners by thermo-couples and found that, under normal running conditions, in these two the liners were 66° C. hotter than those in the other cylinders, at 1 in. below the level of the piston crown when at the inner dead centre. *H* and *C* denote the hottest and coolest cylinders.

Temperatures were determined by the cooling curve method at the centre of the piston and at a point $\frac{1}{4}$ in. from the edge, in line with the sparking plug hole adjacent to this side of the piston. With the engine output 189 B.H.P. at 1,880 r.p.m., and the outlet cooling water 62° C., the temperature at the edge of the piston *H* is about 270° C. nearest the sparking plug, and 220° when the plug is at the opposite side of the cylinder. This change in position of the sparking plug changes the temperature of the top of the cylinder liner from 222° to 173° C., so that the difference in temperature between the cylinder wall and piston at 1 in. below the piston crown is 48° C. At 3·3 in. below the level of the crown the difference varies from 92° nearest the plug to 85° C. with the plug on the opposite side. The temperature at the centre of the piston is in each case lower than at the edge under the sparking plug, and has a mean value 242° C., but the edge temperature changes from 220° to 275°, or a mean value 247° C. This indicates that, with a uniformly hot liner, the temperature would be constant over the piston crown, and tends to show that the piston cooling is mainly due to oil splash, which is excessive in this engine.

The results of the tests indicate that, under normal working conditions, using two sparking plugs, the mean edge temperature of piston *H* is 265° C., and that of the liner 190° C., giving a drop of 75° C. across the oil film at the edge of the piston crown, with a

* Advisory Committee for Aeronautics, L.A.S.C. Report No. 13 (May, 1918).

piston clearance of 0·06 in. at the top lands. Then the remarkable difference between the temperature of the liner and that of the cooling water is 180° C. in cylinder *H* and 120° C. in cylinder *C* ; while at 3·3 in. from the crown level the differences are 130° C. and 80° C., with the mean skirt clearance 0·017 in.

The top land clearance 0·06 in. admits the hot gases between the piston crown and cylinder walls, increasing the temperature of both ; a solid layer of carbon was deposited, and this is a bad thermal conductor. Two pistons were made, having the top clearance reduced to 0·025 in. Then the mean results of tests were : 194 B.H.P. at 1,920 r.p.m., piston edge 218° C., liner mean 152° C., and cooling water 67° C. Comparing similar tests with the larger clearance, the difference of temperature between the edge of piston and the liner was reduced from 85° C. to 66° C.

The *burning of small aluminium pistons* invariably takes place at the periphery and not at the centre, where large pistons are burned through, due probably to a porous casting and excessive temperature. When piston temperatures are measured under normal working conditions, the maximum temperature observed is not nearly high enough to produce burning.

Professor Gibson points out : (1) A burnt piston is always one in which the clearance between the cylinder wall and the piston over the top land is excessive. (2) With only one sparking plug the great majority of failures take place at a point on the diameter opposite the plug, while with two plugs they take place on the diameter joining the plugs and underneath one plug or the other, where the piston is hottest. (3) The tendency to burn appears to be greatest in cylinders fitted with inclined sparking plugs. (4) These factors may contribute to increase the local temperature, either of piston or cylinder wall, to a point at which local breakdown of lubrication is produced and the very thin oil film destroyed, after which the friction or abrasion of the piston against the cylinder wall produces a temperature sufficiently high to burn the aluminium alloy.

Most cases of burning have taken place on engines in which the supply of lubricant to the cylinder was insufficient. Moreover, the burned pistons from three different types of engines by three manufacturers, show that in 95 per cent of them the burning took place on the edge of the piston in a diameter at right angles to the gudgeon pin, just where the contact pressure would be greatest.

Cyclical Fluctuations of Temperature in the Surface of a Piston.

On the assumption that (1) the working fluid in contact with the metal has a cyclical fluctuation of $\pm A°$ C. ; (2) the heat transmission from the gas to the metal is directly proportional to their difference of temperature ; and (3) that this produces a simple harmonic fluctuation of temperature in the metal, it may be shown that, in

the case of a large flat surface, the value of a, the semi-amplitude of the fluctuation of surface temperature, is given by the expression

$$a = \frac{Ae'}{\sqrt{2\pi n\rho sk}} \text{ approximately} \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

when e', the receptivity, is relatively small, that is, the amount of heat transmitted from the gas to the surface per unit area per second, per degree difference of temperature between metal and gas; where n is the number of explosions per second; ρ is the density of the metal; s the specific heat of the metal; and k its thermal conductivity.

Thus, $\dfrac{a}{A} \propto \sqrt{}$ for a piston of given material; that is, the variation of surface temperature is inversely proportional to the square root of the frequency or number of explosions per second. If one engine was running at 2,000 r.p.m. and another with the same piston running at 200 r.p.m., the fluctuations of the surface temperature in the slow speed engine would be $\sqrt{\dfrac{2000}{200}} = \sqrt{10}$, or roughly three times greater than at the higher speed.

The simple assumption (2) cannot hold, under the complex conditions obtaining in a high-speed petrol engine, throughout the cycle in which heat is transferred to and from the piston. There is heat transference to the piston by radiation from the burning gases and hotter surfaces, such as the exhaust valve, which is proportional to the fourth power of the temperature; by convection with continuous variations of density, temperature, and turbulence tending to increase the heat flow from the gas to the metal surface by scrubbing or scouring effect. At the same time the piston is losing heat by radiation and conduction, radial heat flow towards the walls and down the skirt, as well as loss of heat by evaporation of oil mist.

After examination of the measurements of the fluctuations of temperature in the combustion space and piston of the internal combustion engine given above, and the experiments[*] by Sir Dugald Clerk on the rate of cooling in a gas engine, Professor Gibson found the assumption that the heat flow h per unit area per second is equal to eT^2, where T is the difference of temperature between the hot gas and metal surface, agreed very well with the observed results, as a fair approximation, and the semi-amplitude of the cyclical fluctuation of surface temperature is given by the expression

$$a = \frac{e\{(T_1 - T_0)^2 - (T_2 - T_0)^2\}}{2\sqrt{2\pi n\rho sk}} \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

[*] Clerk, *The Gas, Petrol, and Oil Engine*, Vol. I (1909), pp. 204–207.

where T_1 = maximum temperature of the gas
$\quad\quad T_2$ = minimum temperature of the gas
$\quad\quad T_0$ = mean temperature of the metal surface
$\quad\quad eT^2 = h$ = heat transmitted per unit area per second, from gas to surface.

The value of e depends on the temperature and density of the hot gases and on their velocity over the metal surface. An approximation to its value may be deduced from measurements of piston temperatures, for if the temperature gradient across the face of a piston is known, the rate of reception of heat h per unit area of the face, or, rather, the difference between the rate at which heat is being received by the working face and dissipated at the lower face of the piston crown is given very approximately* by

$$h = \frac{4ktT}{r^2 + t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where T = the difference of temperature between the centre and a point at radius r
$\quad\quad t$ = the thickness of the piston crown
$\quad\quad k$ = the conductivity of the material.

If $(T_1 - T_2)$ be the difference of temperatures at two radii r_1 and r_2, this equation (3) becomes

$$h = \frac{4kt(T_1 - T_2)}{r_2^2 - r_1^2}.$$

If over any portion of the crown of a piston, between radii r_1 and r_2, the thickness is proportional to the radius, so that $t = Cr$, then

$$\delta T = \frac{h(r_2^2 - r_1^2)}{4kt} = h \frac{2r.\delta r}{4kt} = h \frac{\delta r}{2kC}, \text{ integrate}$$

and $\quad T_1 - T_2 = \frac{h(r_2 - r_1)}{2kC} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

This applies to the pistons A, A_1, and B, Table 1 (p. 35) over the range of radii 0·5 in. to 1·0 in., that is 1·27 cm. to 2·54 cm. and the value of C is 0·31.

Professor Gibson deduced from temperature gradient measurements and other available data by other observers,† that the value of e, for use in equation (2), increases with turbulence, and under normal working conditions, with the compression ratios usual in practice, varies from 3.6×10^{-6} C.G.S. units in a gas engine of 6 to 12 in. diameter at 200 r.p.m., to about 11×10^{-6} C.G.S. units in a high-speed petrol engine at 1800 r.p.m. These values of e, expressed in C.H.U. per square foot per minute, are respectively 4.4×10^4 and 13.5×10^4.

* See Footnote, p. 339.
† *Proc. R.S.A.*, Vol. 77 (1906), p. 500 ; *Phil. Mag.*, Vol. 47 (May, 1924), p. 883 ; *Proc. Inst. C.E.*, Vol. CXCVI (1913–14), Part 2.

Taking $e = 11 \times 10^{-6}$ C.G.S. units, and assuming $T_1 = 2400°$ C., $T_2 = 250°$ C., $T_0 = 250°$ C. in a petrol engine at 1,800 r.p.m., with an aluminium piston for which $\rho = 2.9$, $s = 0.21$, $k = 0.38$, and substituting in equation (2), gives the semi-amplitude $a = 5.5°$ C., corresponding to a total surface fluctuation of $11°$ C.

For the cast-iron piston, $\rho = 7.8$, $s = 0.11$, $k = 0.11$, and taking the same values for e, T_1, and T_2, but assuming $T_0 = 400°$ C., the fluctuation of surface temperature is $16.4°$ C.

Obviously, from these figures, the fluctuation of surface temperature is negligible, for practical purposes.

To convert the heat inflow, h, to the piston from C.G.S. units to C.H.U. per square foot per minute, multiply by 123, since

$$ h \times \frac{60 \times (2.54)^2}{453.59} \cdot \frac{144}{} = 123h $$

In the case of the 100 mm. $\times$ 140 mm. aluminium pistons, the mean heat inflow over the whole area of the piston is 58.9 C.H.U. per min., that is, the heat equivalent of 2.5 H.P. The average output in the trials, Table I (p. 357), is about 17.9 B H.P., so that the heat given to the piston is $\frac{2.5}{17.9} = 0.14$ of the heat equivalent to the brake horse-power.

The heat supplied to the engine is 1,675 B.Th.U. per min., and the mean value of the heat given to the piston is $\frac{58.9}{1675} \times 100 = 3.5$ per cent of the heat of combustion of the fuel.

In this high-speed petrol engine working on the weakest mixture capable of giving maximum power, and with an air/petrol ratio about 13.5 to 1, the heat given to the piston and flowing to the walls is nearly 3.5 per cent of the heat of combustion of the fuel. When account is taken of the heat dissipated from the under-side of the piston, it appears that the total heat given to the piston is nearly the same fraction of the heat supply as that found in slow speed gas engine tests.

Similar calculations from data given in tests of gas engines give the following mean results —

| Authority | Brake H.P. | Heat to Piston, as a Percentage of— | | Total Loss for stroke bore ratio. |
		Heat of Combustion of Fuel	Total Heat Loss to Surface of Piston and Walls	
Hopkinson	15.6*	3.6	12.2	14.3
Burstall .	11.8*	3.7	16.3	16.4
Coker .	16.2*	4.6	13.1	17.7
Gibson .	13.9	3.5	15.0	15.0

* In these tests the indicated horse-power was measured. The mechanical efficiency is assumed to be 85 per cent in deducing the brake horse-power.

In the tests by Burstall on an engine having a water-cooled, piston, and using producer gas, the compression ratio was varied from 4·36 to 8·07. The proportion of heat given to the piston varied with the compression ratio between 3·03 and 4·95 per cent. having a mean value of 3·7 per cent.

Internal Energy or Specific Heats of Gases at High Temperatures. The physical properties of a gas in chemical equilibrium are completely specified* when we know (1) the relation between the pressure and volume at constant temperature, and (2) the internal energy per unit volume as a function of the temperature and the density.

The internal energy of a gas per unit mass is usually defined as $C_v(T - T_0)$, where T_0 is the standard temperature from which energies are reckoned, and C_v the mean specific heat at constant volume between T and T_0. For all practical purposes in gas engines, the first relation is given by Boyle's law : $pv = $ constant.† It is usual to make the further assumption that the product pv is proportional to the absolute temperature T, i.e. $pv = RT$. If this be true, then the internal energy is a function of the temperature only. In case the perfect gas law does not hold, then the relation between p, v, and T can be deduced from a knowledge of the internal energy, which will be a function of both temperature and density.

The internal energy is expressed in terms of the specific heat and absolute temperature $dE = K_v.dT$, and at any temperature, the slope of the curve giving the relation between E and T is $\dfrac{dE}{dT}$, which measures K_v. For K_v constant, the curve of internal energy would be a straight line and $E = K_v t$, where $t°$ C is the temperature above the arbitrary temperature, from which the internal energy is reckoned. If K_v varies with temperature, then $E = \int K_v.dT$. If $K_v = (K_v)_0 + at$, then $E = (K_v)_0 t + \dfrac{a}{2} t^2$, so that a curve connecting E and t can be drawn when K_v is given.

From a table or curve of internal energy it is possible to construct an ideal indicator diagram corresponding to the cycle of operations used in gas engines for any given combustible mixture, on the assumption that the combustion is instantaneous and complete at the in-centre ; that there is no loss of heat in compression, explosion or expansion ; and that during expansion the gases are at all times in thermal and chemical equilibrium. These conditions can never be completely realized in practice, but can, in theory, be approached by improvements in design within certain defined limits. Such an ideal cycle takes account of the actual physical properties of the

---

* See *Reports of the British Association Committee on Gaseous Explosions.*
† See Witkowski, *Phil. Mag.*, Vol. XLI (1896), p. 309.

working substance, but leaves out of account such imperfections as heat loss to the cylinder walls.

The results of experiments on the energy of gases give the specific heats reckoned per unit mass of the gas. It is easier to measure volumes of gas than their weights, and for most purposes the more convenient form is the specific heats per unit volume, called the *volumetric specific heats*, which may be expressed in the foot-pounds of energy required to raise 1 cub. ft. of the gas through 1° C., or reckoned per gramme-molecule.

Avogadro's Law states that the molecular weights of different gases, regarded as " perfect," have the same volume at standard pressure and temperature. If we take 2 grammes of hydrogen, 32 grammes of oxygen, or 28 grammes of nitrogen, at 0° C. and standard atmospheric pressure, each quantity occupies nearly the same volume, and is called the *gramme-molecule*, or mol. Its volume is 22,400 cub. cm., or $\dfrac{22,400}{28,316} = 0{\cdot}79103$ cub. ft., since 1 cub.ft. $= 28,316$ cub. cm.

One gramme-calorie per gramme-molecule is

$$\frac{1400}{453{\cdot}5926 \times 0{\cdot}79103} = 3{\cdot}902 \text{ ft.-lb. per cub. ft.,}$$

since 1 lb. $= 453{\cdot}5926$ grammes and 1 lb.-calorie (C.H.U.) is equivalent to 1,400 ft.-lb.

Hence *the energy of a gas expressed in gramme-calories per mol. multiplied by 3·9 gives the heat energy in foot-pounds per cubic foot*; also $\dfrac{\text{gramme-calories per mol}}{\text{molecular weight of a gas}} =$ gramme-calories per gramme, or the number of C.H.U. per pound weight.

From the characteristic gas equation $p.v = R.T$; $pmv = mR.T$, and $mR = p.\dfrac{mv}{T}$; where m is the molecular weight; mv represents the volume of a gramme-molecule, which is 22,400 cub. cm. at 0° C. and standard atmospheric pressure, $1{\cdot}0133 \times 10^6$ dynes per square centimetre; the constant $R = K_p - K_v$, the difference between the specific heats of a gas at constant pressure and at constant volume.

The **universal gas-constant,** mR, is also nearly constant for all gases, and represents the work done or change of energy per degree change of temperature, being equal to pressure × change in volume:

thus, $mR = p\,\dfrac{mv}{T} = 1{\cdot}0133 \times 10^6 \times \left(\dfrac{22,400}{273{\cdot}1}\right) = 83{\cdot}11 \times 10^6$ ergs.

Taking the gramme-calorie equivalent to $41{\cdot}868 \times 10^6$ ergs, the gas-constant mR is equivalent to

$$\frac{83{\cdot}11 \times 10^6}{41{\cdot}868 \times 10^6} = 1{\cdot}985 \text{ gramme-calories,}$$

or the same number of (C.H.U.) pound-calories.

Obviously, dividing this gas-constant by m, the molecular weight of a gas, gives the value of R in pound-calories per pound, or gramme-calories per gramme ; and the quotient multiplied by 1,400, the value of J, is the value of R in foot-pounds per pound.

For example, in the case of hydrogen m is 2, then $1 \cdot 985/2$ is $0 \cdot 993$ C.H.U. per pound, and $0 \cdot 993 \times 1400 = 1390$ ft.-lb. per lb., the value of R.

The difference of the specific heats of these gases in gramme-calories per mol. is equal to the gas-constant, so that

$$K_p = K_v + 1 \cdot 985, \text{ and } \gamma = 1 + \frac{1 \cdot 985}{K_v}.$$

In diatomic gases, like N_2 and CO, which nearly satisfy the relation $pv = RT$, the molecule consists of two atoms ; and the specific heats, expressed per unit volume and per gramme-molecule, are found by direct experiment to be very nearly alike. This also agrees with the kinetic theory of diatomic gases, that

$$K_v = \frac{5}{2} R, \text{ and } K_p = \frac{7}{2} R, \text{ so that } \gamma = \frac{7}{5} = 1 \cdot 4.$$

Substitute the value of $R = 1 \cdot 985$, gives $K_v = 4 \cdot 963$, and $K_p = 6 \cdot 948$.

Divide these by the value of $m = 28$, for each gas, we have $K_p = 0 \cdot 248$ and $K_v = 0 \cdot 177$ in gramme-calories per gramme, or C.H.U. per lb.

Increase of Specific Heats with Temperature. Real gases have higher values of K_p and K_v than those of the ideal perfect gas. Swann determined the specific heats of air at atmospheric pressure and temperatures $20°$ C. and $100°$ C. by the continuous electric heating method (p. 72), and the value obtained agrees with that deduced by Callendar* from the results of Joly's experiments on air at constant volume by his steam calorimeter (p. 73), i.e. $K_p = 0 \cdot 2419$ at $55°$ C. ; also given by the formula

$$K_p = 0 \cdot 2413 (1 + 0 \cdot 00005t),$$

for any temperature t between $0°$ C. and $100°$ C. Measured in the same way, Joly's value of K_v for air at $0°$ C. and 1 atmosphere is $0 \cdot 1727$, or $4 \cdot 98$ calories per gramme-molecule, or $4 \cdot 98 \times 3 \cdot 9 = 19 \cdot 4$ ft.-lb. per cub. ft.

The increase of specific heats is more rapid in the triatomic gases, CO_2 and H_2O (steam). Swann's value of K_p for CO_2 at $0°$ C. is $0 \cdot 1973$, and $0 \cdot 2213$ at $100°$ C. ; while the *mean* specific heat, K_p, between $10°$ and $100°$ C., deduced from Joly's experiments at constant volume, is in close agreement, being $0 \cdot 212$, which is nearly 5 per cent higher than Regnault's value at this temperature. Now K_p at $0°$ C. for CO_2 is $0 \cdot 1973$, and taking $m = 44$, gives the equivalent $K_p = 8 \cdot 68$ calories per gramme-molecule, and $K_v = 8 \cdot 68 - 1 \cdot 985 = 6 \cdot 695$, or $26 \cdot 1$ ft.-lb. per cub. ft. ; so that γ is $1 \cdot 3$ approximately.

* Callendar, *Phil. Mag.* (January, 1903), p. 76.

A direct measurement of the specific heat of *steam* by Callendar's electric heating method gave $K_p = 0.497$ at $108°$ C. The experiments of Callendar and Swann make the *ratio* of the specific heat of steam to that of air 2.05 at $100°$ C. By the above formula for air at $100°$ C., $K_p = 0.2425$, so that K_p for steam at $100°$ C. is 0.2425×2.05, or 0.497.

Measurements of specific heat of gases at high temperatures have been made in various ways—

(1) Heating under *constant atmospheric pressure*, by Regnault, Wiedemann, Witkowski, Holborn and Austin, Holborn and Henning.

(2) *Constant volume* explosion, in which the gas is heated by internal combustion : Mallard and Le Chatelier, Clerk, Langen, Petavel, Hopkinson, and others ; also by Joly's steam calorimeter.

(3) Clerk's method by successive compression and expansion of the burnt products in the engine cylinder.

(1) *Constant pressure experiments* have been carried out, by Holborn and Henning,* up to a temperature of $1400°$ C. The gas under atmospheric pressure flows steadily through an electric heater and then through a calorimeter, where it is cooled. The temperature just before entering and just after leaving the calorimeter, and the quantity of heat evolved per gramme molecule of the gas, are measured. This quantity of heat, less the work done in contraction, which is 1.985 times the fall of temperature, is the change of internal energy corresponding to that fall.

At the high temperatures, with a platinum heating tube, the temperature of the hot gas was observed with a thermo-couple near the entrance to the calorimeter. The gain of heat by the calorimeter from the heating tube was partly compensated by surrounding the calorimeter at $115°$ C with a jacket at $40°$ C., by passing a stream of cooling water through it, in order to prevent the calorimeter rising above $115°$ C. when no gas was passing. Under such conditions the calorimetric corrections become so uncertain that the probable error may amount to 10 per cent too low. Consequently, Holborn and Henning consider the rates of increase of the specific heats with temperature are more likely to be correct than the absolute values, and the results are not concordant.

The mean values of C_p between $0°$ and $t°$ given are—

For $N_2 : C_p = 0.235 + 0.000019t$ (a straight line)

　$CO_2 : C_p = 0.201 + 0.0000742t - 0.000000018t^2$

　　　　　　　　　　　　　　　(slightly curved line)

　Steam $C_p = 0.4669 - 0.0000168t + 0.000000044t^2$,

　　　　　　　　　　　　between $100°$ C. and t.

(2) *Constant volume experiments.* The early explosion experiments by Mallard and Le Chatelier† indicated a continuous increase

* *Ann. der Phys.*, Vol 23 (1907), pp. 809-842.

† *Comptes Rendus* (1880-81), Vols. 91, 93 ; *Annales des Mines* (1883), p. 274. *Ibid.* (1887), Vol. 104, p. 1780.

in the specific heat with rise of temperature at constant volume of steam and carbonic acid gas, and a slight increase for the simple gases. When an explosive charge of gas with air or oxygen is fired in an engine cylinder or closed vessel, the actual pressure and temperature attained fall far short of the values calculated from the heat of combustion on the assumption of constant specific heat of the gases, after making allowance for the loss of heat to the enclosing walls. Here the mean temperature of a gas is inferred from its pressure. Corrections amounting to 8 and 14 per cent were made for cooling, and the difference between the equilibrium value of the pressure and the maximum recorded during the explosion. The values of the specific heats at constant volume and high temperature $t°$ C., obtained from the experiments of Mallard and Le Chatelier, are as follow—

For $CO_2 : C_v = 0.1477 + 0.000176t$

$\quad$ $H_2O : C_v = 0.3211 + 0.000219t$

$\quad$ $N_2 : C_v = 0.170 + 0.0000872t$

$\quad$ $O_2 : C_v = 0.1488 + 0.0000763t$

In these experiments a cylindrical vessel 17 cm. $\times$ 17 cm. was used, whereas, later on, Langen used a sphere 40 cm. diameter. The ratio $\dfrac{\text{surface}}{\text{volume}}$ was 2·3 times as great in the first as in the second case. Langen made very few observations on mixtures giving lower temperatures than 1500° C., and takes that as the lower limit of the range of temperature, and the higher limit in these experiments was about 3000° C., reached by the true explosive or electrolytic mixture of hydrogen and oxygen. The results given by Langen are—

Air, $\quad C = 4.8 + 0.0006t$ calories per gramme-molecule

CO_2 $\quad C = 6.7 + 0.0026t$ $\quad$,, $\quad\quad$,, $\quad\quad$,,

$H_2O,$ $\quad C = 5.9 + 0.00215t$., $\quad\quad$,, $\quad\quad$,.

where C is the mean thermal capacity over the range $0°$ to $t°$ C.

These values agree well with observed pressures, except in the case of mixtures of CO and air, for which they are too high. In others the maximum deviation is about 4 per cent.

(3) *Clerk's experiments.** Sir Dugald Clerk employed a method of measuring the fall of temperature due to cooling of the burnt products in a National gas engine cylinder 14 in. diameter by 22 in. stroke, running with a load on the brake 50 H.P. at 160 r.p.m , and water jacket at 71° C. The gases used were the products of the explosion of a mixture of Ashton coal gas and air, with residual products in the engine cylinder, and consisted of 5·2 CO_2, 11·9 steam, 75 nitrogen, and 7·9 oxygen per cent by volume.

* *Proc. Roy. Soc.*, A. Vol. 77, p. 500.

By means of a special arrangement, the exhaust and inlet valves were kept closed at the end of an explosion stroke, and the burnt products were heated by compression on the return stroke and cooled by the next expansion, the speed was kept up by the energy of the flywheel, while a continuous indicator diagram was taken for a series of successive compressions and expansions.

Fig. 142 shows the ordinary compression and explosion curves, with one compression, *ABC*, and expansion, *CD*, of the exhaust gases in the cylinder. The change of internal energy in any portion *BC* of a compression stroke is equal to the work done, less the heat lost to the cylinder walls ; in an expansion stroke *CD* it is the work done plus the heat lost. The loss of heat comes in as a correction,
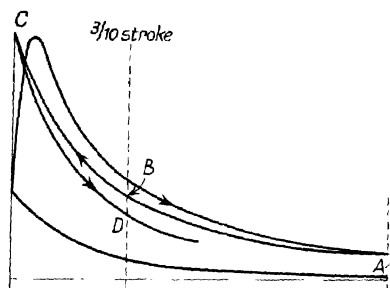
FIG. 142. INDICATOR DIAGRAM OF EXPLOSION, FOLLOWED BY COMPRESSION AND EXPANSION OF THE BURNT PRODUCTS WITHIN THE CYLINDER (*Clerk*)

of some uncertainty, on the work done and was estimated by a comparison of the compression line *BC*, and the following expansion *CD*. The total heat loss in the first partial compression and expansion line in the diagram *BCD*, Fig. 142, is estimated from the fall of temperature and from the net work done, represented by the area *BCD* in the double operation and amounts roughly to half the work done in expansion. This loss has to be divided between compression and expansion. The calculation is based on the assumption that the total heat loss from the hot gases, during any given portion of a stroke is the same in expansion and compression if the mean temperature is the same.

Suppose in the first compression the temperature of the gas rose to about 1100° C. at the point *C*, Fig 142, and during the first three-tenths *CD* of the following expansion stroke, the temperature fell to about 700°C., the work done in this part of the expansion was measured and the heat loss determined was added. Thus the change of internal energy corresponding to the temperature change, 1100° to 700° C., was obtained. The average volumetric heat over

this range is, within errors of experiment, equal to the volumetric heat at the mean temperature 900° C., which is, by this method, determined directly instead of by difference, as is necessarily the case when (as in Holborn and Henning's experiments) the whole internal energy change associated with complete cooling of the gas is measured.

The following table shows the internal energy of the mixed gas with which Clerk experimented, calculated from Holborn and Henning's figures, together with the energy calculated from Clerk's values for the mean volumetric heat. The energies are reckoned from 100° C. and are given in calories per gramme-molecule The energies of an ideal perfect gas with a constant volumetric heat of 4·9 are added for comparison. To reduce to foot-pounds per cubic foot, multiply by 3·9.

Temperature °C.	Holborn & Henning	Clerk	Ideal Gas
400	1580	1720	1470
800	3840	4250	3430
1200	6285	6900	5390
1400	7700	8300	

It will be seen that the values by Holborn and Henning are about 9 per cent lower than Clerk's ; also Clerk and Langen give the same result at 1400° C.

Professor Hopkinson made experiments on the compression and expansion of a charge of cold air in a gas engine which was motored round with the gas cut off The specific heat of air being known, the loss of heat in any part of the compression or expansion stroke can be estimated from the diagram. He found that, while in the latter half of the compression stroke the heat loss to the walls amounted to a considerable fraction of the work done, some part of this loss was actually restored to the gas during the first half of the succeeding expansion, and this notwithstanding the high temperature of the air, which in both expansion and compression was much above that of the walls. Sir Dugald Clerk tried the same experiment, and also found that the resulting value of the specific heat of air is too high, and that the air takes in heat during expansion. The corrections* from further experiments show that the values given above are less than 3 per cent too high.

By this method cooling curves* may be drawn and the effects of varying density and temperature in the engine cylinder considered and the " apparent specific heats " deduced. The apparent specific heats (instantaneous) in foot-pounds per cubic foot of the gas

---

* Second Report of Gaseous Explosions Committee, Section *G* (Winnipeg, 1909) ; see also *The Gas, Petrol, and Oil Engine*, by Clerk (1909), Vol. I, Chap. VIII.

engine burnt mixture of weight 0·07833 lb. per cub. ft. at 0° C. and 760 mm. are given by Sir Dugald Clerk—

erature C.	Specific Heat at Constant Volume (ft.-lb. per cub. ft.)	Temperature ° C.	Specific Heat at Constant Volume (ft.-lb. per cub. ft.)
0	19·6	800	26·2
100	20·9	900	26·6
200	22·0	1000	26·8
300	23·0	1100	27·0
400	23·9	1200	27·2
500	24·8	1300	27·3
600	25·2	1400	27·35
700	25·7	1500	27·45
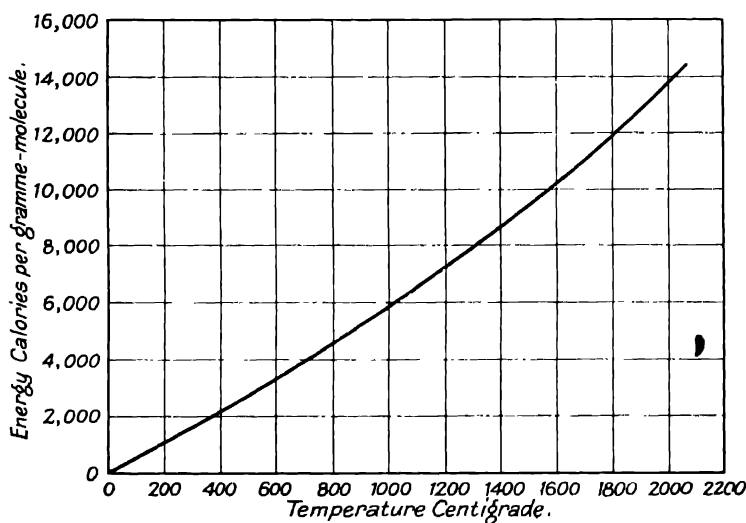
FIG. 143. INTERNAL ENERGY OF BURNT PRODUCTS IN A GAS ENGINE

The curve, Fig. 143, shows the internal energy E of the working fluid in the gas engine (p. 369) taken from that given by the British Association Committee on Gaseous Explosions after discussion of the best available results of experiments by Langen, Mallard, and Le Chatelier at the highest temperatures; Clerk, Holborn, and Henning over the lower range. To reduce the energy from gramme-calories per gramme-molecule to foot-pounds per cub foot, multiply by 3·9.

Professor Burstall gives the probable value of C_v for a gas engine mixture of 1 of gas to 9 of air,

$$C_v = 0·178 + 0·000105t$$

Adiabatic Expansion with Variable Specific Heats. The expression for variable specific heats is often given as a linear function of the absolute temperature T and the constants a, b, and k, thus—

$$C_p = a + kT, \text{ and } C_v = b + kT, \; R = C_p - C_v = a - b, \text{ and } \frac{pv}{R} = T.$$

The energy equation (p. 75) for adiabatic expansion is $C_v\, v.dp + C_p.p.dv = 0$, substituting the variable values of C_p and C_v,

$$(b + kT)v.dp + (a + kT)p.dv = 0, \text{ or } v.\frac{dp}{dv} + \frac{a + kT}{b + kT}.p = 0$$

$$\therefore v.\frac{dp}{dv}(b + kT) + pa + pkT = 0 \qquad . \qquad . \qquad . \qquad (1)$$

Divide by pv, and multiply by dv,

$$\frac{dp}{p}\left(b + k.\frac{pv}{R}\right) + \frac{dv}{v}\left(a + k.\frac{pv}{R}\right) = 0$$

$$b\,\frac{dp}{p} + a.\frac{dv}{v} + \frac{kv}{R}.dp + \frac{kp}{R}.dv = 0$$

$$b.\frac{dp}{p} + a.\frac{dv}{v} + \frac{k}{R}.d(pv) = 0. \quad \text{Integrating, we have}$$

$$b.\log_e p + a.\log_e v + \frac{k}{R}(pv) = \text{constant} \qquad . \qquad (2)$$

or $\quad b.\log_e p + a.\log_e v + k.T = \text{constant}$

Hence the adiabatic equation for variable specific heat may be written

$$p^b\, v^a.v^{k.\frac{pv}{R}} = \text{constant, or } p^b.v^a.v^{kT} = \text{constant} \qquad . \qquad (3)$$

where v is 2·71828, the base of Naperian logarithms.

The increase of specific heat makes the slope of the curve not so steep. Then the change of entropy when 1 gramme of the gas changes from v_0, T_0, to v_1, T_1, is

$$\phi = a \log_e \frac{T_1}{T_0} + R \log_e \frac{v_1}{v_0} + C_v(T_1 - T_0) \qquad . \qquad . \qquad . \qquad (4)$$

The adiabatic curves thus drawn show that in almost all gas engines the expansion lines lie below the adiabatic, showing that there is heat lost to the cylinder walls during expansion of the gas, and that when the ignition is suitably advanced to give a vertical explosion line on the indicated diagram the combustion is practically complete at the point of maximum temperature

Rate of Heat Reception with Variable Specific Heats.

Here $\quad \delta Q = C_v.\delta t + p.dv$ (p. 84)

$$\therefore \frac{dQ}{dv} = C_v\frac{dt}{dv} + p, \text{ but } \frac{dt}{dv} = \frac{1}{R}\left(p + v\frac{dp}{dv}\right) \text{ by differentiating, } T = \frac{pv}{R}.$$

$$\therefore \quad \frac{dQ}{dv} - \frac{C_v}{R}\left(p + v\frac{dp}{dv}\right) + p \qquad . \qquad . \qquad . \qquad . \qquad (5)$$

Now, take $C_p = a + kt$, and $C_v = b + kt$, $R = C_p - C_v = a - b$, and when $t = 0°$ C., $\frac{C_p}{C_v} = \gamma_0 = \frac{a}{b}$

Substituting these values of C_v and R in (5),

$$\frac{dQ}{dv} = \frac{b + kt}{a - b}\left(p + v\frac{dp}{dv}\right) + p$$

$$= \frac{1}{a - b}\left\{pa - pb + pb + pkt + bv\frac{dp}{dv} + ktv\frac{dp}{dv}\right\} \quad . \quad (6)$$

$$= \frac{1}{\frac{a}{b} - 1}\left\{p\frac{a}{b} + v\frac{dp}{dv} + \frac{pkt}{b} + \frac{ktv}{b}\frac{dp}{dv}\right\}$$

$$\therefore \quad \frac{dQ}{dv} = \frac{1}{\gamma_0 - 1}\left(p\,\gamma_0 + v\frac{dp}{dv}\right) + \frac{kt}{a - b}\left(p + v\frac{dp}{dv}\right) \qquad . \qquad (7)$$

For constant specific heats, $k = 0$, and this equation becomes

$$\frac{dQ}{dv} = \frac{1}{\gamma_0 - 1}\left(p\,\gamma_0 + v\frac{dp}{dv}\right), \text{ as in (12) (p. 85).}$$

Rate of heat reception when the expansion or compression follows the law $p.v^n = c$, a constant. Then $v\frac{dp}{dv} = -np$ (p 86), and substituting in (7),

$$\frac{dQ}{dv} = \frac{1}{\gamma_0 - 1}(p.\gamma_0 - np) + \frac{kt}{a - b}(p - np) = \frac{\gamma_0 - n}{\gamma_0 - 1}.p + \frac{kt}{a - b}(1 - n)p$$

$$= \left\{\frac{\gamma_0 - n}{\gamma_0 - 1} - kt.\frac{n - 1}{a - b}\right\}p \qquad . \qquad . \qquad . \qquad . \qquad (8)$$

For constant specific heats, $k = 0$, and this equation becomes

$$\frac{dQ}{dv} = \frac{\gamma_0 - n}{\gamma_0 - 1}.p, \text{ the same as (13) (p. 86).}$$

At the high temperatures in the explosions of compressed charges in the engine cylinder the slope of the curve giving the relation between the specific heat and temperature appears to require a term in T^2, where T is the absolute temperature.

Take $K_v = a + bT + cT_2$

and $K_p = a_1 + bT + cT_2$, where $R = K_p - K_v = a_1 - a$ constant

In adiabatic expansion, the work done by the gas is equal to its loss of internal energy, so that $p.dv = -dE = -K_v.dT$.

Divide by T, $K_v \cdot \dfrac{dT}{T} + p \cdot \dfrac{dv}{T} = 0$, and $\dfrac{p}{T} = \dfrac{R}{v}$

$$\therefore \ K_v \cdot \frac{dT}{T} + R \cdot \frac{dv}{v} = 0$$

Substitute the values of K_v and R gives

$$a \cdot \frac{dT}{T} + b \cdot dT + cT \cdot dT + (a_1 - a) \frac{dv}{v} = 0.$$

Integrating, we have

$$a \log_\varepsilon T + bT + \frac{c}{2} \cdot T^{'2} + (a_1 - a) \log_\varepsilon v = \text{constant,}$$

or $\qquad T^a v^{a1-a} \, \varepsilon^{bT + \frac{c \, T^2}{2}} = \text{constant} \quad . \qquad . \qquad . \qquad . \qquad . \qquad$ (9)

where $\varepsilon = 2 \cdot 71828$, the adiabatic relation between T and v, during adiabatic expansion or compression.

The value of the constant is calculated for the initial temperature and volume of the gas in the engine cylinder.

Then, for expansion, take lower values of the temperature, and find the corresponding values of v. In this way an adiabatic curve can be drawn giving the temperature for any value of the ratio of expansion, and then the pressure p can be found by $\dfrac{pv}{T} = \text{constant}$.

Calculation of the Ideal Efficiency of a Gas Engine. By ideal efficiency is meant the thermal efficiency which would be attained if all the heat losses to the walls were suppressed, and the combustion were complete and instantaneous. Professor Hopkinson gives an example from measurements of the thermal efficiency[*] of the Crossley 40 B.H.P. gas engine (p. 337). With average jacket temperature and the engine exploding every cycle, the volume of mixed gas and air taken in is found by anemometer experiments to be 0·85 of the stroke volume. Assume atmospheric pressure 14·7 lb. per sq. in. and the air temperature 15° C., the *quantity* taken, reckoned in standard cubic feet, is $0 \cdot 85 \times \dfrac{273}{288} = 0 \cdot 805$ of the stroke volume. This is mixed with the gases left in the compression space, and possibly also some of the exhaust products which have backed in from the exhaust pipe. The volume of the compression space is 0·187 that of the stroke volume, the pressure of the gases is atmospheric and their temperature is inferred from the pressure of the gases at release. The release pressure was determined by means of the optical indicator light spring diagrams and varied from 50 to 55 lb. per sq. in., depending on the strength of the mixture. Assume 52 lb. The volume at release is 0·9 times the total cylinder volume.

[*] *Proc. Inst. Mech. Eng.* (April, 1908).

Taking the suction temperature 100° C., or 373° C. (abs.), the temperature just before release is found by the gas equation $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$,

$$\frac{(52 \times 144) \times 0.9}{T_1} = \frac{(14.7 \times 144) \times 1}{373} \quad \therefore T_1 = 1190° \text{ C. (abs.)}$$

The adiabatic expansion down to atmospheric pressure, which occurs very rapidly after release, reduces this temperature to

$$1190 \times \left(\frac{14.7}{52}\right)^{\frac{1.35 - 1}{1.35}}, \text{ if } \gamma = 1.35$$

$$= 1190 \left(\frac{14.7}{52}\right)^{0.26} = 860° \text{ abs., nearly, or say } 600° \text{ C.}$$

Assume that this temperature within the cylinder does not change materially during the exhaust stroke, while the residual gases are in contact with the hot piston, it follows that the contents of the compression space amount to $\frac{273}{860} \times 0.187 = 0.06$ of the stroke volume, reckoned in standard cubic feet. The total cylinder contents at the end of the suction stroke would, therefore, at standard temperature and pressure, occupy $0.805 + 0.06 = 0.865$ of the stroke volume, and the volume they actually occupy is 1.187 times the stroke volume. Thus the mean temperature is

$$\frac{1.187}{0.865} \times 273 = 375° \text{ abs., or } 102° \text{ C.}$$

The calorific value of the coal gas, taken by gas calorimeter, is 600 B.Th.U. per standard cubic foot.

At the end of the suction stroke the valves are all closed and the cylinder is then full of the mixture of coal gas and air (assumed to be dry) which has been drawn in, *plus* products of the previous explosion, amounting to 7 per cent of the whole, at 100° C. and 14.7 lb. per sq. in.

The mixture is compressed adiabatically, and is fired at the in-centre, the combustion being complete and instantaneous. The products are then expanded without loss of heat to the out-centre, when the exhaust valve is opened. The efficiency is calculated for two mixtures, of which the following are particulars—

	A Weak Mixture	B Strong Mixture
Volume of coal gas taken per suction (cubic feet at external temperature and pressure) . . .	0.1	0.13
Percentage of coal gas in mixture, with air drawn in .	9.4	12.2
Percentage of coal gas in mixture in engine . .	8.8	11.4

Products of Combustion of 1 cub. ft. of Mixture—

	A Weak Mixture	B Strong Mixture
Steam	0·125	0·163
CO₂	0·053	0·069
N₂ and O₂	0·793	0·732
Total	0·971	0·964

The analysis of the products is calculated from the average composition of the gas (see p. 348).
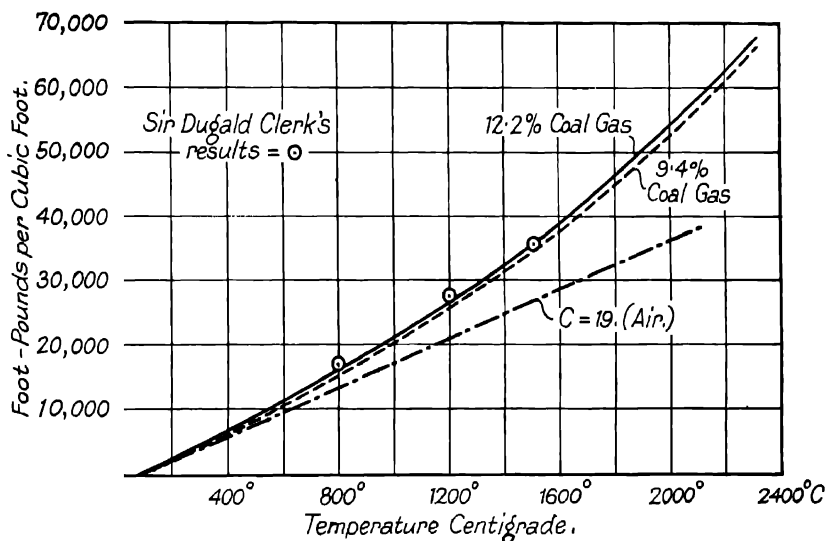
FIG. 144. INTERNAL ENERGY CURVES FOR WEAKEST AND STRONGEST MIXTURES USED (*Hopkinson*)

The internal energy curves, Fig. 144, were calculated from these compositions, using the following values for the specific heats—

Temperature	800° C.	1400° C.	1900° C.
Air	19·9	22·0	23·5
H₂O . . .	26·0	31·0	39·6
CO₂ . . .	35·2	41·4	46·1

The figures are the mean values of the specific heats at constant volume up to the given temperature, expressed in foot-pounds per standard cubic foot of the gas. Those at 800° and 1400° are the

results of Holborn and Austin,* and of Holborn and Henning,†
obtained by external heating at constant pressure. These are
probably correct within 3 per cent. The values between 1500° and
1900° C. are from Langen's explosion experiments,‡ they are prob-
ably rather too high because of incomplete combustion and loss of
heat, but they are the best available. The values given by Clerk
are for mixed gases (p. 369),. and are rather higher than those
calculated from the above figures at 800° and 1400°.

It is most convenient to follow what happens to a standard cubic
foot of the mixture in passing through the engine. The mixture
contains $\frac{0 \cdot 805}{0 \cdot 865} = 0 \cdot 93$ of its volume of gas and air, the rest being
products of combustion. Starting at 100° C., or 373° absolute, it
is compressed adiabatically to the ratio 6·37.

The temperature at end of compression is $373 \times (6 \cdot 37)^{0 \cdot 4} = 780°$

The rise of temperature during compression is $780° - 373° = 407°$C.

The work done during compression is $19 \times 407 = 7700$ ft.-lb.,
since the thermal capacity is constant and equal to 19 ft.-lb. per
cub. ft., nearly. This is the internal energy at the end of compres-
sion with either mixture.

The pressure at the end of compression is

$$14 \cdot 7 \times (6 \cdot 37)^{1 \cdot 4} = 196 \text{ lb. per sq. in. (abs.)}.$$

Strong Mixture (*B*). In this case 12·2 per cent of the mixture
drawn in is coal gas. In the mixture as it exists in the engine (after
mixing with the products of the previous explosion), the percentage
of coal gas is $12 \cdot 2 \times 0 \cdot 93 = 11 \cdot 4$

The heating value of the gas per standard cubic foot is therefore,

$$0 \cdot 114 \times 600 \times 778 = 53,000 \text{ ft.-lb.}$$

Add to this the work of compression $= \ \ 7,700 \ \ \ ,,$

$$\therefore \text{ Internal energy after explosion } = \underline{60,700} \ \ \ ,,$$

After explosion, the standard cubic foot of mixture becomes
0·964 cub. ft. of products.

$\therefore$ the internal energy of the products after explosion, reckoned
from 100° C., is

$$\frac{60,700}{0 \cdot 964} = 63,000 \text{ ft.-lb. per cub. ft.}$$

From the curve, Fig. 144, the corresponding temperature is
2210° C., or 2480° absolute ; and the pressure is -

$$0 \cdot 964 \times \frac{2480}{780} \times 196 = 600 \text{ lb. per sq. in. absolute.}$$

* *Researches of the Reichsanstalt*, Vol. 1V, 1905.
† *Annalen der Physik*, Vol. 23 (1907), p. 809.
‡ *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. XLVII, 1903.

The expansion curve is computed by trial and error. We assume an expansion curve of the form $p.v^n = $ constant. The true adiabatic will not be of this form, because the specific heat is not constant. But if n be chosen so that the gas does not, on the whole, gain or lose heat during expansion, the loss of heat in the first part being balanced by an equal gain in the second, we shall have a sufficiently close approximation to the real adiabatic.

If we take $n = 1.20$, the temperature at the end of expansion is

$$\frac{2480}{(6.37)^{0.2}} = 1713° \text{ absolute, or } 1440° \text{ C.}$$

From the curve, Fig. 144, the internal energy at this temperature is read off to be 33,700 ft.-lb., and the loss of energy in expansion is, therefore, $63,000 - 33,700 = 29,300$ ft.-lb.

The work area under this curve is most simply computed by noting that it is the adiabatic of a gas for which γ is constant and equal to 1.20, and for which the specific heat is therefore

$$\frac{R}{\gamma - 1} \text{ ft.-lb. per lb. (p. 72)}$$

$$= \frac{96}{1.2 - 1} \times 0.0807 \text{ (since 1 standard cub. ft. of air weighs 0.0807 lb.)}$$

$$= 38.7 \text{ ft.-lb. per standard cub. ft.}$$

The fall of temperature during expansion is $2480 - 1713 = 767°$ C. Therefore work done in expansion is $38.7 \times 767 = 29,700$ ft.-lb. per standard cub. ft. of burnt products, which is slightly more than the loss of energy (29,300 ft.-lb.), showing that along this assumed expansion line there must be some gain of heat on the whole. If the index 1.21 be tried, corresponding to an average specific heat of 36.9 ft.-lb. per standard cub. ft., it will be found that the loss of energy in expansion is 30,500 ft.-lb., and the work done 29,500 ft.-lb. corresponding to a slight loss of heat in expansion.

We may take the index 1.20 as sufficiently near.

Since there is only 0.964 cub. ft. of products for every cubic foot of original mixture, the work done in expansion per cubic foot of mixture is $29,700 \times 0.964 = 28,600$ ft.-lb.

The net work done in the cycle, after deducting the work of compression, is $28,600 - 7,700 = 20,900$ ft.-lb.

Since the heating value of the gas is 53,000 ft.-lb.,

$$\text{the thermal efficiency} = \frac{\text{Heat converted into work}}{\text{Heat supplied}}$$

$$= \frac{20,900}{53,000} = 0.394, \text{ or } 39.4 \text{ per cent.}$$

This is the thermal efficiency of an ideal engine using the actual working substance with adiabatic compression, combustion at

constant volume, and in which the expansion line follows the law $p.v^{1\cdot 20}$ = constant. As already pointed out, this is not an adiabatic expansion line, but possesses the property that no heat is lost or gained in the course of it ; the loss of heat in the early parts being balanced by an equal gain of heat towards the end of the expansion. The true adiabatic, for which γ is an increasing quantity, at first slightly less, and afterwards greater than $1\cdot 20$, will be at first above the assumed line, will cross it, and will finally be a little below it. The *final temperature* in true adiabatic expansion will therefore be lower than in the assumed expansion. Since no heat is lost to the walls (on the whole) in either case, it follows that the work area under the true adiabatic curve must be slightly greater than under the line $p.v^{1\cdot 2}$ = constant. Thus the efficiency calculated above is a little lower than that of an engine using real adiabatic expansion, but the difference is inappreciable.

By dividing the whole expansion into stages, and finding an appropriate value of n for each nearer to the values of γ, a still closer approximation to the true adiabatic curve may be obtained.

Weak Mixture (*A*). The following are the results with the weaker mixture, and the reader should work through all the steps of the calculation by the above method—

Compression work (same as before) $=$ 7,700
Heat in gas $= 0\cdot 094 \times 0\ 93 \times 600 \times 778 = 40,700$ Foot-pounds per cubic foot of the Mixture from 100°C.

Internal energy after explosion $= 48,400$

Energy per cubic foot of products $= \dfrac{48,400}{0\cdot 972} = 49,800$ ft.-lb

Corresponding temperature from curve, Fig. 144,

$\qquad = 1940°$ C. $= 2210°$ absolute.

Assuming the expansion curve, $pv^{1\cdot 24}$ = constant, the final temperature is $1418°$ absolute, or $1145°$ C. The energy, from the curve, is then 24,000 ft.-lb.

Loss of energy $= 25,800$

Work area under expansion curve $= 32\cdot 3 \times 792$ $= 25,600$

Work of expansion per standard cubic foot of mixture $= 25,600 \times 0\cdot 975 = 24,950$

Net work $= 17,250$

Heat supply $= 40,700$

Ideal thermal efficiency $= 42\cdot 4$ per cent.

The difference between the ideal efficiencies with the two mixtures is mainly due to the fact that a greater rise of temperature, and therefore of pressure, in proportion to the fuel used, is obtained when exploding the weak than when exploding the strong mixture.

The temperature rises are 1700° and 1430° respectively, and are in the ratio of 1·19, but the corresponding amounts of heat supplied in the fuel are in the ratio of 1·30. The pressure falls rather more rapidly in the adiabatic expansion of the weaker mixture, but the difference in this respect is not very material. The work done in
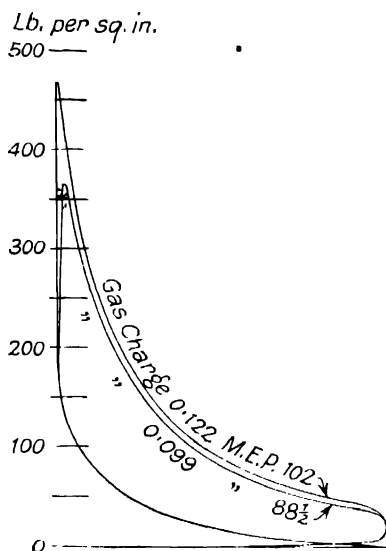
FIG. 145. TYPICAL INDICATOR DIAGRAMS FOR STRONG AND WEAK MIXTURES (*Hopkinson*)

the gas engine cycle is mainly determined by the rise of pressure which occurs on explosion.

The **effect of strength of mixture** on the indicated thermal efficiency, with the engine at full load, is shown in the following table, which gives the results of a series of tests all taken within 2 or 3 hours—

COAL GAS		M.E.P. lb. per sq. in.	Indicated Thermal Efficiency, Per cent	
Cubic Feet per Suction	Percentage of Cylinder Contents			
0·1275	11·0	102·2	32·5	Full load
0·1147	10·0	98·4	34·7	,, ,,
0·1005	8·55	90·2	36·5	,, ,,
0·1275	9·5	108·4	34·5	Light load
0·1140	8·5	101·6	36·1	,, ,,

The gas consumption was measured by the fall of a small standard gas-holder during 40 or 50 explosions, and diagrams, Fig. 145, were

taken with the Hopkinson optical indicator, the explosion line
being nearly vertical, even for the weakest mixture.

" In measuring the indicated power, two diagrams, each covering
about a dozen explosions, were photographed in every test. These
photographs were integrated by planimeter direct from the negatives
by two independent observers, and the average difference was about
1 per cent."

Professor Hopkinson thus obtained, at full load and 11 per cent
of gas present in the cylinder contents, including residual gases, the
indicated thermal efficiency 32·5 per cent, which was increased to
36·5 per cent when the gas in the mixture was reduced to 8·65 per
cent. With a light load and the same gas, 0·1275 cub. ft. per
suction, the efficiency is increased from 32·5 to 34·5 per cent, when
the percentage of coal gas in the
charge is reduced from 11 per cent at
full load to 9·5 per cent at light load,
calculated on the assumption that
suction temperature is 100° C. at full
load and 50° C. at the light load.
Similar results are given by the two
light load tests.

Fig. 146 shows the relation between
gas consumption per suction and the
mean effective pressure ; also the great
increase in pressure, with the same gas
per suction, at full and light loads.

In some trials the indicated power
was calculated from the brake power
and the indicated power at no load,
with a deduction for the difference in
work during the pumping strokes. In
this 40 B.H.P. engine, Hopkinson
found the negative indicated work
2·3 H.P. less at full load than that
indicated in idle cycles when a charge of air is drawn in com-
pressed, and expanded. One such test gave the following results—

Fig. 146. Gas Consumption
and Mean Effective
Pressure (*Hopkinson*)

FULL LOAD

Gas used, 0·1006 cub. ft. per suction at 52° F., and barometer 30·46 in.

Calorific value (measured at the time), 570 B.Th.U. per standard cub. ft.

$$\frac{\text{Gas}}{\text{Total charge}} = 0.0865, \text{ and } \frac{\text{Explosions}}{\text{Cycles}} = 0.896$$

B.H.P. = 34·4.   Efficiency on B.H.P. = 32·2 per cent.

Jacket temperature = 190° F.

LIGHT LOAD

(Taken as soon as the brakes were off)

Gas, 0·1129. Mean pressure calculated from gas consumption, 100 lb.

$$\frac{\text{Explosions}}{\text{Cycles}} - 0 \cdot 138. \quad \text{Indicated horse-power} = 6 \cdot 9$$

Extra pumping work, $0 \cdot 896 \times 2 \cdot 3$ $= 2 \cdot 0$

∴   Mechanical losses $- 6 \cdot 9 - 2 \cdot 0$ $= 4 \cdot 9$

    Indicated horse-power $- 34 \cdot 4 + 4 \cdot 9$ $= 39 \cdot 3$

    Mechanical efficiency $- \dfrac{34 \cdot 4}{39 \cdot 3} \times 100$ $= 87 \cdot 5$ } At full load

    Thermal efficiency $\dfrac{32 \cdot 2}{87 \cdot 5} \times 100$ $- 36 \cdot 8$

Fig. 147 shows the relation between the thermal efficiency at full load and the strength of mixture, including other tests on the same
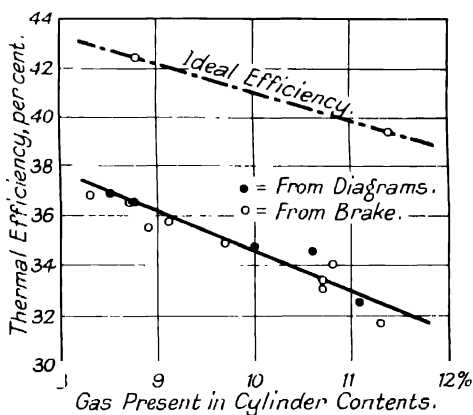
FIG. 147. THERMAL EFFICIENCY AT FULL LOAD *(Hopkinson)*

engine. In the weakest mixture used, the ratio of coal gas to air drawn in was about 1 to 9·5, and the charge in the cylinder contained 8·65 of coal gas. The strongest mixture that could be used in practice, without excessive explosion pressures, was 1 of coal gas to 7·5 of air, the excess of air being 1·5 times the volume of gas.

Within this range the thermal efficiency diminishes steadily as the proportion of gas in the mixture increases, so long as combustion is practically complete, the difference between the weakest and strongest charge being 4·5 per cent in thermal efficiency, or 12 per cent on the work done.

Professor Hopkinson explains that of the 12 per cent additional work done per cubic foot of gas with the weaker mixture, about half is due to lower mean specific heat and half to a smaller heat

loss to the walls in explosion and expansion at lower temperature. Comparing the actual with the ideal efficiency, for a mixture containing 8·5 per cent of coal gas the efficiency ratio is 0·87, but when the gas is increased to 11 per cent, it is only 0·83, so that the weaker mixtures, in addition to giving a higher ideal efficiency, come nearer in practice to realizing that ideal.

Heat Loss in Expansion. Part of the heat in the jacket cooling water passes into the walls after release and should be credited to exhaust.

In a true heat balance, the work done and the energy contained in the gas at the end of expansion should be measured, and the heat loss during expansion obtained by difference. The energy at the end of expansion is calculated from the temperature and quantity of gas present, and the internal energy curve.

The temperature at the end of expansion can be inferred from the pressure of the gas at release, that is, the pressure observed on the indicator diagram at the moment of opening the exhaust valve, which occurs about 45° crank angle before the out-centre. The pressure at end of expansion is that which would be obtained if the expansion had continued to the end of the stroke, on the assumption that the expansion curve would be the adiabatic $p.v^{1·35} =$ constant

The temperature is then calculated from the pressure, taking the suction temperature 100° C. and pressure 14·7 lb. per sq. in., and allowing for a contraction of 3 per cent during explosion.

A series of consecutive tests were made, with gas charges per explosion about 0·1 and 0·13 cub. ft. of calorific value 604 B.Th.U. per standard cubic foot. The following results were obtained—

	Weak Mixture	Strong Mixture
Gas per explosion as measured by gas-holder .	0·1007	0·1294
Gas used per explosion (standard cubic foot). .	0·095	0·122
Percentage of coal gas present in cylinder contents	8·5	11·0
Pressure at release (lb. per sq in. absolute) . .	52	57
Pressure at end of expansion	45	49·5
Temperature at end of expansion (abs. C) . .	1180	1290
Heat supplied per explosion, B Th.U. . . .	57 4	74

From these observations, and taking the volume of stuff present as 1·06 standard cub. ft., the heat energy at the end of expansion (from the internal energy curves) is found to be 24 and 29 B.Th.U. respectively, which are 42 per cent and 39 per cent of the total heat supplied.

At full load, analyses of the exhaust gases show that not more than 1 per cent of the fuel is ever unburnt at the end of the expansion stroke, and may be neglected.

The heat balance is as follows—

	Weak Mixture, Per Cent	Strong Mixture, Per Cent
Indicated work (from curve)	37	33
Heat in exhaust (from release pressure)	42	39
Heat loss in expansion (by difference)	21	28
	100	100

Radiation.   An estimate of the radiation was made by comparing the jacket loss with the same gas charge with a hot and cold jacket, other conditions being kept the same.   The result of several tests showed that when the jacket water at outlet was 70° C., the heat taken away by the water is less than at 40° C., by between 100 and 150 B.Th.U. per min., that is, between 2 and 3 per cent of the whole heat supply, when the engine is running at full load.   Since there is some radiation at 40° C., it is probable that the radiation at 70° C. is about 4 per cent.

Professor Burstall, in the early stage of his research work, found the radiation from the outer wall of the cylinder amounted to about 4 per cent of the total heat when the jacket water was at 65° C. His results also agree with those given above in showing an increase in economy as the charge gets weaker.

The air cycle thermal efficiency, 52·2 per cent, for constant specific heat, with no heat loss, corresponding to the compression 6·37, would be represented by a horizontal straight line on the diagram, Fig. 147.

The ideal thermal efficiency nearly follows a straight line law, like the actual efficiency which has the steeper slope.   Professor Hopkinson pointed out that these lines, when continued towards the left of the diagram, meet the vertical of no heat supply at nearly the same point ; so that, if it were possible to burn weaker mixtures by stratification of the charge, and if the slope of the ideal and actual efficiency continued in the same relation to the gas consumption, these efficiencies would tend to become equal to one another and to that of the air cycle, as the heat supply is reduced to a very small quantity.   (See Fig. 150, p. 388.)

Ideal Thermal Efficiency with any Liquid Hydrocarbon Fuel. Messrs. Tizard and Pye have investigated the *ideal thermal efficiency* in the high speed petrol engine, assuming no loss of heat to the cylinder walls, but allowing for losses due to increase of specific heat at high temperatures ; also for dissociation with re-combination during explosion and expansion.   They show that, within narrow limits, the efficiency obtainable, with any hydrocarbon fuel, is the same when burnt at the same compression ratio ; and that for a

correct explosive mixture the highest ideal efficiency attainable is $E = 1 - \left(\dfrac{1}{r}\right)^{0.258}$

Messrs. Tizard and Pye assume 20 per cent excess of air in the weakest possible mixture consistent with complete combustion, for which the ideal efficiency would be $E = 1 - \left(\dfrac{1}{r}\right)^{0.295}$

In an engine having the most perfect design of compact combustion chamber, so that the direct heat loss to the jacket walls
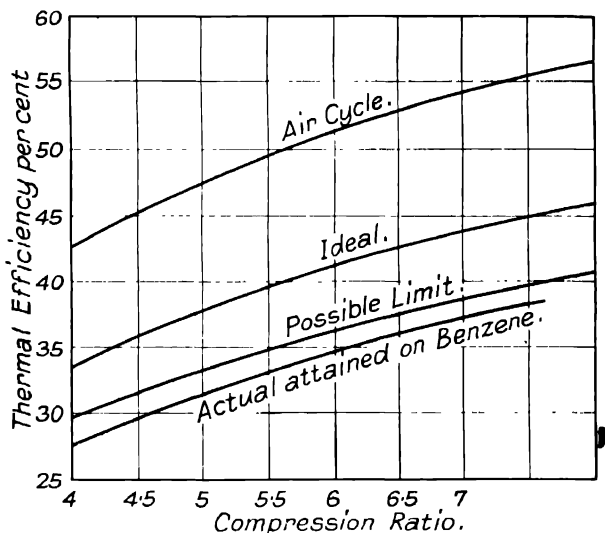
FIG. 148.   IDEAL AND INDICATED THERMAL EFFICIENCIES

during explosion and expansion is reduced to the minimum possible, Mr. H. R. Ricardo gives the highest indicated thermal efficiency attainable.*

$$E = 1 - \left(\frac{1}{r}\right)^{0.25}$$

This formula represents the *limiting thermal efficiency* obtainable under the best possible conditions in a cylinder of comparatively large capacity and the speed of the engine not less than 1,500 r.p.m., assuming (1) perfect carburation and distribution; (2) that the compression and expansion ratios are equal; and (3) that the mixture is homogeneous and of the most economical strength. Here the index, 0·25, is taken as $n - 1$, instead of $\gamma - 1$.

* Society of Automotive Engineers, U.S.A.; also *The Automobile Engineer*, September, 1922.

The following table and Fig. 148 give the ideal thermal efficiencies for a range of compression ratio from 4 : 1 up to 8 : 1, and the air standard efficiency.

IDEAL THERMAL EFFICIENCY

Com- pression, Ratio, r	Air Cycle $\gamma = 1.4$	Tizard & Pye Correct Mixture $n = 1.258$	Tizard & Pye 20% weak $n = 1.295$	Ricardo Limit $n = 1.25$	Results Observed for Benzine 15% weak, by Ricardo
4·0	0·4256	0·300	0·336	0·296	0·277
4·5	0·4521	0·322	0·359	0·314	0·297
5·0	0·4747	0·340	0·378	0·332	0·316
5·5	0·4944	0·356	0·396	0·348	0·332
6·0	0·5116	0·370	0·411	0·361	0·346
6·5	0·5270	0·383	0·424	0·375	0·360
7·0	0·5398	0·395	0·437	0·386	0·372
7·5	0·5534	0·406	0·449	0·396	0·383
8·0	0·5647	0·416	0·460	0·406	- -
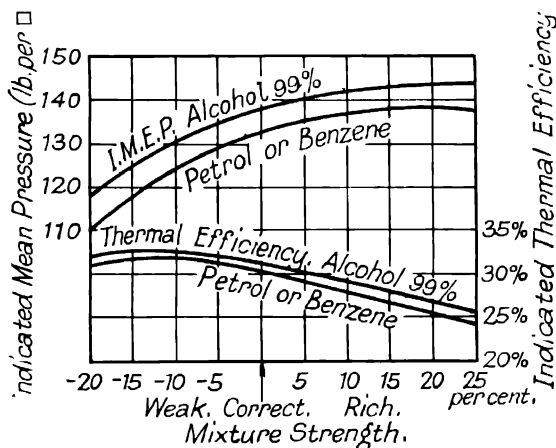
FIG. 149. OBSERVED VARIATION IN MEAN PRESSURE AND THERMAL EFFICIENCY (*Ricardo*)

Heat input to carburettor = 65 B.Th.U. per min.

Speed = 1,500 r.p.m.   Compression ratio = 5 : 1.

Mr. Ricardo showed by experiment that in practice the highest thermal efficiency is always obtained with about 15 per cent excess of air in homogeneous mixtures of any volatile liquid fuel or gas, except hydrogen and ether. No matter how their chemical or physical properties may vary, they all apparently have this characteristic in common.

The observed variation in thermal efficiency is shown in Fig. 149

over a wide range of mixture strength. For each fuel the maximum efficiency is with a mixture 15 per cent weak, when it is necessary to advance the ignition 15° earlier (from 32° to 47°) than that required for the correct mixture giving complete combustion. At this compression ratio and speed, *with still weaker mixtures*, the thermal efficiency and mean pressure are reduced by slow and incomplete combustion ; flame lingers in the cylinder, and the fresh charge is ignited on entry.

The increase of specific heat and direct loss of heat to the cylinder walls depend upon the temperature in the cylinder. If it were
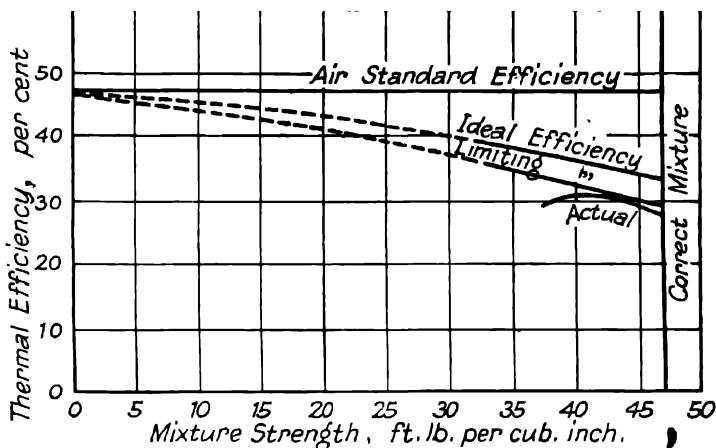
FIG. 150 IDEAL THERMAL EFFICIENCY (COMPRESSION RATIO 5 : 1)

possible to diminish the temperature by control of the fuel alone, with a weaker mixture as the load is reduced, the direct heat loss would diminish and the thermal efficiency rise in nearly a straight line until, at the point of no heat supply and no load, the limiting efficiency would coincide with that of the air cycle, as shown in Fig. 150.

By *stratification*, it is possible to reduce the mixture strength by using a small charge of combustible mixture and admitting separately a large charge of air, and prevent them mixing until after ignition ; then turbulence is needed in the combustible part of the charge to ensure rapid combustion.

Mr. Ricardo succeeded experimentally in doing so, on two engines, obtaining the whole range from dead light to full load of 24 B.H.P. at 750 r.p.m. Fig. 151 shows the actual efficiency 37 per cent at about one-third full load, corresponding to a fuel consumption of just under 0·36 pint of benzol per indicated horse-power hour.

Not only is the efficiency on reduced loads far higher than could be attained by other means, but the heat loss is so low that a

water-cooled engine can be run continuously at reduced loads without cooling water. The engine has a water-cooled bulb, but no water jacket or fins on the cylinder of 5 in. bore. In the top of
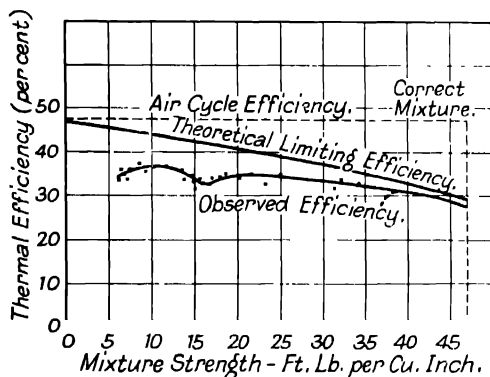
FIG. 151. EFFICIENCY AND MIXTURE STRENGTH FOR STRATIFIED CHARGE ENGINE WITH COMPRESSION RATIO 4·8 : 1 (*Ricardo*)

the bulb an automatic inlet valve is fitted for admission of a little fuel and air, the igniter being at the side. The bulb is connected by a narrow neck with the main combustion chamber, which is conical, and fitted with the main air inlet and exhaust valve. As
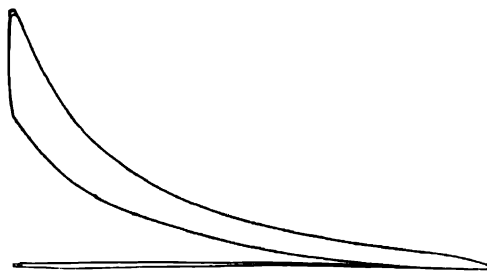
FIG. 152. INDICATOR DIAGRAM FROM STRATIFIED CHARGE ENGINE (*Ricardo*)

the piston descends during the suction stroke, the automatic inlet valve in the bulb opens first, and a mixture of air and fuel is drawn in through a throttle valve. At about 20 per cent down the stroke, the main air valve is opened and cold air enters the cylinder below the bulb, and probably at first some of the rich mixture flows into the depression. On the compression stroke some air is driven up through the narrow neck into the bulb, causing great turbulence. At the end of compression the rich mixture in the bulb is fired, and the burning mixture rushes out through the narrow neck, mixes

with and ignites any fuel in the very weak mixture containing a large excess of air.

Detonation. The efficiency of the ordinary petrol engine working on the explosion cycle is limited by the tendency of the fuel mixture to detonate and ultimately to pre-ignite when the compression ratio is increased above a certain limit, depending on the self-ignition temperature and time rate of the self-propagating flame through the fuel and air mixture ; also upon the nature and *chemical constitution* of the liquid hydrocarbon fuel. Detonation appears to be the setting up of extremely rapid rise of pressure, like an " explosion wave," which passes through the mixture at a velocity hundreds of times greater than the normal spread of inflammation.

When an explosive mixture of oil vapour and air is compressed and ignited at any point, the flame at first spreads by the ordinary process of flame propagation, aided by turbulence, and by its expansion compresses before it the unburnt portion of the charge. Unless the latter can get rid of its heat with sufficient rapidity, by conduction and convection to the cylinder walls, it is liable to be compressed above its self-ignition or self-propagating flame temperature, and to ignite spontaneously throughout its whole mass. Thus an " explosion wave " is set up, which strikes the cylinder walls with a ringing hammer-like blow and noise known as " pinking," and reacting in its turn further compresses the portion of the charge first ignited, raises its temperature and that of the igniter points or other parts insulated from the water jacket in the vicinity from which ignition first started to so high a degree as eventually to cause persistent pre-ignition, which may stop the engine.

The investigations of Messrs. Ricardo and Tizard have led to the following conclusions—

(1) Detonation depends primarily upon the *time rate of burning* of that portion of the charge first ignited.

(2) That the rate of burning increases very rapidly with slight increase of *flame temperature*, and that whether it will prove sufficiently rapid to produce detonation or not depends upon the ratio between the rate of evolution of heat by the burning portion of the mixture and the *rate of heat loss* to the cylinder walls. An engine with water-cooled exhaust valves, or with sleeve-valves and combustion chamber without side pockets, and hemispherical cylinder head with ample water cooling, is comparatively exempt from detonation, and will stand a high compression ratio.

(3) Detonation also depends upon the *distance* the flame has to travel from the ignition point before it has passed through the whole mixture. Thus the more nearly hemispherical in shape the *combustion chamber*, the better to avoid detonation ; and the *smaller the cylinder* the less the tendency to detonation of the charge.

(4) For any given mixture strength the maximum flame temperature which causes detonation depends upon the *compression*

temperature and the proportion of *residual inert exhaust products* present in the cylinder, which, like CO_2, exert a great influence in diluting the charge, *retarding the rate of burning*, and so lowering the temperature of the flame that a higher compression can be used.

This theory is supported by various tests : (*a*) the residual products were cleared away by scavenging with air, when it was found that " detonation " at once became severe, even with very low compression pressures ; (*b*) varying quantities of additional cool exhaust products were added by way of the carburettor, and the compression could be raised.

It has been the practice to add to paraffin petrols benzol or benzine, which reduces the tendency to pre-ignition and so permits of a higher compression and more efficient use of the fuel. Toluene is a still more effective " anti-detonator," while the alcohols are the best of all. A small proportion of alcohol to petrol stops detonation and allows a higher compression to be used, but the alcohol must not contain water. It has been discovered that tetra-ethyl lead suppresses detonation, even when added in very small quantities. In America lead ethide is widely used for this purpose.

Commercial " 90 per cent benzol " is a heavy spirit of specific gravity 0·88 at 60° F., and consists of about 70 to 75 per cent benzene (C_6H_6), 24 to 29 per cent toluene (C_7H_8), and 1 per cent xylene (C_8H_{10}). Its latent heat of evaporation is 164 B.Th.U. per lb. ; it requires 13·5 lb. of air per lb. benzol for complete combustion, and has a lower calorific value about 17,250 B.Th.U. per lb. For instance, a petrol consisting mainly of fractions of the paraffin series with the most efficient mixture strength and ignition timing, " detonates " at a compression ratio of 4·85 : 1. By adding 20 per cent of toluene, the compression ratio can be raised to 5·57 : 1, the increase in thermal efficiency on actual test is found to be from 31·1 per cent to 33·5 per cent, and in mean effective pressure from 131·8 to 140 lb. per sq. in. ; while the toluene adds less than 2 per cent to the weight of the fuel per unit of heat.

In his research work Mr. Ricardo designed a *variable compression engine** in which the ratio of compression could be quickly changed from 3·7 to 8, while running at full power, without disturbing the temperature conditions, by raising or lowering the whole cylinder, together with carburettor, camshaft, and valve gear. A micrometer is used to measure and record the compression ratio. Every expedient is adopted to attain the highest possible thermal efficiency and power output and to ensure that all losses, whether thermal or frictional, are reduced to a minimum.

By means of this engine the highest compression ratio was determined for each fuel to give the best results, without risk of pre-ignition during compression. Table VI shows the compression ratio at which the samples of fuel were found just to detonate ; also

* *The Automobile Engineer*, August, 1921.

the temperature, when a mixture of fuel and air of a strength to give complete combustion, was suddenly compressed, without initial turbulence, until self-ignition or spontaneous ignition just occurred.

TABLE VI (*H. R. Ricardo*)

Fuel	Specific Gravity at 15° C.	Detonation Point in Engine			Self-ignition Temperature by Adiabatic Compression with Air Mixture, °C.
		Compression Ratio	Compression Pressure, lb. per sq. in. (Gauge)	Compression Temperature °C.	
Petrol *A* . . .	0·782	6·1	148·5	430	367
Petrol *F* . . .	0·704	5·05	111·5	400	---
Petrol *G* . . .	0·750	4·55	96	381	—
Kerosene . . .	0·813	4·2	86	369	—
Aromatic Series					
Benzene, 98% . .	0·884	6·9	179	450	419
Toluene, 99% . .	0·870	⌐ 7	⌐ 183	> 452	422
Alcohol Group					
Ethyl Alcohol, 98%	0·798	⌐ 7·5	⌐ 204	⌐ 424	514
Methyl Alcohol (Purified Wood Naphtha)	0·829	5·2	116·5	342	457
Methylated Spirits	0 821	6·5	163·5	382	—
Ether	0·735	2 95	47·5	305	256

Table VII gives the results of numerous experiments oft-repeated with a very high degree of accuracy in measurement, all the conditions being checked before and after each trial run. It was found from experiments that the characteristic of the torque curve over the range of speed 800 to 1,800 r.p.m. was the same for every fuel tested with the correct mixture strength and ignition setting for each speed. In each series of tests the engine was run on wide-open throttle at 1,500 r.p.m., with the best ignition setting, mixture strength, and compression ratio for the development of the maximum power obtainable on each sample of fuel ; the temperature of the circulating water and the heat input to the carburettor being kept constant.

Effect of Latent Heat of Evaporation. Starting with a mixture of fuel and air at atmospheric temperature, heat is added by (*a*) contact with the inlet valve and hot walls of the cylinder, (*b*) admixture with the residual exhaust gases in the combustion chamber, and by (*c*) external heating of the carburetter or induction system. Heat is absorbed by (*d*) the latent heat of evaporation of the fuel ; (*a*) and (*b*) may be regarded as constant.

The weight of charge taken into the cylinder will be inversely proportional to its absolute temperature when the inlet valve closes. Experiment proves that, with the exception of alcohol, all fuels

boiling below 200° C. are completely evaporated at the end of the suction stroke, except a very small negligible proportion entering the cylinder in coarse drops, which are neither evaporated nor burned completely. The temperature, and therefore the weight of the charge taken into the cylinder, depends upon the quantity of the fuel and its latent heat; also upon the heat added to it before entry. With a suitable fixed amount of pre-heating, the volumetric efficiency, and therefore the *power output, will vary directly with the latent heat of the fuel*, since it is immaterial, as regards power output, whether the pre-heating is expended in raising the temperature of the mixture or in vaporizing the fuel at low temperature, provided it is completely evaporated before the inlet valve closes. The fall in temperature of the mixture due to evaporation of the liquid is shown (p. 234). Variations due to the differences in the latent heat of evaporation were most marked in the alcohol group.

TABLE VII

RESULTS OF TESTS (*H. R. Ricardo*)

FUEL	Specific Gravity at 15° C	Compression Ratio 5·1 Maximum I.M.E.P. lb. per sq. in.	Highest Useful Compression Ratio, Total Vol Clearance — *r*	Highest Useful Compression Pressure, lb per sq. in. (Gauge)	Maximum I M.E P at highest Useful Compression lb./sq. in.	Minimum Con-sumption lb. per I H.P. hour	Thermal Efficiency at highest Useful Compression Per Cent
A Petrol . . .	0·782	131 2	6·0	148 5	140·1	0·393	31 9
B ,, . . .	0·723	131 5	5·7	133 5	137 5	0 393	34·1
G ,, . . .	0 727	131 0	5·25	118 0	133 9	0 410	32 5
D ,, . . .	0·760	131·2	5·35	121 5	134 9	0·407	33 1
F ,, . . .	0 704	131 8	5·05	111 5	132·7	0 412	32 1
H ,, . . .	0·767	131·0	5 9	140·5	139·5	0 389	34 6
Heavy Aromatics	0 885	130·7	6·5	163·5	142·5	0·447	31·5
Paraffin Series							
Hexane (80% pure)	0 685	132·3	5 1	113·5	133·1	0·405	32 4
Heptane (97% pure)	0·691	131 2	3·75	72 0	119	0·491	26·7
Aromatic Series							
Benzene (pure).	0 884	131 6	6 9	179·0	146·5	0 392	37 2
Toluene (99% pure)	0 870	131·5	⟩·7	>183	147	0 385	37 5
Naphthene Series							
Cyclohexane (93% pure) . .	0 786	131·3	5 9	140 5	139 2	0 385	34 9
Hexahydrotoluene (80%) . .	0·780	131 0	5·8	136 5	137·9	0 394	34 3
Olefines.							
Cracked Spirit(53% unsat.) . .	0·757	131·6	5·55	128 0	136	0 405	33·9
Alcohol Group							
Ethyl Alcohol (98%) . .	0·798	137·8	>7 5	>204	156·5	0·532	40 4
Methyl Alcohol (Wood Naphtha)	0·820	144·8	5·2	116·5	146·6	0·725	35·1

A constant supply of heat was added to the air at the carburettor by electrical resistance coils, giving 65 B.Th.U. per min. when running at 1,500 r.p.m.

All the tests proved that the power obtainable was proportional to the internal energy of the fuel. It will be seen from Table VII that the observed power, given in terms of the indicated mean effective pressure, was practically the same for all fuels boiling below 200° C., except alcohol, *when used at the same safe suitable compression ratio* 5 : 1, with the ignition fully advanced. The three columns on the right give the indicated mean effective pressure, the fuel consumption, and thermal efficiency when full advantage was taken of increasing the compression ratio up to the limit set by detonation or by pre-ignition for each particular fuel, under the same constant conditions of pre-heating.

In the case of alcohol, the very high latent heat of evaporation, and the larger proportion of fuel to air in the mixture, caused the evaporation to continue throughout the compression stroke, which was nearly isothermal. The low temperature in the cylinder increased the weight of charge, or volumetric efficiency, and the small rise of temperature at the end of compression, combined with the low calorific value of alcohol, conspired to keep the cylinder temperature low throughout the cycle. Hence the direct heat losses by conduction and increase of specific heat were reduced. The power obtainable also increased as the mixture strength was enriched and continued to increase until the over-rich mixture was about 20 per cent more than that required for complete combustion ; apparently because more fuel was then evaporated and the temperature of the charge lowered, the gain in weight of charge or volumetric efficiency more than counter-balanced the increased loss due to change in the specific heat of the products of combustion.

Although ethyl alcohol gives greater maximum power, and can be used with a much higher compression ratio and thermodynamic efficiency than either petrol or benzol, yet the calorific value per unit weight or per unit volume of the liquid is so much lower than that of petrol or benzol, that the rate of fuel consumption of alcohol per horse-power hour is greater.

The **advantages of alcohol** as a motor fuel are enumerated in the 1907 Report of the Fuels Committee of the Motor Union, U S.A. : (*a*) high compression pressure over 200 lb. per sq. in. before ignition ; (*b*) the wide range of explosive mixtures of alcohol vapour with air, from 4 to 13·6 per cent by volume ; also the small range of boiling point at which industrial alcohol distils from 80° to 110° C., whereas that of petrol extends at least 50° to 150° C. or higher, and benzol 80° to 120° C. ; (*c*) high thermal efficiency and safety in use, the flash point being 60° C. as compared with that of petrol − 10° C., and a fire from alcohol can be extinguished by water, which only spreads the flames of petrol.

It is obvious from the results in the above tables that ethyl alcohol is a more suitable fuel for power purposes, at high compression, than methyl alcohol or wood spirit, because of the

low calorific value and high latent heat of evaporation of the latter.

An admixture of industrial alcohol with some highly volatile spirit appears necessary not only for easy starting from cold, but also to increase the heating value of the mixture, in order to obtain, with high compression, a lower rate of fuel consumption per horse-power hour.   Industrial or power alcohol is never pure, but nearly always contains water and, in order to render it undrinkable, a small proportion of acetone, pyridine, or other unpalatable substance not easily separated, is added as a denaturant. The aromatic hydrocarbons, benzol or benzene and toluene, are readily soluble in alcohol, even at low temperatures, have comparatively high heating values, and can be used to best advantage at high compression with good excess of air, giving complete combustion and tending to prevent acetic acid being formed. Thus, a mixture of industrial alcohol, with 20 or 30 per cent of benzol, has a high ignition temperature, and can be used in an engine with a compression ratio of about 8 : 1.   The high percentage of carbon in benzol compensates for the low carbon content of alcohol.   Ethyl alcohol is obtained by the fermentation and distillation of any starchy or sugary substance like sugar-cane or edible grains.

The supply of alcohol is inexhaustible while the sun shines, but it is not likely to prove a commercial success to a large extent until it can be produced cheaply on a large scale from some form of vegetation without destroying foodstuffs.

In Natal, South Africa, an alcohol-ether fuel, called Natalite,* was introduced in 1915, consisting of 54·3 per cent alcohol, 45 per cent ether, 0·5 per cent ammonia, with 0·2 per cent white arsenic as denaturant.   This fuel is obtained cheaply as a by-product, but only in rather small quantities, and is reported to have given cool running in a car engine designed to use petrol, and even easier starting than with petrol.   The high percentage of ether tends to cause detonation.   The heat value is about 14,000 B Th.U. per lb., as against 18,500 B.Th.U. per lb. of petrol.   On the other hand, it can be used at about 10 per cent higher efficiency.   It can only compete commercially with petrol so long as it is a by-product.

In France and Germany the use of alcohol-petrol, alcohol-benzol mixtures was encouraged prior to and during the Great War.   The large, specially designed carburettor is well heated, and the cylinder walls kept near the boiling point of water.   One type of alcohol engine is started and stopped on petrol.   Trials of alcohol engines using a mixture of alcohol with 50 per cent benzol gave high thermal efficiency.

Extensive research and experimental work was carried out by the Empire Motor Fuels Committee on various alcohol mixtures, and the results reported in the *Proceedings of the Institution of Automobile Engineers.*

* *The Autocar*, May, 1919.

Air Measurement. Mr. Ricardo measures the air consumption of engines on the test bed, in order to determine the combustion efficiency, and from the observed maximum mean effective pressure developed, so ascertain the thermal efficiency. The air measurement is made to a very high degree of accuracy by means of a calibrated gas-holder and an electro-magnetically operated counter, recording every 0·2 cub. ft. When the engine is developing its normal output and the temperature and all other conditions are normal and steady, the observations give : (1) the number of revolutions during the consumption of a measured quantity of hydrocarbon fuel ; (2) the number of cubic feet of air taken in by the engine and its temperature ; (3) the time taken during the consumption of fuel and air.

TABLE VIII

AIR CONSUMPTION (*H. R. Ricardo*)

FUEL	Effective Lower Calorific Value, B.Th.U. per lb.	Air to Fuel Ratio at Correct Mixture Strength	Heat Liberated by 1 lb. of Air, B.Th.U.
Petrol or Gasoline—			
Sample *A* . . .	19,200	15·05	1275
,, *B* . . .	19,020	14·7	1295
,, *C* . . .	19,120	14·8	1293
,, *D* . . .	18,900	14·6	1295
,, *F* . . .	19,250	15·0	1285
,, *G* . .	18,920	14·7	1288
Kerosene . .	19,100	15·0	1275
Hexane	19,390	15·2	1275
Heptane . . .	19,420	15·1	1285
Benzene . . .	17,460	13·2	1320
Toluene	17,660	13·4	1315
Cyclohexane . .	18,940	14·7	1290
Ether . .	16,830	13·0	1295
Ethyl Alcohol, 99%	11,950	8·95	1335

Table VIII gives the total effective heat energy evolved by combination with the oxygen in 1 lb. of air when fully saturated with various fuels, irrespective of the calorific value of the fuel. The values in the last column do not include the weight of fuel in the mixtures, and it will be seen that, whatever fuel be used, the energy liberated by 1 lb. of air will always be 1,300 B Th.U., approximately, The variation over the range of petrols is only from 1,275 to 1,295, depending upon the proportion of aromatics present in the fuel.

Ethyl alcohol gives the greatest amount of heat energy per pound of air.

TABLE IX

PETROL, SAMPLE *D* COMPRESSION RATIO, 5 : 1 (*H. R. Ricardo*)

Mixture Strength		Total Air lb. per hr.	Indicated Mean Pressure, lb. per sq. in.	Indicated Horse-power	Lb. of Air per I.H.P. Hour	Indicated Thermal Efficiency Per Cent
Correct	. . .	196·0	132·0	32·0	6·13	32·1
Excess fuel	5%	196·5	135·0	32·7	6·0	32·8
,,	,, 10%	197·0	136·0	33·1	5·95	33·1
,,	,, 15%	197·5	137·5	33·3	5·93	33·2
,,	,, 20%	198·1	138·0	33·4	5·94	33·15
,,	,, 25%	198·5	138·0	33·4	5·96	33·05
,,	,, 30%	199·5	137·5	33·3	5·98	32·85
,,	,, 35%	200·0	136·5	33·1	6·04	32·7

TABLE X

ETHYL ALCOHOL, 99% COMPRESSION RATIO, 5 : 1 (*H. R. Ricardo*)

Mixture Strength.		Total Air lb. per hr.	Indicated Mean Pressure, lb. per sq. in.	Indicated Horse-power	Lb. of Air per I.H.P. Hour	Indicated Thermal Efficiency, Per Cent
Correct	. . .	199·0	141·0	34·2	5·82	32·9
Excess fuel,	5%	199·5	143·0	34·6	5·77	33·3
,,	,, 10%	200·0	144·5	35·0	5·72	33·65
,,	,, 15%	201·0	145·5	35·25	5·70	33·7
,,	,, 20%	202·0	146·5	35·5	5·69	33·75
,,	,, 25%	203·0	147·0	35·6	5·70	33·7
,,	,, 30%	204·0	147·3	35·7	5·72	33·65
,,	,, 35%	205·0	147 6	35·8	5·73	33·6

Tables IX and X give the results of experiments over a range of mixture strength from the correct mixture for complete combustion with no excess of air up to 35 per cent rich.   The constant heat given to the entering charge by electric heater was 65 B.Th.U per min., and the engine kept at a constant speed of 1,500 r.p m.   It is remarkable that in each case, no matter what the excess of fuel present, there is (1) so little variation of air consumption, expressed in pounds per indicated horse-power hour ; and (2) the thermal efficiency, based on the air consumption or on that portion of the fuel which is burnt, is nearly the same, and actually reaches a maximum with the mixture containing about 20 per cent excess fuel.   Part of the petrol above 20 per cent excess in the mixture cannot be properly burned or converted into useful work, and the waste heat left over makes trouble by excessive heat flow to the exhaust valves.

In multi-cylinder engines, by irregular distribution in the induction system from the carburettor, each cylinder may not receive the mixture of exactly the same strength. Sometimes part of the lubricating oil may be partially burnt or carbonized, to an unknown amount, causing the troubles of gummed up piston-rings and burnt valves or pistons. Consequently, the most efficient internal combustion engine of the constant volume explosion type is that which makes the best use of the air and fuel consumed, and is therefore the most reliable and durable.

Compression Ratio. Table XI shows the thermal efficiency, as determined from the air consumption when the engine was running on benzol with a mixture strength varied from 10 to 30 per cent rich, and the ignition correctly timed to give the best result in each case.

<div align="center">

TABLE XI

VARIED COMPRESSION RATIO. BENZOL ABOUT 20 PER CENT RICH
(*H. R. Ricardo*)

</div>

Compression Ratio	Total Air lb. per Hour	Indicated Mean Pressure, lb. per sq. in.	Indicated Horse-power	Lb. of Air per I.H.P. Hour	Indicated Thermal Efficiency by Air Measurement. Per Cent	Indicated Thermal Efficiency by Fuel, 15% Weak. Per Cent
4	200·5	125·0	30·3	6·62	29·2	27·7
5	194·0	136·5	33·1	5 32	33 2	32·0
6	188·0	145·0	35·2	5·34	36 2	35·0
7	164·0	152·0	36·8	5·0	38 5	37·3

In the last column is shown the maximum thermal efficiency as found from the fuel consumption, with the mixture adjusted in each case to give highest economy, i.e. about 15 per cent weak. The efficiency calculated from the air consumption is always slightly higher than that from the fuel consumption, because of the layer which escapes complete combustion by adhering to the cold cylinder walls.

The indicator diagrams in Fig. 152 (*a*) are from the Ricardo variable compression engine at a speed of 1,500 r.p.m., and show clearly the relative increases in both maximum and mean pressure as the compression ratio is increased.

The diagram on a time base shows the angle turned through by the crankshaft. It is not desirable to have the explosion strictly at constant volume, or to obtain the peak of maximum pressure until 10° to 15° after the top centre (see also Fig. 136, p. 345). The piston is nearly stationary during this brief period of time, and there is this saving of the time during which both the temperature

and density of the charge are at a maximum, and when the loss of heat is most rapid. Any heat liberated by combustion during the expansion stroke can be utilized only at a reduced efficiency corresponding to that of the expansion remaining available. In the limit, any heat liberated at the end of the expansion stroke can do no useful work at all, but will heat the exhaust valve

Table XI shows that as the compression ratio is raised, so also is the weight of air taken in by the engine per cycle reduced. Observations at widely varying speeds showed this decrease in volumetric efficiency to be substantially the same at all speeds. This decrease
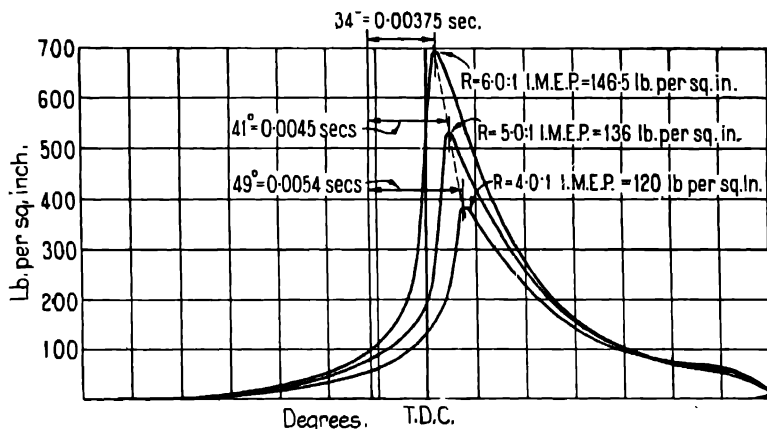
Fig. 152 (a)

accounts for the result found in general experience, that, while the use of a high compression ratio greatly increases the thermal efficiency, yet the gain in power output is much less marked.

Heat Distribution in the High-Speed Petrol Engine. The following results of observations, taken with a high degree of accuracy, in tests of the Ricardo variable compression engine, show the distribution of heat in that engine under different conditions of working on the same sample of petrol (A). In the first test at compression ratio 3·8 : 1, with constant fuel to air ratio, but varying speed, the results obtained were—

Revolutions per minute	975	1500	1700
Piston Speed, feet per minute . .	1300	2000	2266
Heat converted to indicated horse-power, per cent	25·9	26 1	26·1
Heat lost to cooling water, per cent . .	30·4	28·0	27·0
Heat lost in exhaust, radiation, etc., per cent.	43 7	45 9	46·9
	100·0	100·0	100·0

Similar tests with ethyl alcohol (95 per cent) give a thermal efficiency up to 38·3 per cent, with a compression ratio 7 : 1.

In the second test with petrol (*A*) at compression ratio 5·45 : 1, with constant fuel to air ratio, and at constant speed 1,500 r.p.m and piston speed 2,000 ft. per min., but the power varied by throttling, gave—

Percentage of maximum indicated horse-power	100%	80%	60%	40%
Heat converted to indicated horse-power, per cent	33·5	34·0	34·1	33·5
Heat lost to cooling water, per cent . .	26·5	28·2	31·8	35·5
Heat lost in exhaust, radiation, etc , per cent .	40·0	37·8	34·1	31·0
	100·0	100·0	100·0	100·0

In both cases the circulating water was at constant temperature 60° C. ± 2° C. The heat input to the carburettor was adjusted to bear a constant proportion to the weight of petrol used. The fuel-air ratio was such as to give nearly 10 per cent of air in excess of that required for complete combustion of the petrol, the air consumption being measured and adjusted in each case. No readings were recorded until all temperature conditions had been steady for a considerable time after each change of working condition.

The heat lost in the exhaust, radiation, etc., was obtained by difference.

The indicated thermal efficiency may be taken as accurate to within about half of 1 per cent, and the heat to the cooling water to within 1 per cent. In both tests the heat produced by piston friction and that lost by radiation balanced at nearly 1,500 r p.m ; the cooling water temperature at which the readings were taken being that which the cylinder attains when motored continuously at 1,500 r.p.m , i.e. 45° C. above atmospheric temperature.

Thermal Efficiency under Reduced Loads. Tests made on the Ricardo single cylinder research engine showed : That, as the load is reduced by throttling and with the ignition set to give the best results on full load, the fuel consumption per indicated horse-power hour increases slightly as the load is reduced, but that, if at each throttle opening, the ignition timing is advanced as the load is reduced, then the fuel consumption per indicated horse-power hour remains nearly constant throughout the range from 30 per cent to 100 per cent full load torque.

" In all these tests the same procedure was adopted, viz. at each throttle opening the whole range of mixture strength was explored. The circulating water was maintained throughout at the same temperature ; the heat input to the carburettor was proportional to the load, i.e. at full load the heat input was at the rate of 65 B.Th.U. per.min ; at half load, 32·5 B.Th.U. per min., etc. ; also, at each

throttle position the mechanical losses were measured at intervals by motoring, which was done by simultaneously switching off the ignition and changing over the armature circuit of the dynamometer, the combined operation taking less than 1 sec. and being effected without any appreciable change in speed."

Curves A, A_1, Fig. 152 (b), show the results obtained with fixed ignition; and curves B, B_1 with ignition adjusted for each change in load. The gain in thermal efficiency by advancing the spark when running on light loads is considerable.

In 1908, the British Association Committee on Gaseous Explosions gave a curve (p. 372) of the internal energy and temperature for the burnt products of a coal gas and air mixture which formed the
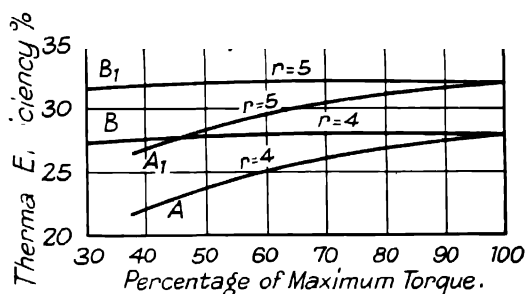
FIG. 152 (b). POWER AND THERMAL EFFICIENCY CURVES FOR A RICARDO THROTTLE IN A PETROL ENGINE

working fluid in the gas engine used by Sir Dugald Clerk in his experiments on specific heat. More recent research indicates that the values of the specific heats at the higher temperatures then available are somewhat too high ; but, as shown above, the curve is useful in gas engine calculations.

Mr. Ricardo gives a similar curve,* Fig. 152 (c), for a petrol or benzene-air mixture in the petrol engine, constructed from the results of investigations by Messrs. Tizard and Pye.

Although this curve applies strictly only to the chemically correct benzene-air mixture for complete combustion at a compression ratio of 5 : 1, it may be taken as applicable within negligible error to any other hydrocarbon fuel at the compression ratios commonly used in the ordinary petrol engine working on the constant-volume cycle. Owing to the different specific heat of the products of combustion of alcohol or ether and air mixtures, the curve does not apply to these, nor to very rich or weak mixtures, though, with these, the error is very small within the working range of a homogeneous mixture.

The curve gives the heat energy in equivalent foot-pounds per

* *The Automobile Engineer*, November, 1922. Reproduced by the kind permission of Mr. H. R. Ricardo.

standard cubic inch plotted against the temperature on the horizontal scale. Zero energy is taken at 100° C., this being the average temperature of the charge at the beginning of compression. The

FIG. 152 (c). INTERNAL ENERGY OF A PETROL OR BENZINE-AIR MIXTURE

difference between the upper part of the two curves, on a vertical line, shows the chemical energy stored in the products of dissociation.

A graphic construction, by Mr. Alcock, is given by which the range of temperature throughout the expansion stroke may be

estimated from the heat of combustion of the mixture and thermal efficiency of the engine. The ideal thermal efficiency of the petrol constant-volume cycle is taken $= 1 - \left(\dfrac{1}{R}\right)^{0.258}$

An example is worked out to explain the use of the diagram—

Given the compression ratio $R = 5$; energy content of charge $= 46 \cdot 2$ ft.-lb. per standard cub. in. ; assuming heat loss to the cylinder walls during combustion, 6 per cent ; and heat loss during expansion, 6 per cent. The heat added to the mixture during compression is allowed for various ratios by the scale PP_1, near the bottom of the diagram.

The point $R = 5$, representing $3 \cdot 6$ ft.-lb. heat of compression, is joined to that value above the energy content of the mixture on the vertical O_1Y_1 from the 100° C. starting-point, i e. $46 \cdot 2 + 3 \cdot 6$, or $49 \cdot 8$ ft.-lb. From this gross energy content per cubic inch must be deducted : (a) The loss due to the weakening of the mixture by dilution with the residual exhaust products, assumed to be at 1000° C ; and (b) the loss to the walls of the combustion chamber during combustion.

At the top of the diagram on the horizontal scale C is marked the decrease in energy due to dilution, and scale E shows the loss due to cooling during combustion. A line is drawn between the two points on these scales representing 6 per cent, and the point of intersection of this line with the scale D gives the total loss due to these two causes, $11 \cdot 5$ per cent.

Drop a vertical from this point of intersection to the line joining the total energy content $49 \cdot 8$ ft.-lb., and the heat of compression on the scale PP_1. Then a horizontal line is drawn to the energy scale O_1Y_1 on the left, and to the energy curve OA. The point on the energy scale shows the net energy available for expansion, $44 \cdot 5$ ft.-lb. per standard cub. in. of charge, and from the curve OA can be read off the actual maximum temperature, 2475° C.

The drop in temperature during expansion depends on (1) the work done on the piston, and (2) the heat loss to the walls. The net power output as a percentage of the heat content of the mixture given on scale F, does not cover losses to the walls during expansion, which is laid off on scale E. A line drawn between the points on scales E and F gives their sum on scale D, as before. A vertical from this on D is drawn to meet a line from the net energy point on O_1Y_1 to the suitable compression point on PP_1. As the gross work done during expansion is the sum of the net work and that during compression, the latter ($3 \cdot 6$ ft.-lb. in example) must be laid off below the point of intersection to find, by the horizontal, the energy content at the end of expansion, $24 \cdot 5$ ft.-lb. ; and the corresponding final temperature, 1675° C., is read off the curve OA.

In an actual test of the variable compression engine, having compression ratio 5 : 1, the observed indicated thermal efficiency is

31 per cent; 5 per cent of loss during expansion is due to the change of specific volume of the mixture during combustion, so that the heat drop is $46.2 \times 0.31 \times \dfrac{100}{105} = 13.6$ ft.-lb. per cub. in. Add to this the 3.6 ft.-lb. of compression work restored during expansion, and 6 per cent of 46.2, or 2.8 ft.-lb. of wall loss, makes the total heat drop during expansion : $13.6 + 3.6 + 2.8 = 20$ ft.-lb. per cub. in., leaving a final energy content, $44.5 - 20 = 24.5$ ft.-lb. per cub. in., which agrees with the result found graphically, under the same conditions.

EXAMPLES VIII

1. Prove that the ideal efficiency of an internal combustion engine working on the Otto cycle is $1 - \left(\dfrac{1}{r}\right)^{\gamma - 1}$, where r is the ratio of compression. Calculate this efficiency in the case of an engine having a stroke 16 in., a piston diameter 12 in., and a clearance volume of 485 cub. in. ; and find the gas consumption per indicated horse-power hour if the gas has a calorific value of 260 C.H.U. per cub. ft., and the efficiency ratio of the engine is 56 per cent.
(U.L., B.Sc. (Eng.), 1923.)

2. An engine, working on the four-stroke constant volume cycle, has compression ratio 6. In a brake test at 1,200 r.p.m., the rope of 1 in. diameter all round the brake wheel of 24 in. diameter, had a difference of vertical pulls on its ends 254 lb. The fuel used per hour was 34 lb. of petrol having lower calorific value 18,500 B.Th.U. per lb. Mechanical efficiency with this load, 80 per cent. Calculate (a) the average torque on the crankshaft ; (b) the brake horse-power ; (c) the indicated and brake thermal efficiencies ; and (d) the efficiency relative to that of the ideal air cycle for this engine.

3. Work out the trial made on a gas engine in which the following observations were made : Cylinder diameter, 7 in. by 15 in. stroke ; clearance volume, 0·0824 cub. ft. ; explosions per minute, 78·5 ; M.E.P., 77 lb. per sq. in. ; brake horse-power, 6·15. Gas used per minute, 2·77 cub. ft. at 12·2° C., and 2·2 in. of water above atmospheric pressure of 29·8 in. mercury. Cooling water per minute, 20·25 lb., having temperature rise 17·4° C. The calorific value of gas, 282 C.H.U. per cub. ft. at 0° C., and 30 in. mercury. Draw up a balance sheet for the engine, and determine the mechanical, thermal, and relative efficiencies of the engine. *(U.L., B.Sc. (Eng.), 1922.)*

4. A trial of an internal combustion engine, during *one hour*, gave the following results : Indicated horse-power, 24·7 ; brake horse-power, 20·5 ; total oil fuel used, 1·35 gallon of specific gravity 0·750, and lower calorific value 18,700 B.Th.U. per lb. ; total cooling jacket water, 523 lb., of temperature at inlet 60° F. and at exit 150° F. The exhaust gases leaving the cylinder were passed through an exhaust calorimeter, raising 923 lb. of water from 60° F. to 140° F.
Calculate (a) the mechanical and (b) thermal efficiencies of the engine, and draw up a heat balance sheet for the engine, showing the distribution of heat in B.Th.U. per minute.

5. A gas engine, working on the four-stroke constant volume cycle, gave the following results when loaded by a friction brake during a test of 1 hour duration : Cylinder diameter, 9·5 in. ; stroke, 19 in. ; clearance volume, 272 cub. in. ; effective circumference of brake wheel, 12·8 ft. ; net load on brake, 277 lb., at average speed 226·7 r.p.m. ; average explosions per minute, 77 ; and mean effective pressure of indicator cards, 106 lb. per sq. in. Gas used, 455·5 cub. ft. at 15° C., and pressure 771 mm. mercury. Lower calorific

value of gas, 329 C.H.U. per cub. ft. at N.T.P. Cooling jacket water, 1,380 lb., raised 34·2° C.

Calculate, (a) the indicated horse-power ; (b) brake horse-power ; (c) the mechanical efficiency and indicated thermal efficiency ; (d) the ideal air cycle efficiency ($\gamma = 1·4$) and efficiency ratio ; and (e) give the heat balance for this engine per minute.

6. A gas engine, working on the Otto cycle, has clearance 0·25 cub. ft. and the volume swept by piston 1·25 cub. ft. From the indicator cards the compression curve is $p.v^{1·35} =$ constant, and the expansion $p.v^{1·3} =$ constant. When the compression stroke begins, the charge is at 14 lb. per sq. in. absolute and 100° C. The maximum pressure of explosion at constant volume is 400 lb. per sq. in. (abs.), and at 110 explosions per minute the engine takes in 10 cub. ft. per min. of gas, having lower calorific value 275 C.H.U. per cub. ft. as used.

Calculate : (a) the net indicated work per cycle ; (b) the indicated horse-power ; (c) the thermal efficiency ; and (d) the efficiency relative to that of the ideal air cycle for the engine ($\gamma = 1·4$).

7. An air engine works on an ideal cycle in which heat is received at constant pressure and rejected at constant volume. The pressure at the end of the suction stroke is 14 lb. per sq. in. (abs.) ; the ratio of compression is 15·3, and the ratio of expansion is 7·5. If the expansion and compression curves are given by $p.v^{1·4} =$ constant, find the mean pressure for the cycle.

An oil engine working on this cycle, and with the above expansion and compression ratios, and initial pressure, gives a mean pressure of 114 lb. per sq. in. Account for the difference. (*U.L., B.Sc. (Eng.).*)

8. Deduce an expression for the ideal efficiency of the Diesel engine cycle, and calculate this efficiency in the case of an engine having a compression ratio of 13·5, and in which the fuel is cut off at 0·05 stroke.

(*U.L., B.Sc. (Eng.), 1922.*)

9. The following results were obtained during a test of a gas engine loaded by a friction brake : Cylinder diameter, 8 in. ; stroke, 17 in. ; dead weight, 165 lb. ; spring balance reading, 23·8 lb. ; brake wheel diameter, 5 ft. ; revolutions per minute, 215 ; explosions per minute, 98 ; M.E.P. of indicator card, 82 lb. per sq. in. ; gas per minute, 7·16 cub. ft. at 29·9 in. of mercury and 14·8° C. ; cooling water per minute, 37·7 lb. raised 25·8° C. ; calorific value of gas, 275 C.H.U. per cub. ft. measured at N.T.P. Calculate the indicated and brake horse-powers of the engine, and find the mechanical and thermal efficiencies. Draw up a balance sheet for the engine per minute.

(*U.L., B.Sc. (Eng.), 1924.*)

10. Calculate the stroke and diameter of a gas engine which is to develop 15 H.P. at 300 r.p.m. Assume a compression ratio of 4·5, adiabatic compression, a pressure at the end of explosion of 350 lb. per sq. in., and an expansion index of 1·3. The stroke is to be 1·5 times the diameter.

(*U.L., B.Sc. (Eng.), 1923.*)

11. A gas engine has to develop 20 B.H.P. with a mechanical efficiency of 80 per cent, when there are 80 explosions per min. Determine the necessary diameter of the cylinder if the stroke is 2 ft. The equations to the expansion and compression curves are of the form $pv^{1·25} =$ constant ; the clearance is one-fourth of the volume swept by the piston, and the maximum pressure after ignition is three times the compression pressure.

(*U.I., B.Sc. (Eng.).*)

CHAPTER IX

STEAM

Properties of Steam. In steam we have to deal with a fluid which may be either in the liquid or gaseous condition under normal conditions of temperature and pressure, and we can no longer even approximately apply the simple laws of perfect gases. Under normal equilibrium conditions, a vapour, such as steam, if in contact with its liquid, is said to be in a saturated condition. If the mixture is now heated, the liquid will gradually change into vapour, until just at the point where all the liquid has turned into vapour, the stuff is called dry saturated vapour. Any further addition of heat to the dry saturated steam will superheat it, and we have what is called a superheated vapour. For any particular pressure, there is only one temperature at which the liquid will change into vapour, this temperature (called the saturation temperature) remaining constant, provided the pressure is kept constant, so long as there is any liquid left. As soon as all the liquid has been converted into vapour, the temperature will rise with any further addition of heat at constant pressure. The behaviour of the superheated vapour will now approximate more and more to that of a perfect gas the higher the temperature is removed from the saturation temperature corresponding to the pressure to which the vapour is subjected.

We have to consider the production of steam from water, but it may be mentioned that the general principles which will be applied to steam can be applied to other fluids with which we have to deal in both the liquid and vapour conditions.

In steam we are by no means dealing with a perfect gas, and therefore the simple characteristic equation

$$PV = RT$$

does not apply.

Various characteristic equations have been formulated to express the properties of a fluid in any state, none of which are applicable over an unlimited range of temperature and pressure. The subject is beyond the scope of this work, and it must suffice to mention the characteristic equation given above (p. 67) as formulated by Callendar, which may be written

$$V = \frac{RT}{P} + b - c$$

where R and b are constants and c is a function of the temperature; and $V =$ volume in cubic feet per pound; $P =$ Pressure in pounds per square foot; $T =$ absolute temperature in degrees C.

The properties of steam which have been calculated by Callendar are embodied in his Steam Tables, excerpts from which are included at the end of this volume. If values from these tables are compared with those of other authorities, such as the Marks and Davis, or the Peabody Tables, discrepancies will be found. It may be pointed out, however, that any such tables must be calculated from empirical formula based on experimental results, and it is found that the Callendar Tables are not only in good agreement with modern research, but also give values which are mutually consistent among themselves.

Relation of Pressure and Temperature in Saturated Steam. The temperature at which a liquid is converted into vapour by the addition of heat depends on the pressure to which the fluid is subjected, and the liquid boils when heated to the saturation temperature corresponding to the external pressure to which it is subjected. Regnault carried out classical investigations on the relation between temperature and pressure, when water was being converted into steam. The values determined, up to a temperature of 220° C., agree within narrow limits with those of later experimenters. Modern applications, however, require values at higher temperatures, and a considerable amount of work has been carried out since Regnault's time, culminating in that of Callendar, who has determined the corresponding values of saturation temperature and pressure from his characteristic equation for steam. These values are given in the first two columns of the Steam Tables. Thus, in Table 1, at a pressure of 14·689 lb. per sq. in. absolute, water boils at a temperature of 100° C, while at 100 lb. per sq. in. absolute, water boils at 164·28° C.

Various attempts have been made from time to time to formulate an empirical formula expressing the relation between temperature and pressure for saturated steam. An empirical formula being based on experimental results, its justification is that it gives values consistent with those results. A formula given by Callendar, consistent with his characteristic equation, is

$$\log p = 21 \cdot 07449 - \frac{2903 \cdot 39}{T} - 4 \cdot 71734 \log T + 0 \cdot 4057(c - b)\frac{p}{T}$$

where p = saturation pressure in pounds per square inch, T = absolute saturation temperature C., and b and c have the values given above (p. 68). This formula is somewhat cumbersome and, for elementary purposes, intermediate values not given in the tables may be obtained by direct interpolation.

The Specific Volume of Steam. Direct experimental determination of the volume per unit mass of dry saturated steam is difficult, although a certain number of measurements have been carried out, the best known being those of Knoblauch, Linde, and Klebe. Values of V_{sa} for the Steam Tables are more easily found by

27—(5434)

calculation from experimental values of other properties, the direct experimental results serving as a check on these values with which they are in very close agreement.

The specific volume of dry saturated steam may be found from Clapeyron's equation (p. 100),

$$V_{sa} = V_w + \frac{JL}{T} \cdot \frac{dT}{dP}$$

where $\frac{dT}{dP}$ is the rate of change of saturation temperature with pressure at the absolute temperature T; and V_w is the specific volume of water at temperature T, and L is the latent heat.

Various empirical formulæ expressing the relation between the specific volume of dry steam and the saturation pressure have been put forward, among which are Rankine's formula

$$P V_{sa}^{\frac{17}{16}} = \text{constant};$$

and Mollier's formula, deduced from Callendar's equation,

$$p V_{sa}^{\frac{16}{15}} = 490$$

where p is in pounds per square inch, and V_{sa} is in cubic feet per pound.

The specific volume of steam, either dry, saturated, or superheated, may be calculated from the Callendar characteristic equation

$$V = 154 \cdot 168 \frac{T}{P} - 0 \cdot 4213 \left(\frac{373 \cdot 1}{T}\right)^{\frac{10}{3}} + 0 \cdot 01602,$$

where T is in Centigrade absolute and P is in pounds per square foot.

Latent Heat. In the formation of steam from 1 lb. of water at $0°$ C. under constant atmospheric pressure of $14 \cdot 689$ lb. per sq. in., as heat is given to the water the temperature rises gradually to $100°$ C. According to the usual definition of the pound calorie, the heat necessary to raise the temperature to $100°$ C. would be 100 calories. When the temperature has reached $100°$ C. it remains stationary, any further addition of heat converting the water, at $100°$ C., into steam at $100°$ C. The amount of heat necessary to just change the 1 lb. of water at $100°$ C. into steam at $100°$ C. is called the " latent heat " at $100°$ C.

If steam is to be formed at any other pressure P, the water must first be heated to the corresponding saturation temperature T. When the water is at this temperature, then any further addition of heat will change the water into steam at the same temperature. The values of the latent heat at various pressures are given in Table I of the Steam Tables.

Formation of Steam at Constant Pressure. Let us imagine that we have 1 lb. of water at $0°$ C. at the bottom of an upright cylinder of cross-sectional area 1 sq. ft. On this water rests a frictionless,

weightless piston, the constant pressure on the top side of the piston being that due to normal atmospheric pressure, namely, 14·689 lb. per sq. in. By definition (p. 42) the total heat of this water is

$$E_0 + APV_0 = 0 + \frac{144 \times 14\cdot689 \times 0\cdot01602}{1400}$$

$$= 0\cdot0242 \text{ calories (approximately).}$$

E_0 is really not quite zero, as the pressure is not the vapour pressure at $0°$ C., but the increased pressure has no appreciable effect on E_0.

Let the water now be given a quantity of heat Q calories, which changes the temperature of the water from $0°$ C. to $100°$ C., the pressure above the piston remaining the same throughout, and the water expanding from V_0 to V_{100}. The total heat of the water will now be $h_{100} = E_{100} + APV_{100}$.

As the heat has been taken in at constant pressure, then from page 43,

$$Q = h_{100} - h_0$$
$$= E_{100} + APV_{100} - (E_0 + APV_0)$$
$$= E_{100} - E_0 + AP(V_{100} - V_0)$$

Now as the area of the piston is 1 sq. ft., it will rise a distance $V - V_0$, due to the expansion of the water, and therefore $AP(V_{100} - V_0)$ is the external work done by the water expanding when taking in the quantity of heat Q. This external work

14·689
lbs. per
sq. inch.

1 lb.
Water.

Area 1 sq. foot.

Fig. 153

is equal to $\dfrac{144 \times 14\cdot689\, (\cdot01671 - \cdot01602)}{1400} = \cdot00105$ calories, which is negligible compared to Q. Thus practically the whole of Q goes to increase the internal energy of the water. It will be noticed that the equation $Q = (E_{100} - E_0) + AP(V_{100} - V_0)$ directly follows from the energy law $dQ = dE + dW$, for $E_{100} - E_0$ is the increase in internal energy and $AP(V_{100} - V_0)$ is the external work done.

At the point where the water is just at a temperature of $100°$ C., we have the total heat of the water $h_{100} = Q + h_0 = Q + APV_0$, if E_0 is taken as zero. Any further addition of heat will now gradually change the water at $100°$ C. into steam at $100°$ C. In order to just change all the water at $100°$ C. into dry saturated steam at $100°$ C. we must give a quantity of heat, L, to the water, L being the latent heat at $100°$ C. $= 539\cdot3$ calories.

The total heat of the dry saturated steam $H_{100} = h_{100} + L$, as the heat has been taken in at constant pressure, and hence the change in total heat is equivalent to the heat taken in. During the formation of the steam from the water, the piston will rise a distance
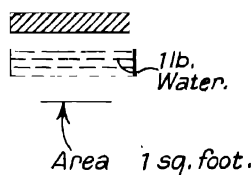
$(V_{sa} - V_{100})$ where V_{sa} is the specific volume of dry saturated steam at 100° C., namely, 26·788 cub. ft. per lb.

Thus the part of the latent heat L which is used up in doing the external work due to the expansion of the water at 100° C. into steam at 100° C. equals $AP(V_{sa} - V_{100})$

$$= \frac{144 \times 14\cdot689\,(26\cdot788 - 0\cdot01671)}{1400} = 40\cdot45 \text{ calories.}$$

It will be noticed that the volume of the water is practically negligible compared to the volume of the steam, and hence it can be neglected in elementary calculations. The increase in internal energy of the stuff as it changes from water at 100° C. to dry steam at 100° C. is, therefore, $539\cdot3 - 40\cdot45 = 498\cdot85$ calories.

If dry saturated steam is to be formed at any other constant pressure from water at 0° C., then the above calculations still hold if the values for V and L, corresponding to the pressure under which the steam is being formed, are taken.

Thus, if V_0 = Specific volume of water at 0° C.

V_w = Specific volume of water at saturation temperature corresponding to the pressure P at which steam is to be formed.

V_{sa} = Specific volume of dry saturated steam at pressure P.

E_0 = Internal energy of water at 0° C.

E_w = Internal energy of water at saturation temperature.

E_{sa} = Internal energy of dry saturated steam

L = Latent heat of steam at pressure P

h = Total heat of water at saturation temperature,

H_{sa} = Total heat of dry saturated steam.

we have from the fact that heat taken in at constant pressure is equivalent to the change in total heat, and also from the basic energy law,

$$\left. \begin{array}{l} E_0 + AP_0V_0 \qquad\qquad = h_0 \\ (E_w - E_0) + AP(V_w - V_0) = Q \\ (E_{sa} - E_w) + AP(V_{sa} - V_w) = L \end{array} \right\} \left. \begin{array}{l} \\ = h \\ = L \end{array} \right\} = H_{sa}$$

Also from the definition of total heat.

$$H_{sa} = \left\{ \begin{array}{l} APV_s\, V_{sa} - AP(V_{sa} - V_w) + AP(V_w - V_0) + APV_0 \\ E_{sa} - (E_{sa} - E_w) + (E_w - E_0) + E_0 \end{array} \right.$$

We see that the total heat of dry saturated steam $H_{sa} = h + L$, where h is the total heat of the liquid at saturation temperature and pressure, and L is the latent heat of steam at the saturation pressure. It should be noticed that by our definition the total heat of dry steam is not the latent heat plus the heat given to the water while

its temperature rises from $0°$ C. to the saturation temperature. It is made up of these two quantities together with the term APV_0, although this term for normal pressures is small compared with the values of Q and L.

Specific Heat of Water and the Mean Thermal Unit. The variation of specific heat of water with temperature, as determined by Callendar and Barnes, is shown above in Fig. 20 (p. 40). Hence the heat taken in by water while being raised from temperature t_1 to temperature t_2 is not exactly equal to $(t_1 - t_2)$. The minimum specific heat is now taken as ·99666 and it has this value at a temperature of $37.5°$ C. Owing to this variation of specific heat, it has been found convenient to formulate a mean thermal unit. The mean pound calorie may be defined as the one-hundredth part of the amount of heat necessary to raise the temperature of 1 lb. of water from $0°$ to $100°$ C. under a constant pressure of 1 standard atmosphere.

In view of the fact that the heat is assumed to be taken in *at constant pressure*, this unit may also be defined as the one-hundredth part of the change of the total heat of 1 lb. of water when heated from $0°$ to $100°$ C., under constant pressure of 1 standard atmosphere. By this definition, if the total heat of water at $100°$ C. under a pressure of 1 standard atmosphere (14.689 lb./sq. in. London) is taken as 100 lb. calories, then water would have zero total heat at $0°$ C. and under a pressure of 14.689 lb. per sq. in. But from the last paragraph we see that, by the definition, the total heat of this water will be equal to

$$APV_0 = \frac{144 \times 14.689 \times 0.01602}{1400} - 0.0242 \text{ calorie}.$$

Callendar therefore defines the mean pound calorie as the one-hundredth part of the change in total heat of water between $0°$ C. and $100°$ C., when the water is heated *under the varying pressure of its own vapour only*.

By the first definition we have

$$100 = E_{100} + AP_{100}V_{100} - (E_0 + AP_{100}V_0)$$

By Callendar's definition,

$$100 = E_{100} + AP_{100}V_{100} - (E_0 + AP_0V_0)$$

Callendar's definition therefore exceeds the usual definition by the term $A(P_{100} - P_0)V_0 = \dfrac{144(14.689 - 0.0892)0.01602}{1400}$

$$= 0.024 \text{ lb. calorie}.$$

This difference between the usual definition and the Callendar definition of the mean unit of heat, namely, 0.024 lb.-calorie in 100 lb.-calories, is beyond the limit of experimental accuracy in the determination of the heat unit, and hence, at the expense of this

error, we have the convenience that the total heat of water at 0° and 100° C., is zero and 100 lb.-calories respectively.

Formation of Superheated Steam at Constant Pressure. If dry saturated steam takes in heat at constant pressure the temperature will rise and the steam will become superheated. If the specific heat C_p of steam were constant for all temperatures and pressures, then the heat taken in would be $C_p(t_{su} - t_{sa})$, where t_{su} is the temperature to which the steam is superheated and t_{sa} is the saturation temperature corresponding to the pressure at which the steam is being heated. The specific heat of superheated steam, however, falls with rise of temperature and rises with rise of pressure, as shown in Fig. 154.

An approximate value of the heat taken in may be found by putting for C_p in the above expression the mean value between the temperatures of saturation and superheat for the particular pressure in question. The required values of C_p may be found from a chart or from tables. Thus, if dry saturated steam at an absolute pressure of 100 lb. per sq. in. (saturated temperature 164·28° C.) be superheated to 300° C., the mean specific heat will be $\dfrac{0\cdot562 + 0\cdot504}{2} = 0\cdot533$.
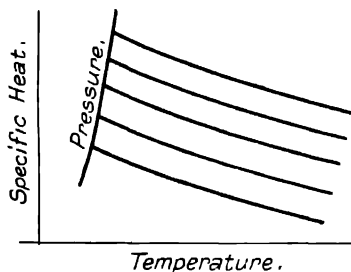
Temperature.

Fig. 154

The heat taken in while superheating 1 lb. of dry saturated steam by this amount will therefore be approximately $0\cdot533(300 - 164\cdot28) = 72\cdot34$ lb. calories.

As the heat taken in by a fluid at constant pressure is equal to the change in total heat of a fluid, an approximate expression for the change of total heat of a fluid while being heated from the dry saturated state is $C_p(t_{su} - t_{sa})$. Hence the total heat of superheated steam is approximately $H_{su} = h + L + C_p(t_{su} - t_{sa})$. More exact values of H_{su} are given in the steam tables (Table III), and hence the amount of heat Q_2 taken in during the superheating from the dry state may be calculated from the equation $Q_2 = H_{su} - H_{sa}$. For example, the amount of superheat in steam at 100 lb. per sq. in. and 300° C. is $t_{su} - t_{sa} = 300 - 164\cdot28 = 135\cdot72°$ C. Hence, from Table III, the total heat H_{su} is, by interpolation, 733·15 calories, while the total heat of dry saturated steam at the same pressure is 661·83. Hence the heat taken in during the superheating is $733\cdot15 - 661\cdot83 = 71\cdot32$ lb. calories per lb.

The heat Q_2 is used in raising the internal energy of the steam, and also in doing external work as the steam expands. The external work done will be $AP(V_{su} - V_{sa})$ where V_{su} is the specific volume of the superheated steam. The volume of 1 lb. of

superheated steam at 100 lb. per sq. in. and 300° C. is 6·0509 cub. ft. (Table V). The external work done in the above example is therefore $\dfrac{144 \times 100(6\cdot0509 - 4\cdot4509)}{1400} = 16\cdot46$ lb. calories per lb. The change in internal energy will therefore be $71\cdot32 - 16\cdot46 = 54\cdot86$ lb. calories per lb.

The mean specific heat of superheated steam can be obtained from Table III. Thus, suppose the specific heat of steam at 100 lb. per sq. in. superheated 20° C. above the saturation temperature is required. The total heat of dry saturated steam at 100 lb. per sq. in. $= H_{sa} = 661\cdot83$ lb. calories ; while the total heat of the steam if at 100 lb. per sq. in. and superheated 40° C. $= H_{su} = 683\cdot70$ lb. calories. Hence, as heat taken in at constant pressure is measured by the change in total heat, the mean specific heat

$$\frac{H_{su} - H_{sa}}{40} = \frac{683\cdot70 - 661\cdot83}{40} = 0\cdot547$$

This is, therefore, the approximate value of the specific heat of superheated steam at 100 lb. per sq in. pressure, and at a temperature of $164\cdot28 + 20 = 184\cdot28°$ C.

Empirical Formulæ for Total Heat. The formula used by Callendar to represent the total heat of water is

$$h = st + L\,\frac{V_w}{V_{sa} - V_w} - 0\cdot003$$

The constant 0·003 is the value of the term $L\,\dfrac{V_w}{V_{sa} - V_w}$ at 0° C., namely, $\dfrac{594\cdot27 \times 0\cdot01602}{3275\cdot9 - 0\cdot01602}$, hence the formula gives zero value for h at 0° C. For particulars as to derivation of this formula, reference should be made to Professor H. L. Callendar's work.[*]

An oft-quoted formula for dry saturated steam is that of Regnault

$$H_{sa} = 606\cdot5 + 0\cdot305t \text{ calories}$$

where t is the temperature in degrees Centigrade.
• This equation, however, while giving a fair value between 100° C. and 200° C., is inconsistent below about 100° C., and is more or less useless for exact calculations.

The formula[†] of Professor E. H. Griffiths for saturated steam at t° C.

$$H_{sa} = 596\cdot73 + 0\cdot4t°$$

agrees more closely with the results of experiment, taking the thermal unit at 15° C. ; and with Joly's determinations gives consistent results.

* *Properties of Steam and Thermodynamic Theory of Turbines* by H. L. Callendar.
† *Phil. Trans.* Vol. CLXXXVI, 1895, p. 319 ; and Proc. Phy. Soc. 26, 1913.

A formula given by Callendar for steam either dry or superheated

is
$$H = 0.4772T - \left(\frac{13}{3}c - 0.016\right)\frac{144}{1400}p + 464 \text{ calories}$$

$$= 0.4772T - \frac{(c - 0.0037)}{2.2436}p + 464$$

where p is in pounds per square inch.

From this equation a value for H in terms of pressure and volume can be derived,

$$H_{sa} = p\,\frac{(V_{sa} - 0.0123)}{2.2436} + 464 \text{ calories.}$$

By inverting this formula a convenient expression for the specific volume of dry steam is obtained,

$$V_{sa} = \frac{2.2436(H_{sa} - 464)}{p} + 0.0123 \text{ cub. ft.}$$

(See Examples, p. 69.)

Entropy of Water. The increase of entropy of a fluid when taking in a quantity of heat dQ in a reversible manner at a constant absolute temperature T, is measured by $d\phi = \dfrac{dQ}{T}$. If the heat is taken in at a temperature varying from T_1 to T_2, then the change of entropy is

$$\phi_2 - \phi_1 = \int_{T_1}^{T_2}\frac{dQ}{T}$$

When heat is taken in at constant pressure

$$\phi_2 - \phi_1 = \int_{T_1}^{T_2}\frac{dQ}{T} = \int_{T_1}^{T_2}\frac{C_p dT}{T}$$

where C_p is the specific heat of the fluid at constant pressure. If C_p is assumed constant for all temperatures, this quantity reduces to

$$C_p\int_{T_1}^{T_2}\frac{dT}{T} = C_p \log_e\frac{T_2}{T_1}$$

Entropy is generally taken as zero for a fluid in the liquid state at 0° C. Hence, taking the specific heat of water as constant and equal to unity, the entropy of 1 lb. of water ϕ_w at absolute temperature T is approximately $\log_e\dfrac{T}{273}$. The change in entropy while heating 1 lb. of water from T_1 to T_2 will be $\log_e\dfrac{T_2}{T_1}$.

Values of entropy of water measured from 0° C. are given in Table I of the Steam Tables.

Entropy of Steam. When 1 lb. of water at the saturation temperature is changed into 1 lb. of dry saturated steam, the latent

heat L is taken in at the constant saturation temperature T_{sa}. Hence the change of entropy in evaporating 1 lb. of water at T_{sa} to 1 lb. of dry saturated steam is $\dfrac{L}{T_{sa}}$. This quantity is sometimes called the "entropy of evaporation." The entropy of dry saturated steam is therefore $\phi_{sa} = \phi_w + \dfrac{L}{T_{sa}} = \log_e \dfrac{T_{sa}}{273} + \dfrac{L}{T_{sa}}$.

Suppose the 1 lb. of dry saturated steam is now superheated at constant pressure to a temperature T'_{su}. The increase of entropy during superheating will be

$$\phi_{su} - \phi_{sa} = \int_{T_s}^{T_{su}} \frac{C_p \, dT}{T}.$$

As has already been pointed out, the specific heat for superheated steam is by no means constant, but a fairly approximate value for the increase of entropy may be obtained by taking for the value of C_p the mean value between the temperature of saturation and superheat. With this assumption

$$\phi_{su} - \phi_{sa} \quad C_p \log_e \frac{T_{su}}{T_{sa}}$$

and the entropy of superheated steam at a temperature T'_{su} may be written

$$\phi_{su} \quad = \log_e \frac{T_{sa}}{273} + \frac{L}{T_{sa}} + C_p \log_e \frac{T_{su}}{T_{sa}}.$$

Values of entropy of dry saturated steam are given in Table I, and values of entropy of superheated steam are given in Table IV of the Steam Tables.

Wet Steam. When steam vapour is in contact with water it is said to be wet. The dryness fraction of steam is measured by the proportion of the stuff which consists of steam vapour. Thus, 1 lb of steam of 0·9 dryness fraction would contain 0·9 lb. steam vapour and 0·1 lb. water, all, of course, at the same temperature. From this it follows that to produce 1 lb. of steam, of dryness fraction say 0·9, from 1 lb. water at the temperature of saturation corresponding to the external pressure, the heat required would be 0·9 L, when L is the latent heat of steam at the pressure considered. Hence the total heat of wet steam is

$$H_x = h + xL$$

where x is the dryness fraction and h is the total heat of the water at the saturation temperature.

As $\quad H_{sa} = h + L$ and $H_x = h + xL$

then $\quad x = \dfrac{H_x - h}{L} = \dfrac{H_x - h}{H_{sa} - h}$

an expression for dryness fraction in terms of total heat.

The specific volume of wet steam must obviously be

$$xV_{sa} + (1 - x)V_w$$

As the specific volume V_w of the water is small compared to the specific volume of steam, it is for elementary purposes generally sufficiently accurate in the case of nearly dry steam to take the specific volume of wet steam as equal to xV_{sa}.

In the formation of 1 lb. of wet steam of dryness fraction x, the heat taken in is xL, and hence the entropy of 1 lb. of wet steam will be $\phi_x = \phi_u + \dfrac{xL}{T_v}$

As $\phi_{sa} = \phi_w + \dfrac{L}{T_{sa}}$, $\phi_x = \phi_u + \dfrac{xL}{T_{sa}} = \phi_{sa} - (1 - x)\dfrac{L}{T_{sa}}$.

From these equations, remembering that $H_x = h + xL$, we get

$$\phi_x - \phi_w = \frac{H_x - h}{T_{sa}}$$

and

$$\phi_{sa} - \phi_x = \frac{H_{sa} - H_x}{T_{sa}}$$
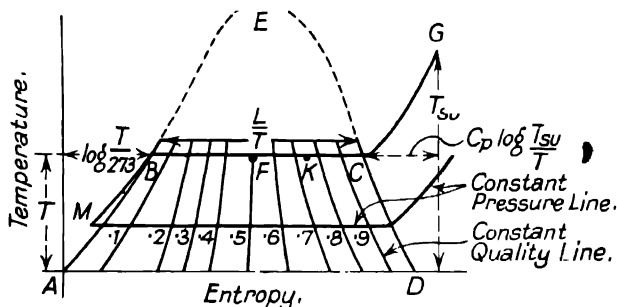
FIG. 155

The Temperature-Entropy Diagram for Steam. In this diagram values of entropy for 1 lb. of fluid are plotted horizontally and values of temperature vertically. If the values of entropy of 1 lb. of water at various temperatures are plotted against the corresponding temperatures, the line AB (water line), Fig. 155, is obtained. In the same way, plotting corresponding values of entropy and temperature for dry saturated steam will produce the line CD (saturation line). These two lines merge into one another at E, the temperature of which is the critical temperature for steam (365° C.). Charts for ordinary practical purposes, however, are not carried up as far as this point. In the absence of tables, the approximate value of any point B on the water line at an absolute temperature T can be calculated from the expression $\log_\varepsilon \dfrac{T}{273}$.

If water at $T°$ is heated at constant pressure it gradually changes into steam at the same temperature, and attains a dry saturated condition when it has received a quantity of heat L, its entropy increasing by the amount $\dfrac{L}{T}$. Hence the horizontal line BC of length $\dfrac{L}{T}$ represents the conversion of water into steam at constant pressure, the point B representing 1 lb. of water and C 1 lb. of dry steam, any intermediate point representing wet steam. Thus at F, half-way between B and C, the difference in entropy between F and B is $\frac{1}{2}\dfrac{L}{T}$, the heat given to the water in passing from B to C is $\frac{1}{2}L$, and the dryness fraction is 0·5. Thus, by dividing the line BC into, say, 10 equal parts, the various points so obtained will represent steam of dryness fractions varying by one-tenth. A little thought will show that the dryness fraction at any point K will be measured by the ratio $\dfrac{BK}{BC}$.

If the dry saturated steam at the point C is heated at constant pressure it becomes superheated to some temperature T_{su}. The actual value of the entropy of superheated steam at the temperature T_{su} and pressure P, can be obtained from the steam tables and hence various points on the constant pressure line CG may be found. An approximate value of the change in entropy as the fluid passes along the constant pressure line from C to G can be calculated from the expression

$$\phi_g - \phi_c = \phi_{su} - \phi_{sa} = C_p \log_e \frac{T_{su}}{T_{sa}}.$$

By drawing constant pressure lines such as BCG for various pressures, then dividing each horizontal portion between the boundary curves into 10 equal portions, and joining correspondingly numbered points, we can obtain constant quality lines as shown in Fig. 155.

Constant volume lines may be drawn on the chart in the following manner—

At a pressure of 152 lb. per sq. in., the volume of 1 lb. of dry saturated steam is practically 3 cub. ft. If AB, Fig. 156, is therefore the 152 lb. per sq. in. pressure line, the specific volume of the fluid at the point B is 3 cub. ft. Neglecting the volume of the water compared to that of the steam, the volume of the fluid at the point A may be taken as zero. Now, in passing from A to B, the volume of the fluid increases as steam
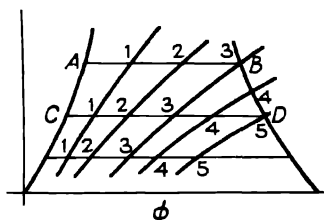
Fig. 156

is formed, in proportion to the heat taken in, and hence in proportion to the increase of entropy. Hence if AB is divided into three equal parts, points corresponding to volumes of 1 and 2 cub. ft. are obtained. At 89 lb. per sq. in. the specific volume of dry steam is practically 5 cub. ft. Hence, if CD is the 89 lb. per sq. in. constant pressure line, and it is divided into 5 equal parts, points representing volumes from 1 to 5 are obtained. By proceeding in the same way for other pressures and joining correspondingly numbered points, the various constant volume lines may be obtained. It must be remembered that if exact conditions are required, the volume of the water must be taken into account, although this correction is very small under normal conditions.

On a temperature entropy chart (p. 523), it will be seen that for any point between the boundary lines the fluid is wet steam, and its dryness fraction, pressure, temperature, specific volume, and entropy may be read directly. Points to the right of the saturation line represent superheated steam, and pressure, temperature and entropy may be read off the chart directly.
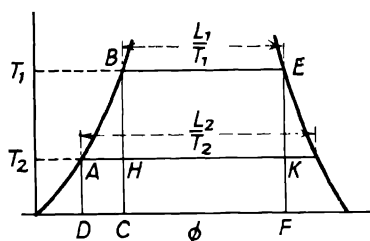
Fig. 157

Points to the left of the water line represent water. If any constant pressure line such as GCB is produced into the water area it will come down some such curve at BM, Fig 155. Owing to the incompressibility of water, however, these constant pressure curves in the water region are practically indistinguishable from the water curve AB, so that, for all practical purposes, in heating water from one temperature to another at constant pressure we can assume we pass along the curve AB.

Let us now consider what the various areas on the chart represent. It will be remembered that areas under a temperature entropy curve represent the heat taken in or given out as the state of the substance passes along the curve.

Thus, in Fig. 157, the area $ABCD$ represents the heat necessary to change 1 lb. of water from the state A to the state B. If A corresponds to a temperature T_2 and B to a temperature T_1, then the area $ABCD$ practically represents the heat necessary to raise the temperature of 1 lb. of water from T_2 to T_1. If the heating is carried out at constant pressure this is equal to $h_1 - h_2$ where h_1 and h_2 are the values of the total heat of water at T_1 and T_2 respectively, the pressure being the same in each case. Hence area $ABCD = h_1 - h_2$. Neglecting the variation in the specific heat of water, this may be taken as approximately equal to $T_1 - T_2$.

The area $BEFC$ will represent the heat necessary to convert 1 lb. of water at T_1 to 1 lb. of dry saturated steam at the same temperature. This quantity is, of course, L_1. This also follows from the fact that

$$BEFC = BE \times EF = \frac{L_1}{T_1} \times T_1 = L_1$$

It may also be noted that AH is the increase of entropy of water when it is heated from T_2 to T_1. Hence $AH = \log_\varepsilon \frac{T_1}{T_2}$ approximately, and therefore $AK = AH + HK = \log_\varepsilon \frac{T_1}{T} + \frac{L_1}{T_1}$.

The Total Heat-Entropy Diagram for Steam (Mollier Chart). Fig. 158 shows an $H\phi$ chart devised by Mollier, in which total heat
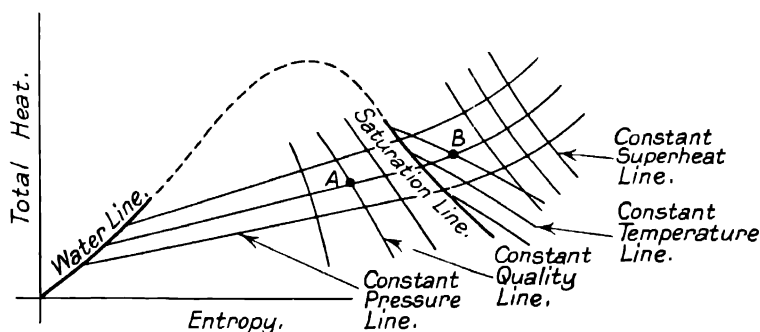
Fig. 158

is plotted vertically and entropy horizontally. The water and saturation curves can be plotted directly from the tables, the two portions really forming one curve if plotted far enough. Within the wet region, between the two boundary curves, any point A, corresponding to a saturation pressure p and dryness fraction x, can be determined from the expressions

$$H_r = h + xL$$

$$\phi_x = \phi_w + \frac{xL}{T}.$$

By joining up points of corresponding dryness fraction, the constant quality lines are obtained, while a constant pressure line passes through all points representing the same pressure.

The slope of a constant pressure line is obviously measured by $\frac{dH}{d\phi}$. Now it has been shown (p. 43) that when heat is taken in by a fluid at constant pressure, the heat taken in is equivalent to the change

of total heat of the fluid. Hence $\dfrac{dH}{d\phi} = \dfrac{dQ}{d\phi} = T$ by the definition of entropy. Therefore the slope of a constant pressure line at any point measures the temperature at that point. In the wet region, as water changes to steam at constant pressure, the temperature remains constant until the fluid becomes dry steam. Constant pressure lines in the wet region are therefore straight, and their slope measures the saturation temperature corresponding to the pressure.

To the right of the saturation line is the superheated region. Any such point as B at a temperature T_{su} and pressure P can be plotted from the tables of total heat and entropy for superheated steam. In the absence of such tables, approximate values may be obtained from the equations
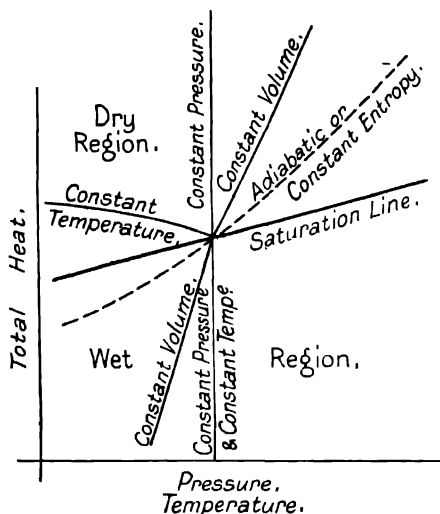
$$H_{su} = H_{sa} + C_p(T_{su} - T_{sa})$$

$$\phi_{su} = \phi_{sa} + C_p \log_\varepsilon \frac{T_{su}}{T_{sa}}.$$

By joining up points of corresponding temperatures and corresponding pressures respectively, constant temperature and constant pressure lines are obtained.

It should be noticed that, as an adiabatic expansion of a fluid is one in which the entropy remains constant, adiabatics on this chart, as well as on the temperature entropy chart, are vertical straight lines.

Fig. 159

Only the region round the saturation curve is included in an $H\phi$ chart, as this is all that is necessary for practical purposes, the very wet region not being often required.

The Callendar Steam Chart. In this diagram* pressure is plotted horizontally, not on a linear scale but to a logarithmic scale, and total heat is plotted vertically to the usual linear scale. Alongside the pressure scale is the corresponding saturation temperature scale, the temperatures on this scale being the saturation temperatures at the pressures coinciding on the pressure scale. By the use of the logarithmic scale, equal horizontal intervals on the diagram represent equal ratios of pressures, a great convenience in

---

* The "Callendar Steam Diagram," and the "$H\phi$ chart," from *Steam Power* by Professor Dalby, published by Messrs. Edward Arnold & Co., London, can be purchased separately, price 6d. each.

practice. The diagram also gives a more open scale for pressure than is the case in the Mollier diagram.

For the sake of clearness, one line of each type included in this diagram is shown in Fig. 159. The saturation line with the wet region below and the superheated region above is nearly straight, and slightly inclined to the horizontal, the constant quality lines running below and more or less parallel to it. Constant pressure lines are obviously vertical straight lines through the diagram. As water changes into steam at constant temperature if the pressure is kept constant, the constant temperature lines in the wet region will also be vertical straight lines. They become slightly curved in the superheated region and run nearly horizontal, any such constant temperature line in the superheated region, of course, starting on the saturation line at a point vertically above where its particular temperature is indicated on the horizontal temperature scale. The constant volume lines are slightly curved and change their direction at the saturation line. The adiabatics or lines of constant entropy are curved and run down from right to left.

Interpolation can be easily carried out by the use of scales Logarithmic scales are required for interpolating between the pressure lines and also between the volume lines. For interpolating between the adiabatic lines, a curved paper template may be used. It will be found that these curves are not all of the same curvature, but a template cut to the mean curvature of the various adiabatics on the chart will be found sufficiently accurate for most purposes.

Example 1. Find the internal energy of a cubic foot of steam at 150 lb. per sq. in. absolute pressure and of dryness 0·7. Calculate the change in internal energy if this steam is expanded until its pressure is 1 lb. per sq. in. absolute, (1) if the expansion is hyperbolic ; (2) if the expansion is at constant entropy. (*U.L., B.Sc. (Eng.).*)

Total heat of 1 lb. of steam at 150 lb. per sq. in. and 0·7 dryness fraction by Table I,

$$H_x = h + xL = 183·59 + 0·7 \times 482·9 = 521·62 \text{ lb. calories.}$$

This value can be taken direct from the $H\phi$ chart by finding the point where the 150 lb. per sq. in. line intersects the 0·7 dryness line and passing horizontally across to the total heat scale. From Table I the volume of 1 lb. of dry steam at 150 lb. per sq. in. = 3·041 cub. ft. = V_{sa}.

The saturation temperature at 150 lb. per sq. in. = 181·31°, and at this temperature the volume of 1 lb. water = 0·018 approx. = V_w.

∴ volume of 1 lb. of steam at 150 lb. per sq. in. of 0·7 dryness = $xV_{sa} + (1 - x)V_w = 0·7 \times 3·041 + 0·3 \times 0·018 = 2·1341$ cub. ft.

∴ 1 cub. ft. of this steam weighs $\dfrac{1}{2·13}$ lb.

$\therefore$ total heat $H_1 = \dfrac{H_x}{2\cdot13} = \dfrac{521\cdot62}{2\cdot13} = 244\cdot9$ lb. calories.

$\therefore$ internal energy $E_1 = H_1 - AP_1V_1 = 244\cdot9 - \dfrac{144}{1400} \times 150 \times 1$

$$= 244\cdot9 - 15\cdot43 = 229\cdot47 \text{ lb. calories per cub. ft.}$$

(1) Before hyperbolic ($PV = $ constant) expansion $P_1V_1 = 150 \times 1$
$\therefore$ after hyperbolic expansion to 1 lb. per sq. in.

$$\text{Volume } V_2 = \frac{150 \times 1}{1} = 150 \text{ cub. ft.}$$

This volume is part steam and part water.

Volume of 1 lb. dry steam at 1 lb. per sq. in. $= 333\cdot1$ cub. ft. $= V_s^1$
Volume of water at saturated temperature corresponding to 1 lb.
per sq. in. $= 0\cdot0161$ cub. ft.

$$\therefore 150 = xVs_1 + (1-x)V_w = \frac{x \times 333\cdot1 + (1-x)0\cdot0161}{2\cdot13}$$

$$\therefore x = 0\cdot96.$$

$\therefore$ total heat after hyperbolic expansion

$$H_2 = \frac{h + xL}{2\cdot13} \text{ (from tables at 1 lb. per sq in)}$$

$$= \frac{38\cdot62 + 0\cdot96 \times 573\cdot84}{2\cdot13} = 276\cdot76 \text{ lb. calories.}$$

This value can be obtained more quickly from the Callendar chart
thus : The volume after expansion is 150 cub. ft.

$\therefore$ the volume of 1 lb. is $150 \times 2\cdot13 = 320$ cub. ft. approximately.

The constant volume line corresponding to 320 cub. ft. is found
on the chart by interpolation. From the point of intersection of
the line thus found with the 1 lb. per sq. in. pressure line pass
horizontally to the total heat scale on the left. The value of total
heat thus obtained must be divided by $2\cdot13$.

The internal energy $E_2 = H_2 - AP_2V_2$

$$= 276\cdot76 - \frac{144}{1400} \times 1 \times 150$$

$$= 276\cdot76 - 15\cdot43 = 261\cdot33 \text{ lb. calories.}$$

$\therefore$ change in internal energy due to hyperbolic expansion

$$= E_2 - E_1 = 261\cdot33 - 229\cdot47 = 31\cdot86 \text{ lb. calories increase.}$$

(2) The value of the total heat after expansion at constant entropy
can be calculated or obtained direct from the $H\phi$ chart thus : From
the point of intersection of the 150 lb. per sq. in. line with the 0·7
dryness line, drop a vertical line to cut the 1 lb. per sq. in. line.
The value of the total heat of this point read off on the H scale is

thus $H_3 = 388 \cdot 5$ lb. calories per lb. The dryness fraction is read off the chart as $0 \cdot 61$. Hence the volume V_3 is $0 \cdot 61 \times 333 \cdot 1$ (neglecting the volume of the water) $= 203 \cdot 2$ cub. ft. per lb.

$\therefore$ internal energy $\quad E_3 = H_3 - AP_3V_3$

$$= 388 \cdot 5 - \frac{144 \times 1}{1400} \times 203 \cdot 2$$

$$= 388 \cdot 5 - 20 \cdot 9 = 367 \cdot 6 \text{ lb. calories per lb.}$$

$\therefore$ internal energy per original cubic foot

$$E_4 = \frac{367 \cdot 6}{2 \cdot 13} = 172 \cdot 58 \text{ lb. calories.}$$

$\therefore$ change in internal energy due to expansion at constant entropy

$$= E_1 - E_4 = 229 \cdot 47 - 172 \cdot 58 = 56 \cdot 89 \text{ lb. calories, decrease.}$$

Example 2. Calculate the internal energy of 1 cub. ft. of steam at 128 lb. per sq. in., absolute pressure, and having dryness 0 95. If this steam expand hyperbolically until it is just dry and saturated, find the final pressure of the steam. How much heat must be added to the steam during expansion ?

(*U.L., B.Sc. (Eng.)*, 1925.)

Volume of 1 lb. of dry steam at 128 lb. per sq. in.

$$= 3 \cdot 531 \text{ cub. ft. (from tables).}$$

$\therefore$ volume of 1 lb. of this wet steam $= 3 \cdot 531 \times 0 \cdot 95 = 3 \cdot 354$ cub. ft.

$\therefore$ weight per cubic foot $= \dfrac{1}{3 \cdot 354}$ lb.

Total heat of 1 lb. of this steam

$$H_x = h + xL = 176 \cdot 37 + 0 \cdot 95 \times 488 \cdot 28$$
$$\text{(by interpolation from tables)}$$

$$= 640 \cdot 24 \text{ lb. calories per lb.}$$

$\therefore$ total heat per cubic foot, $H_1 = \dfrac{640 \cdot 24}{3 \cdot 354} = 191$ lb. calories

$\therefore$ internal energy, $\quad E_1 = H_1 - AP_1V_1$

$$= 191 - \frac{144}{1400} \times 128 \times 1$$

$$= 177 \cdot 8 \text{ lb. calories per cubic foot.}$$

To find pressure P_2 after hyperbolic expansion, we have

$$P_1V_1 = 128 \times 1 \times 3 \cdot 354 = 429.$$

From the tables we have, for dry saturated steam—

P	V	PV
100	4·451	445·1
70	6·218	435·3
50	8·520	426
40	10·5	420
20	20·08	401·6

By plotting the value of PV against the corresponding value of P we find that $PV = 429$ for dry saturated steam when the pressure $P_2 = 56$ lb. per sq. in.

It is interesting to compare this with the value obtained from the equation connecting pressure and volume for dry saturated steam,

$$PV^{\frac{16}{15}} = 490. \quad \text{We also have } PV = 429.$$

From these two equations, $P = \dfrac{429^{16}}{490^{15}}$

whence $P_2 = 58 \cdot 4$ lb. per sq. in.

At 56 lb. per sq. in. the total heat H_2 of this dry steam
$$= 654 \cdot 95 \text{ lb. calories per lb.}$$

The specific volume $V_2 = 7 \cdot 663$ cub. ft. per lb.

The heat added

$$Q = \int_{E_1}^{E_2} dE + \int_{V_1}^{V_2} P dv = E_2 - E_1 + A P_1 V_1 \log \frac{V_2}{V_1}$$

$$= (H_2 - P_2 V_2) - (H_1 - P_1 V_1) + A P_1 V_1 \log_\varepsilon \frac{V_2}{V_1}$$

$$= H_2 - H_1 + A P_1 V_1 \log_\varepsilon \frac{V_2}{V_1}, \text{ since } P_1 V_1 = P_2 V_2$$

$$= \frac{654 \cdot 95 - 640 \cdot 24 + \dfrac{144}{1400} \times 429 \times 2 \cdot 3 \times \log_{10} \dfrac{7 \cdot 663}{3 \cdot 354}}{3 \cdot 354}$$

$$= 15 \cdot 3 \text{ lb. calories.}$$
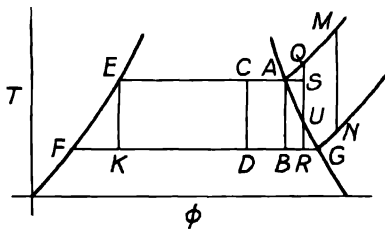
FIG. 160

The Adiabatic Expansion of Steam. The normal adiabatic expansion of steam is best shown by means of the temperature entropy chart. Let A, Fig. 160, represent the state of 1 lb. of dry saturated steam at pressure P_1 and saturation temperature T_1. Let this steam expand adiabatically to B at some lower pressure P_2 and temperature T_2. Since in adiabatic expansion the entropy remains constant, AB is a vertical straight line, the point B being in the wet region, thus showing that on adiabatic expansion, dry steam becomes wet, the dryness fraction being measured by the ratio $\dfrac{FB}{FG}$.

In order to obtain a general expression for the dryness fraction after expansion, let wet steam, represented by the point C, expand adiabatically to D. Then, if 1 refers to the higher temperature T_1 and 2 to the lower temperature T_2, we have

Entropy at C = entropy at D

$$\phi_{w1} + \frac{x_1 L_1}{T_1} = \phi_{w2} + \frac{x_2 L_2}{T_2}$$

Therefore $\quad x_2 = \left(\phi_{w1} - \phi_{w2} + \frac{x_1 L_1}{T_1}\right) \frac{T_2}{L_2}.$

If the steam is originally dry, then $x_1 = 1$.

In the absence of tables, remembering that the entropy of water is approximately given by $\log_\varepsilon \dfrac{T}{273}$

$$x_2 = \left(\log_\varepsilon \frac{T_1}{273} - \log_\varepsilon \frac{T_2}{273} + \frac{x_1 L_1}{T_1}\right) \frac{T_2}{L_2} \text{ approximately.}$$

$$\log_\varepsilon \frac{T_1}{T_2} + \frac{x_1 L_1}{T_1}\right) \frac{T_2}{L_2}.$$

It is instructive to obtain this expression direct from Fig. 160. The length FK represents the change of entropy as water is heated from T_2 at a pressure P_2 to T_1 at a pressure P_1, that is $FK = \phi_{w1} - \phi_{w2}$

Now $\quad x_2 = \dfrac{FD}{FG} = \dfrac{FK + KD}{FG} = \dfrac{FK + EC}{FG} = \dfrac{\phi_{w1} - \phi_{w2} + \dfrac{x_1 L_1}{T_1}}{\dfrac{L_2}{T_2}}.$

In order to investigate the adiabatic expansion of superheated steam, let AM and GN be the constant pressure lines for pressures P_1 and P_2 respectively in the superheated region. Then, after expansion from some superheat temperature T_{su}, the steam may be either still superheated, as in the expansion MN, just dry, or wet, as in the expansion QR. In order to calculate the dryness fraction of the point R, we have

Entropy at Q = entropy at R

$$\phi_{su} = \phi_{w2} + \frac{x_2 L_2}{T_2}$$

$$x_2 = (\phi_{su} - \phi_{w2}) \frac{T_2}{L_2}.$$

If we use the approximations given in a previous paragraph, we have

$$\phi_{su} = \log_\varepsilon \frac{T_1}{273} + \frac{L_1}{T_1} + C_p \log_\varepsilon \frac{T_{su}}{T_1}, \text{ and } \phi_{w2} = \log_\varepsilon \frac{T_2}{273}$$

whence $\log_\varepsilon \dfrac{T_1}{273} + \dfrac{L_1}{T_1} + C_p \log_\varepsilon \dfrac{T_{su}}{T_1} = \log_\varepsilon \dfrac{T_2}{273} + \dfrac{x_2 L_2}{T_2}$

$$x_2 = \left\{ \log_\varepsilon \dfrac{T_1}{T_2} + \dfrac{L_1}{T_1} + C_p \log_\varepsilon \dfrac{T_{su}}{T_1} \right\} \dfrac{T_2}{L_2}.$$

As in the previous case, this expression may be derived direct from the diagram for

$$x_2 = \dfrac{FR}{FG} = \dfrac{FK + KB + BR}{FG} = \dfrac{FK + EA + AS}{FG}$$

$$= \dfrac{\log_\varepsilon \dfrac{T_1}{T_2} + \dfrac{L_1}{T_1} + C_p \log_\varepsilon \dfrac{T_{su}}{T_1}}{\dfrac{L_2}{T_2}}.$$

If a pressure volume curve for the adiabatic expansion of wet or dry saturated steam from some pressure P_1 is required, it may be obtained by calculating from the foregoing equations the dryness fraction x due to expansion from the pressure P_1 down to some lower pressure. The volume at this lower pressure may then be calculated from the equation

$$V_x = x V_{su} + (1 - x) V_w$$

This may be done for expansion from the pressure P_1 down to various pressures, and hence the curve may be plotted.

An approximation to this curve may be obtained by assuming that adiabatic expansion of steam follows the law $PV^m =$ a constant, where m can be calculated from Zeuner's equation, $m = 1.035 + 0.1x$, where x is the dryness fraction at the beginning of the expansion. This method will obviously be only an approximation, for m depends on the value of the dryness fraction at the start of the expansion, and therefore curves obtained in this way starting at various dryness fractions will not coincide.

A curve correct at the initial and final pressures will be obtained if we make $P_1 V_1{}^m = P_2 V_2{}^m$. whence

$$\log P_1 + m \log V_1 = \log P_2 + m \log V_2$$

$$m = \dfrac{\log P_1 - \log P_2}{\log V_2 - \log V_1}.$$

In the case of the adiabatic expansion of steam which is initially superheated and finally wet, we have two stages, QU and UR, Fig. 160. In the first stage, down to the point U where it is just dry, the fluid behaves more or less as a gas, and Callendar shows that the expansion closely follows the law

$$P(V - b)^{1.3} = \text{a constant.}$$

where $b = 0.016$ cub. ft. per lb. and for ordinary work is negligible,

except for high pressures. The second stage, UR, of the expansion, proceeds as described above for wet steam

Experiments at McGill, in 1895, on the law of condensation of steam, led to the conclusion that, for the adiabatic expansion of dry steam, the law connecting pressure and temperature was given by $\dfrac{P}{T^{\frac{13}{3}}} =$ constant.

Now Callendar's characteristic equation for steam can be written

$$(V - b) = \frac{RT}{P} - c \; ; \text{ where } R \text{ is a constant, and } c \text{ varies as } \frac{1}{T^{\frac{10}{3}}}$$

Combining these two equations, we obtain for the adiabatic expansion of dry steam

$$\frac{P}{T^{\frac{13}{3}}} = \text{constant} \; ; \quad P(V - b)^{1\cdot3} = \text{constant} \; ;$$

$$T^{\frac{10}{3}}(V - b) = \text{constant} \; ; \quad \frac{P(V - b)}{T} = \text{constant}.$$

It may be therefore assumed that superheated steam expanding adiabatically closely follows these equations as long as it remains dry.

Example 3. Calculate as accurately as the steam tables permit the final dryness fraction of steam which has expanded adiabatically from an initial state of 100 lb. per sq. in. absolute and 200° C. to a pressure of 15 lb. per sq. in. absolute. The specific heat of steam at 100 lb. per sq. in. is 0·55.

If this expansion be represented by the equation $PV^n =$ constant, what is the value of n, satisfying initial and final conditions

(*a*) Saturation temperature at 100 lb. per sq. in.

$$= 164\cdot28° \text{ C.} = 437\cdot38° \text{ C. (abs.)}.$$

∴ steam is superheated $200 - 164\cdot28 = 35\cdot72°$ C.

Entropy per pound of this steam (reckoned from water at 0° C.)

$$= \phi_1 = \log_\varepsilon \frac{T_{sa}}{273\cdot1} + \frac{L}{T_{sa}} + C_p \log_\varepsilon \frac{T_{su}}{T_{sa}}$$

$$= 2\cdot3 \log_{10} \frac{437\cdot4}{273\cdot1} + \frac{496\cdot1}{437\cdot4} + 0\cdot55 \times 2\cdot3 \log_{10} \frac{473\cdot1}{437\cdot4}$$

$$= 1\cdot6477$$

This entropy can be obtained direct from Table IV thus—

ϕ for steam at 100 lb. per sq. in. and 30° C. of superheat $= 1\cdot6447$

ϕ „ „ „ „ 40° C. „ $= 1\cdot6560$

Difference for 10° C. „ $= 0\cdot0113$

∴ ϕ_1 for steam at 100 lb. per sq. in. and 35·72° of superheat

$$= 1\cdot6447 + 0\cdot00113 \times 5\cdot72 = 1\cdot6512$$

Entropy after expansion $= \phi_2 = \log_\varepsilon \dfrac{T_2}{273\cdot1} + x\dfrac{L_2}{T_2}$

where $T_2 =$ saturation temperature at 15 lb. per sq. in. $= 373\cdot7°$ C. (abs.)

$L_2 =$ latent heat at $T_2 = 538\cdot95$

∴ $\qquad \phi_2 = 2\cdot3\log_{10}\dfrac{373\cdot7}{273\cdot1} + x\dfrac{538\cdot9}{373\cdot7}$

But $\qquad \phi_2 = \phi_1 = 1\cdot65$; hence $x = 0\cdot93$.

This dryness fraction can be taken direct from the $H\phi$ chart, thus : From the point of intersection of the 100 lb. per sq. in. pressure line, with the 200° C. temperature line in the superheated region, draw a vertical line to cut the 15 lb. per sq. in. line. The dryness fraction of the point thus obtained is then read off as 0·926.

(b) To find the value of n we first require the specific volume V_1 of superheated steam at 100 lb. per sq. in. and 200° C. This can be obtained from tables, $V_1 = 4\cdot8901$ cub. ft.

In the absence of tables it can be calculated from the Callendar equation : $V_1 = \dfrac{2\cdot2436\ (H - 464)}{p} + 0\cdot0123$

$$= \dfrac{2\cdot2436\ (681\cdot4 - 464)}{100} + 0\cdot0123 = 4\cdot89 \text{ cub. ft.}$$

The specific volume of dry steam at 15 lb. per sq. in. $= 26\cdot27$ cub. ft

∴ specific volume of wet steam after the adiabatic expansion

$$= V_2 = 0\cdot93 \times 26\cdot27 = 24\cdot4 \text{ cub. ft.}$$

∴ $\qquad n = \dfrac{\log P_1 - \log P_2}{\log V_2 - \log V_1} = \dfrac{\log 100 - \log 15}{\log 24\cdot4 - \log 4\cdot89}$

∴ $\qquad n = 1\cdot183$

Supersaturated Steam. In the last paragraph we have dealt with adiabatic expansion where the fluid is always in a state of equilibrium at any point during the expansion. As the expansion proceeds the steam condenses and there is always a stable mixture of steam and water. We have now to deal with an expansion which is adiabatic in the sense that no heat passes into or out of the mixture, but during which there is no separation of water as the expansion proceeds.

As we have already seen, if a dry saturated vapour is expanded under adiabatic conditions, the vapour will normally condense more and more as the expansion proceeds. It can be shown, however, that if a dry saturated vapour be suddenly expanded, it may continue for a time to remain in the state of a homogeneous vapour,

although the conditions are such that it should normally be partly liquid. Under these conditions, the vapour is said to be supersaturated. The matter will perhaps be made clearer by means of the $T\phi$ chart (Fig. 161).

Consider dry saturated steam expanding adiabatically from a pressure P_1 (saturation temperature T_1) to a pressure P_2 (saturation temperature T_2). Normal adiabatic expansion is represented by AB (Fig. 161), where any point on AB lower than A lies in the wet region, the dryness fraction at any point G of the expansion being measured by $\dfrac{FG}{FK}$.

Now, if the expansion proceeds under supersaturated conditions, we may look on the saturation line AD, that is, the boundary line
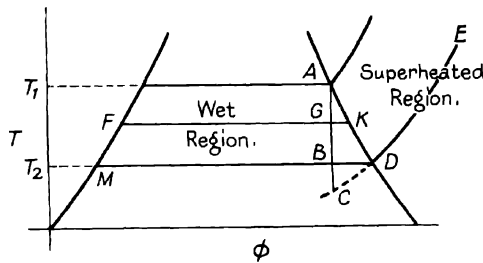
FIG. 161

between the wet and superheated regions, as not being in existence. If, therefore, DE is the continuation of the constant pressure line MD, supersaturated expansion will proceed from A to C, where DC is the continuation of the constant pressure line ED, into what is normally the wet region. Thus, while AB represents normal equilibrium adiabatic expansion from pressure P_1 to P_2, AC represents supersaturated expansion from pressure P_1 to P_2. It will be noticed that the temperature at C, after supersaturated expansion, is lower than the saturation temperature T_2, corresponding to the pressure P_2, that is, the supersaturated vapour is said to be supercooled or under cooled.

In this supersaturated condition, the density at any pressure is higher than the density of saturated vapour at that particular pressure. As the expansion proceeds, therefore, not only does the actual temperature become considerably lower than the corresponding saturation temperature, but the density increases above the density of saturated vapour. As the supersaturated condition is metastable, there will be a limit of expansion below which the supersaturated condition can no longer hold, and condensation takes place.

During supersaturated adiabatic expansion of steam, as the steam

remains dry, the same equations hold as in the case of the adiabatic expansion of superheated steam, namely,

$$P(V-b)^{1.3} = \text{constant} \; ; \quad \frac{P}{T^{\frac{13}{3}}} = \text{constant} ; \; T^{\frac{10}{3}}(V-b) = \text{constant}.$$

From these equations the actual temperature T of the super-saturated steam can be calculated, and hence the degree of under-cooling can be determined ; that is, the difference between this temperature and the saturation temperature corresponding to the pressure of the steam.

It will be seen in a later chapter that supersaturated expansion can take place through a suitably shaped nozzle.

The Throttling of Steam. If steam is allowed to pass through a small orifice so that the pressure drops as it passes through, it is said to be throttled. In an ideal throttle there should be no change in velocity on the two sides of the orifice ; also no external work will be done by the steam, and no heat should be lost to external bodies.

Under these circumstances the equation of steady flow (p. 43)

$$H_1 - H_2 + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = AW + Q$$

gives us $H_1 - H_2 + 0 = 0 + 0$
that is, $H_1 = H_2$

Fig. 162

Hence in the ideal throttling of steam the total heat remains constant

Let us consider what happens when dry saturated steam at, say, 100 lb. per sq. in. is throttled to 50 lb. per sq. in. The total heat of dry steam at 100 lb. per sq. in. = 661·83, while the total heat of dry steam at 50 lb per sq. in. is only 653·6. But during throttling the total heat of the fluid remains constant. Hence the fluid must become superheated. Similarly, throttling wet steam tends to dry it. This may best be shown on the total heat entropy chart.

In this chart, as total heat is plotted vertically, throttling will be represented by a horizontal straight line. Hence, in Fig. 162, dry saturated steam at 100 lb. per sq. in. will be represented by A where the 100 lb. per sq. in. constant pressure line crosses the saturation line. In being throttled to 50 lb. per sq. in , the state of the steam will pass along a horizontal straight line to the point B where this line cuts the 50 lb. per sq. in. constant pressure line. This point will obviously be in the superheated region, and its exact state can be read off from the chart.
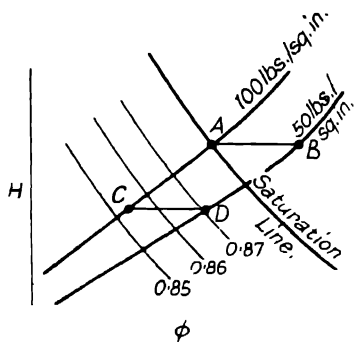
Similarly, steam of dryness fraction equal to 0·854 at a pressure of 100 lb. per sq. in. will be represented by the point C. In throttling, the state point of the steam will pass along the horizontal straight line until it cuts the 50 lb. per sq. in. pressure line at D. The dryness fraction of the point D is read off the chart as equal to 0·87. Hence throttling wet steam dries it.

In order to calculate the condition of steam after throttling, we may consider two cases. Let suffixes 1 and 2 refer to the state of the steam before and after throttling.

Case I. Wet steam of dryness fraction x_1 still wet after throttling. Total heat before throttling = total heat after throttling.

$$h_1 + x_1 L_1 = h_2 + x_2 L_2$$

$$\therefore x_2 = \frac{h_1 + x_1 L_1 - h_2}{L_2}$$

Case II. Wet steam of dryness fraction x_1 superheated to T_{su} after throttling.

$$h_1 + x_1 L_1 \qquad L_2 + C_p(T_{su} - T_{sa}) \text{ (assuming } C_p \text{ is constant)}.$$

$$T_s \qquad \frac{h_1 + x_1 L_1 - h_2 - L_2}{C_m} + T_s$$

The Throttling Calorimeter. The use of this instrument to find the dryness fraction of steam depends on the constancy of total heat during a throttling process.

Steam from the main steam pipe passes by way of a valve F (Fig. 163) through the orifice A into the calorimeter B, and so away at C. The pressure of the steam after throttling at A can be determined from the manometer at D. The temperature of the steam after throttling is given by the thermometer E. If the steam is nearly dry in the main steam pipe it will become superheated after passing through the orifice.
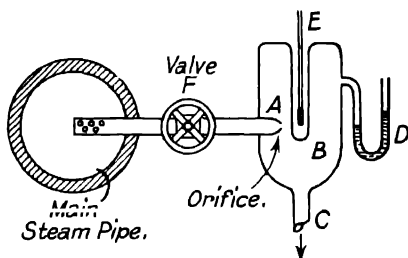
FIG. 163. THROTTLING CALORIMETER

Let p_1 = pressure in main steam pipe

h_1 = total heat of water at p_1

x_1 = dryness fraction in main steam pipe

L_1 = latent heat at p_1

p_2 = pressure in calorimeter B, obtained from manometer

H_2 = total heat of dry steam at p_2

T_{su} = temperature from thermometer E

T_{sa} = saturation temperature at pressure p_2

Then $\qquad h_1 + x_1 L_1 = H_2 + C_p(T_{su} - T_{sa})$

hence $\qquad x_1 = \dfrac{H_2 + C_p(T_{su} - T_{sa}) - h_1}{L_1}$

Care must be taken that the valve F is fully opened to prevent the steam from being throttled in passing through it.

This instrument will only be of use if the steam, after expansion, is superheated. It is useless if the steam in the main steam pipe is so wet that it remains wet after expansion. The limiting case will be when the steam is just dry after expansion, that is, when the temperature on the thermometer E is just the saturation temperature corresponding to the pressure p_2. In this case $h_1 + x_1 L_1 = H_2$. Hence one may know when the instrument is not superheating by the fact that the thermometer E is not giving a temperature greater than the saturation temperature corresponding to the pressure in the calorimeter B.

Provided that a true sample of the steam is obtained, this method is an accurate one for determining the dryness fraction of steam within the limits of its functioning. The main difficulty is that of obtaining a true sample. A common method is as shown in Fig. 163, where a pipe closed at the end and perforated with holes facing the direction of the steam, lies across the main steam pipe. In the case of a horizontal steam pipe, however, an error may be caused by a large proportion of the water in the steam being carried along the bottom of the pipe.
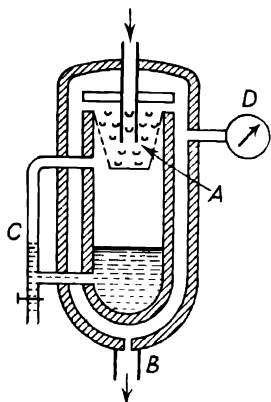
The Separating Calorimeter. The dryness fraction of steam too wet for use in a throttling calorimeter can be determined by means of a separating calorimeter. This consists of a double-walled chamber, the inner chamber containing a perforated metal cup A (Fig. 164). The inner chamber is connected to the outer through an opening at the top of the perforated cup. The outer chamber has an outlet at B. The steam to be tested enters the inner chamber at the top and, in discharging against the bottom of the perforated cup, is forced to make a sharp turn, thus separating the water from it. The steam then passing through the sides of the cup passes to the outer chamber, and so to the outlet B. The separated water collects at the bottom of the inner chamber, and its amount may be determined by the graduated gauge glass C. The amount of steam passing through the outer chamber can be determined by condensing it in a weighed quantity of cold water, as it passes from the outlet B. For quick measurements, which may be somewhat

unreliable, the flow may be determined by a pressure gauge *D* carrying a second scale, which shows directly the weight of steam flowing in a certain time   This scale can be calibrated by weighing the steam actually passing in this time, for several rates of flow. The whole calorimeter must be well lagged.   Then, assuming that the steam passing into the outer chamber is dry, if

w = weight of water removed from steam in a certain time

W = weight of dry steam passing in same time

$$\text{Dryness fraction} = \frac{W}{W + w}$$

If the steam passing into the outer chamber is not quite dry, the dryness fraction can be determined by the throttling calorimeter, and allowance made accordingly.

Example 4. Describe carefully any method of determining the dryness fraction of the steam supplied to an engine.   Discuss fully the precautions which have to be adopted and state what degree of accuracy you expect to obtain.

Steam passes through a sampling pipe in a steam pipe to a separator and from the separator it passes through a wire drawing calorimeter.   The following observations were made—

(1) Temperature of the wet steam in the main, 168° C.

(2) Temperature of the steam after wire drawing to atmospheric pressure, 108° C.

(3) Weight of water caught in the separator per minute, 0·035 lb.

(4) Weight of water passing through the wire drawing orifice per minute, 0·81 lb.

Determine the original dryness fraction of the steam.

Saturation pressure corresponding to 168° C. = 110 lb. per sq. in. absolute (very nearly).

Let x = dryness fraction as the steam leaves the separator and before it is wire drawn.

Before being wire drawn—

Total heat at 110 lb. per sq. in. $= h_{110} + xL_{110}$

$$= 169{\cdot}75 + x \times 493{\cdot}18$$

After being wire drawn—

Total heat at 14·7 lb. per sq. in. $= H_{14{\cdot}7} + C_p(t_{su} - t_{sa})$

$$= 639{\cdot}3 + 0{\cdot}5\,(108 - 100)$$

$$\therefore\ 169{\cdot}75 + 493{\cdot}18x = 639{\cdot}3 + 0{\cdot}5 \times 8$$

$$\therefore x = 0{\cdot}96$$

Hence 0·81 lb. of steam of dryness fraction 0·96 leaves the separating calorimeter per minute.

Hence amount of dry steam leaving the separator per minute

$$= 0{\cdot}96 \times 0{\cdot}81 = 0{\cdot}778 \text{ lb.}$$

Total stuff passing per minute $= 0.035 + 0.81 = 0.845$

$\therefore$ original dryness fraction $= \dfrac{0.778}{0.845} = 0.92$.

It should be noted that the dryness fraction before wire drawing can be determined direct from the $H\phi$ chart thus : From the point of intersection in the superheated region of the 14·7 lb. per sq. in. pressure line with the 108° C. temperature line, draw a horizontal line to cut the 110 lb. per sq. in. pressure line. The dryness fraction of the point so obtained can then be read.

Gain of Entropy During Throttling of Steam. From Fig. 162 it is obvious that the entropy of steam increases due to throttling. To calculate this increase of entropy we can make use of the temperature-entropy chart, and we will consider two cases.

Case I. Steam of dryness fraction x_1 still wet after throttling.
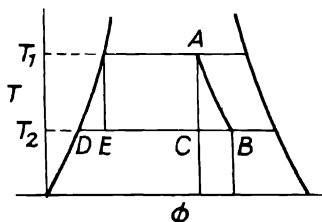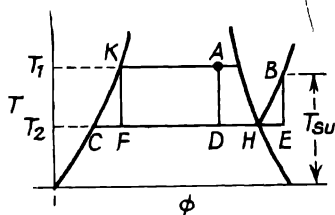
FIG. 165   FIG. 166

Let the steam represented by the state point A, Fig. 165, be throttled from a pressure P_1 (saturation temperature T_1) to a pressure P_2 (saturation temperature T_2). Since entropy increases the final state, B will be to the right of A on the chart. Let the final dryness fraction be x_2.

Then entropy at $B = \phi_{w2} + \dfrac{x_2 L_2}{T_2} = \log_e \dfrac{T_2}{273} + \dfrac{x_2 L_2}{T_2}$ (approx.)

entropy at $A = \phi_{w1} + \dfrac{x_1 L_1}{T_1} = \log_e \dfrac{T_1}{273} + \dfrac{x_1 L_1}{T_1}$

Hence increase of entropy due to throttling

$$= \phi_{w2} - \phi_{w1} + \dfrac{x_2 L_2}{T_2} - \dfrac{x_1 L_1}{T_1}$$

$$= \dfrac{x_2 L_2}{T_2} - \dfrac{x_1 L_1}{T_1} - \log_e \dfrac{T_1}{T_2} \text{ (approx.)}.$$

This result may be obtained direct from the diagram.

Increase in entropy $= CB = DB - DE - EC$

$$= \dfrac{x_2 L_2}{T_2} - \log_e \dfrac{T_1}{T_2} - \dfrac{x_1 L_1}{T_1}$$

In order to find x_2 in terms of the initial conditions, we have, since total heat remains constant during throttling,

$$h_2 + x_2 L_2 = h_1 + x_1 L_1$$

$$\therefore x_2 = \frac{h_1 - h_2 + x_1 L_1}{L_2}$$

$\therefore$ increase of entropy $= \phi_{w2} - \phi_{w1} + \dfrac{h_1 - h_2 + x_1 L_1}{T_2} - \dfrac{x_1 L_1}{T_1}$

$$= \frac{T_1 - T_2 + x_1 L_1}{T_2} - \frac{x_1 L_1}{T_1} - \log_e \frac{T_1}{T_2} \text{ (approx.).}$$

Case II. Steam of dryness fraction x_1 superheated after throttling.

Let the steam represented by the state point A, Fig. 166, of dryness fraction x_1 be throttled from a pressure P_1 (saturation temperature T_1) to a pressure P_2 (saturation temperature T_2).

As we are assuming that the steam becomes superheated, the final state B will be on the continuation of the constant pressure line for P_2 in the superheated region. Let the temperature at this point be T_{su}. The increase in entropy required will therefore be represented by the distance DE.

Increase in entropy $= CE - CD = (CH + HE) - (CF + FD)$

But CH represents the change in entropy when 1 lb. of water at T_2 is changed into 1 lb. of dry steam at $T_2 = \dfrac{L_2}{T_2}$; and HE represents the change in entropy when 1 lb. of dry steam at saturation temperature T_2 is superheated at constant pressure to a temperature $T_{su} = C_p \log_e \dfrac{T_{su}}{T_2}$, approximately; and $CF =$ change in entropy when 1 lb. of water is heated from T_2 to $T_1 = \log_e \dfrac{T_1}{T_2}$, approximately; and $FD = KA =$ change in entropy when 1 lb. of water at T_1 is changed into dry saturated steam of dryness fraction x_1 at the same temperature $= \dfrac{x_1 L_1}{T_1}$.

$\therefore$ Change in entropy during throttling

$$= \frac{L_2}{T_2} + C_p \log_e \frac{T_{su}}{T_2} - \log_e \frac{T_1}{T_2} - \frac{x_1 L_1}{T_1}$$

In order to find the value of T_{su} in terms of the conditions before throttling, we have, as total heat remains constant during throttling,

$$h_1 + x_1 L_1 = H_2 + C_p \log_e \frac{T_{su}}{T_2}$$

from which T_{su} can be calculated and the value inserted in the previous equation.

EXAMPLES IX

1. Determine from the steam tables the amount of heat in 1 lb. of steam—
(a) At a pressure of 152 lb. per sq. in. abs. and 0·9 dry.
(b) At a pressure of 215 lb. per sq. in. abs. with 250° F. (139° C.) of superheat.
If in both cases the steam was generated from water at 90° F. (32° C.), estimate the respective gains of internal energy.   (*I.Mech.E.*, 1925.)

2. Estimate the amount of heat necessary to convert 10 lb. of water at 212° F. into steam at a pressure of 190 lb. per sq. in. abs. with 280° F. of superheat. Determine the proportion of this heat used in doing external work.   (*I.Mech.E.*, 1923.)

3. Calculate the total energy of 1 lb. of steam at 126 lb. per sq. in. abs. pressure and dryness 0·985, and find also its internal energy. If steam in this state be expanded in a cylinder to 3 lb. per sq. in abs., the expansion being hyperbolic, find the final temperature of the steam. Use Callendar's equation

$$V - 0·016 = 1·0706\frac{T}{p} - 0·4212\left(\frac{373·1}{T}\right)^{\frac{10}{3}}$$

where p = pressure in pounds per square inch

V = volume in cubic feet

T = absolute temperature of the steam.

(*U.L.*, *B.Sc.* (*Eng.*), 1923.)

4. Steam at a pressure of 15 lb. per sq. in. abs., but slightly superheated, reaches a surface condenser at 232° F. (111° C.) and leaves as water at 202° F. (94·4° C.). The circulating water enters at 55° F. (12·8° C.), and the quantity has to be regulated so that its maximum outlet temperature does not exceed 160° F. (71° C.). Calculate the amount of circulating water that will be required per pound of steam.   (*I.Mech.E.*, 1925.)

5. Steam originally dry is allowed to fall in pressure from 150 to 60 lb. per sq. in. abs. under the following conditions—
(a) In a closed vessel which loses heat by radiation and conduction.
(b) By passing through a throttle valve.
Estimate the final condition of the steam (superheat or dryness fraction) in both cases.   (*I.Mech.E.*, 1924.)

6. Steam at a pressure of 190 lb. per sq. in. abs., with 200° F. (111° C.) of superheat, has its pressure lowered to 80 lb. per sq. in. abs. by passing through a reducing valve. Estimate the temperature of the low pressure steam. If 10 lb. of steam per second are passing through the pipes, calculate their diameters so that the steam speed should not exceed 120 ft per sec.   (*I.Mech.E.*, 1925.)

7. Steam at a pressure of 200 lb. per sq. in. abs., with 260° F. of superheat, is throttled to 120 lb. per sq. in. abs. and then expanded adiabatically to 50 lb. per sq. in. abs. Determine the temperature of the steam (a) after throttling, (b) after expansion.   (*I.Mech.E.*, 1923.)

8. Define the term "total heat" of a fluid and prove that this quantity does not change during a throttling process.
Steam, after throttling, has a pressure of 15 lb. per sq. in. abs. and a temperature of 150° C. If the pressure of the steam before throttling was 200 lb. per sq. in. abs., find its dryness fraction.
What is the minimum dryness fraction which may be determined by means of a throttling calorimeter if the steam is throttled from 200 lb. per sq. in. abs. to 15 lb. per sq. in. abs. ?   (*U. L.*, *B.Sc.* (*Eng.*), 1926.)

9. A pipe line between a boiler and engine is equivalent to a straight length of 200 ft. of 4 in. external diameter piping covered with lagging. The engine takes 4,860 lb. of steam per hour. The steam pressure is 120 lb. per sq. in. abs. and at the boiler end of the pipe the steam is by test $\frac{3}{4}$ per cent wet. A throttling calorimeter at the engine steam chest expands a steam sample to 15 lb. per sq. in. abs., at which pressure the temperature reading is 134° C. The drainage from the pipe line is 30 lb. per hour. Estimate the heat loss per square foot of metal surface, per hour, per degree Centigrade difference between steam and air temperature (22° C.) for the pipe.

(*U.L.*, *B.Sc.* (*Eng.*), 1921.)

CHAPTER X

The Carnot Cycle with Steam as Working Substance. It will be seen above (pp. 94 and 98) that the Carnot cycle consists of an isothermal and adiabatic expansion and an isothermal and an adiabatic compression. Let us consider this cycle, using water and steam as the working substance.

Let us have 1 lb. of water at an absolute temperature T_1 in the cylinder, the state being represented on the pressure volume diagram by the point a, and on the temperature entropy diagram by the point A (Fig. 167).
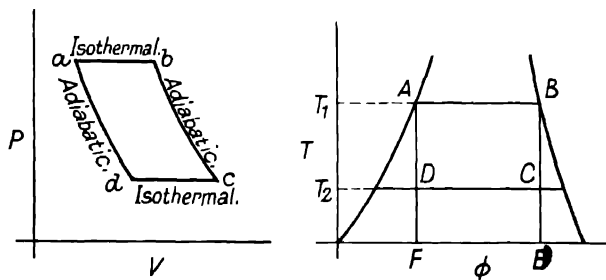
FIG. 167

Stage 1.   Path ab on pv diagram, and AB on $T\phi$ diagram.

The water at T_1 is gradually changed into dry saturated steam at the same temperature, the dry state being represented by b and B. The isothermal expansion of the substance takes place at constant pressure (the saturation pressure corresponding to T_1).

The heat taken in is L_1 and the change of entropy $AB = \dfrac{L_1}{T_1}.$

Stage 2.   Path bc on pv diagram, BC on $T\phi$ diagram.

The dry steam expands adiabatically, the pressure and temperature falling and the volume increasing. No heat is transferred to or from the working substance, and there is no change in entropy. BC is therefore a vertical straight line and the state at the end of expansion, as represented by C, is that of wet steam. The temperature of the steam falls to some lower temperature T_2.

Stage 3.   Path cd on pv diagram, CD on $T\phi$ diagram.

The wet steam is compressed at constant temperature T_2, i.e. isothermally. The pressure remains constant at the saturation pressure corresponding to T_2, while the steam gradually changes to water, heat being rejected from the working substance. Let this

438

compression be completed at some point (d and D), such that the subsequent adiabatic compression completes the cycle.

Stage 4. Path *da* on *pv* diagram and *DA* on *Tφ* diagram.

The wet mixture at D is compressed adiabatically to A, pressure and volume returning to the original state (a and A). No heat is transferred to or from the working substance, and there is no change in entropy. The temperature rises, and the final state is once again that of water at T_1.

It will be noticed that the whole cycle is reversible and the heat is taken in at one temperature and rejected at one lower temperature, and hence the efficiency is a maximum and should be equal to $\dfrac{T_1 - T_2}{T_1}$.

The heat taken in during stage 1 is represented by the area

$$ABEF = AB \times BE = \frac{L_1}{T_1} \times T_1 = L_1.$$

The heat rejected in stage 3 is represented by the area

$$DCEF = DC \times CE = \frac{L_1}{T_1} \times T_2.$$

The work done in the cycle = heat taken in – heat rejected

$$= \text{Area } ABCD = AB \times BC = \frac{L_1}{T_1}(T_1 - T_2)$$

$$\therefore \text{ Efficiency of cycle} = \frac{\text{Work done}}{\text{Heat taken in}}$$

$$= \frac{\dfrac{L_1}{T_1} \cdot (T_1 - T_2)}{L_1} = \frac{T_1 - T_2}{T_1}.$$

Clapeyron's Equation. The equation which has already been referred to (pp. 99 and 100) gives a means of calculating the specific volume of dry saturated steam, a quantity very difficult to measure experimentally.

Consider a Carnot engine working between two temperatures differing by a small amount δT, the corresponding difference of saturation pressure being δP. Let this cycle be represented by *abcd*, Fig. 168. The efficiency of this cycle is $\dfrac{\delta T}{T}$ and the work done per pound of working substance $= \dfrac{L\delta T}{T}$. This work is represented by the area of the diagram *abcd*. If the temperature difference is very small this area approximates to a rectangle of height δP and width ($V_{sa} - V_w$) where V_{sa} is the specific volume of dry saturated steam and V_w is the specific volume of water.

Hence $(V_{sa} - V_w)\delta P = JL \frac{\delta T}{T}$, approximately

i.e. $V_{sa} - V_w = \frac{JL}{T} \frac{\delta T}{\delta P}$

In the limit, as δT and δP approach zero, this equation becomes exactly true and hence

$$V_{sa} - V_w = \frac{JL}{T} \frac{dT}{dP}, \text{ or } V_{sa} = V_w + \frac{JL}{T} \frac{dT}{dP}$$

From this equation the volume of 1 lb. of dry steam may be calculated for any pressure. The values of L and T can be obtained from the steam tables, the value of $\frac{dT}{dP}$, which is the rate of change
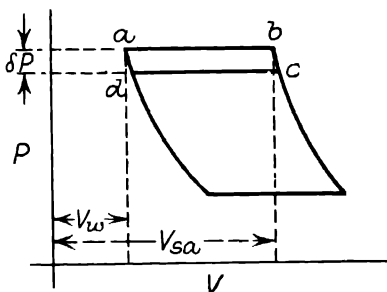
FIG. 168

of saturation temperature with pressure, being found by calculation from the steam tables or by finding the slope of the saturation temperature pressure curve for the temperature T.

Example 1. Establish the equation $V = \frac{JL}{T} \frac{dT}{dP}$, where V is the volume of 1 lb. of dry steam, L latent heat, J is 1,400 ft.-lb. per C.H.U., T the absolute temperature, and P the pressure in pounds per square foot. From the tables supplied work out the volume of 1 lb. of steam using the above formula, taking the case of steam at 5 lb. per sq. in. pressure, and give any reasons you consider valid to account for the difference between your answer and that given in the steam tables supplied. (*U.L., B.Sc. (Eng.)*, 1920.)

It will be noticed that the small quantity $V_w = 0.01602$ cub. ft. has been neglected in this question.

From the tables, when $P = 4$ lb. per sq. in. $T = 340.33°$ C.

$P = 6$ lb. per sq. in. $T = 349.82°$ C.

$$\therefore \frac{\delta T}{\delta P} = \frac{349.82 - 340.33}{2 \times 144} = \frac{4.745}{144}.$$

This is not the true value of $\frac{dT}{dP}$, for if values of T and P are plotted, it will be seen that the slope of the curve, that is $\frac{dT}{dP}$, varies considerably, especially at the low pressures.

Taking this value, and finding L and T for 5 lb. per sq. in. from the tables, we have

$$V = \frac{1400 \times 555\cdot38}{345\cdot48} \times \frac{4\cdot745}{144}$$

$$= 74\cdot16 \text{ cub. ft.}$$

The chief reason for the difference between this value and that in the tables is due to the inaccuracy of the determination of the value of $\frac{dT}{dP}$. If a smaller difference of pressure δP is taken, the value will be nearer the true one, which is exactly given by the tangent to the pressure temperature curve at the point where $P = 5$ lb. per sq. in.

With some steam tables there may also be another reason for the difference. In the older steam tables the various properties of steam have been calculated from various empirical formulæ obtained from experimental results. These formulæ are not necessarily mutually consistent. In the Callendar steam tables the various formulæ have all been derived to conform to the Callendar characteristic equation for steam, and hence the values for the various properties of steam in these tables are mutually consistent.
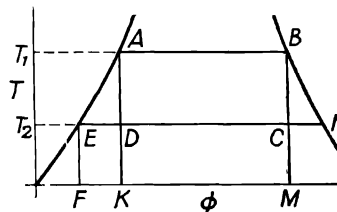
Fig. 169

The Rankine Cycle. An obvious practical difficulty in the Carnot cycle is the stopping of the isothermal compression of the working substance at the correct point D (Fig. 167).

A simple modification is to continue the compression CD to E (Fig. 169) until all the steam has been condensed to water and then to convert this water at the low pressure and temperature T_2 to water at the higher pressure and temperature T_1. The cycle would then be $ABCE$, Fig. 169, where CE represents the condensation of the wet steam at C to water at E, and EA represents the conversion of the water at low pressure and temperature T_2 to high pressure and temperature T_1.

The ideal cycle thus modified will be as follows—

Stage 1. Path AB. Water at temperature T_1 and saturation pressure P_1 is converted into dry saturated steam at constant temperature and pressure. The heat taken in by working substance is represented by the area $ABMK$ and is equal to L_1.

Stage 2. Path BC. The dry saturated steam expands adiabatically from pressure P_1 to P_2

The dryness fraction of the working substance after expansion

$$= \frac{EC}{EN}.$$

Stage 3. Path CE. The steam is now condensed by the removal of heat at constant pressure P_2 and constant saturation temperature T_2, until the working substance is wholly water.

The heat given out by the working substance is represented by the area $ECMF$.

Stage 4. Path EA. The water at temperature T_2 and saturation pressure P_2 is raised by the addition of heat to temperature T_1 and saturation pressure P_1, the pressure at any stage being the saturation pressure corresponding to the temperature at that stage. The heat taken in by the working substance is represented by the area $EAKF$.

The work done in the complete cycle is therefore equal to heat taken in – heat rejected, and is represented by the area $EAKF + ABMK - ECMF =$ area $ABCE$.
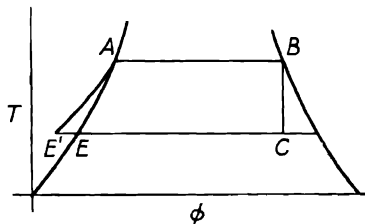
The efficiency of the cycle $=$

$$\frac{\text{Work done}}{\text{Heat taken in}} = \frac{\text{Area } ABCE}{\text{Area } EABMF}$$

In the cycle just considered the working substance is assumed to remain in the cylinder throughout the whole cycle. We will now consider what modification is necessary to conform more nearly to practice, where heating is carried out in a boiler and the condensation of the steam is carried out in a condenser apart from the working cylinder. Under these circumstances, the fourth stage just considered can be assumed to be carried out in two operations.

Fig. 170

The condensed water at E, Fig. 169, at P_2 and T_2 will first be pumped by means of a feed pump into the boiler, where the pressure is P_1. The water then takes in heat in the boiler at constant pressure P_1 until its temperature rises to T_1. These two stages are represented by EE' and $E'A$, Fig. 170, where AE' is the continuation in the water region of the P_1 constant pressure line AB. Owing to the small compressibility of water, however, the points E and E' are practically coincident, and hence, to all intents and purposes, the cycle is still represented by $ABCE$.

We will now consider this cycle, assuming it is carried out not all in the working cylinder, but consecutively in the boiler, working cylinder, condenser, and feed pump. The cycle is shown at *abce* in the *pv* diagram and $ABCE$ in the $T\phi$ diagram, Fig. 171. The line *ea* in the *pv* diagram really slopes to the left as it goes from *e* to *a*, owing to the compression of the water reducing the volume.

This is, however, so slight that ea may be taken as a vertical straight line. The cycle is as follows—

Stage 1. Path ab and AB. Water is converted in the boiler into steam at constant pressure P_1 and constant temperature T_1, and enters the working cylinder as it is produced. Work done by steam as it enters cylinder, $fbmo$, during formation from water. Heat taken in by working substance, $ABUW$.

Stage 2. Path bc and BC. Boiler is shut off from the cylinder and the steam expands adiabatically in cylinder. Work done by steam, $bcnm$. Heat taken in, *nil*. Pressure falls to p_2 and volume increases.

Stage 3. Path ce and CE. Steam is rejected from cylinder and is ejected to the condenser, where it is condensed at constant pressure p_2 and constant temperature T_2 to water. Work done on steam, $gcno$. Heat given out by working substance, $ECUS$.
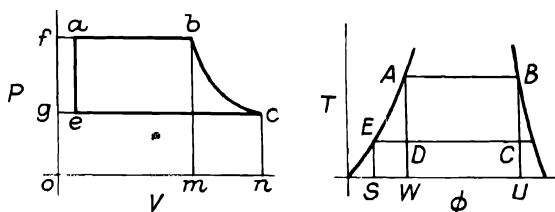
Fig. 171

Stage 4. Water at pressure p_2 and temperature T_2 is pumped into boiler, where the pressure is p_1. It takes in heat in the boiler and its temperature rises to T_1. Work done by feed-pump, $faeg$. Heat taken in by water $EAWS$.

The net work done in the cycle is represented by

$$fbmo + bcnm - gcno - faeg = abce$$

on the pv diagram. On the $T\phi$ diagram this work, which is equal to the difference between the heat taken in and the heat rejected by the working substance, is represented by

$$ABUW + EAWS - ECUS = ABCE.$$

In order to obtain expressions for the work done in the cycle, it will be remembered that on p. 44 it was proved that the area $fbcg$, Fig. 171, which represents $\int VdP$ over the adiabatic expansion bc is measured by the heat drop during this expansion. The area $fbcg$ represents the value of $H_B - H_c$ in work units, where H_B and H_C are the values of total heat of the working substance before and after expansion. Now the area $faeg$ represents the work done by the feed-pump, namely, $V_w(P_1 - P_2)$, where V_w is the volume of the water at e.

Hence the work done during the cycle

$$= H_{\mathrm{B}} - H_{\mathrm{C}} - \frac{V_w(P_1 - P_2)}{J} \text{ heat units.}$$

The heat taken in is the heat necessary to change the water at P_1 and T_2 into dry saturated steam at P_1 and T_1. As this heat is given at constant pressure, its value will be $H_{\mathrm{B}} - h_{\mathrm{E}'}$, Fig. 170, where $h_{\mathrm{E}'}$ is the total heat of water at P_1 and T_2. Now $h_{\mathrm{E}'}$ will be greater than h_{E}, the total heat of water at P_2 and T_2, by the thermal equivalent of the amount of work necessary to bring the water at P_2 and T_2 to P_1 and T_2, that is, by $\dfrac{V_w(P_1 - P_2)}{J}$.

Hence the heat taken in $= H_{\mathrm{B}} - \left(h_{\mathrm{E}} + \dfrac{V_w(P_1 - P_2)}{J} \right)$.

The efficiency of the cycle is therefore

$$\frac{H_{\mathrm{B}} - H_{\mathrm{C}} - \dfrac{V_w(P_1 - P_2)}{J}}{H_{\mathrm{B}} - h_{\mathrm{F}} - \dfrac{V_w(P_1 - P_2)}{J}}$$

The term $\dfrac{V_w(P_1 - P_2)}{J}$ is so small compared to the other quantities that it can usually be ignored, and hence the efficiency which is represented on the $T\phi$ diagram by $\dfrac{\text{area } ABCE}{\text{area } EABUS}$ may be taken as equal to $\dfrac{H_{\mathrm{B}} - H_{\mathrm{C}}}{H_{\mathrm{B}} - h_{\mathrm{E}}}$. In this expression H_{B} is the total heat of dry saturated steam at P_1, and h_{E} is the total heat of water at P_2, both values being taken from the steam tables. In order to calculate the value of H_{C}, the dryness fraction x_c at C must first be calculated from the expression $x_c = \left(\phi_{\mathrm{A}} - \phi_{\mathrm{E}} + \dfrac{L_1}{T_1} \right) \dfrac{T_2}{L_2}$ where A and E are the values of the entropy of water at A and E respectively.

There is no need for this calculation, however, if the $H\phi$ chart is used, for H_{B} can be read off direct for the point where the P_1 constant pressure line cuts the saturation line. If a vertical straight line is now drawn from this point to cut the P_2 constant pressure line, the second point so obtained will give the value of H_{C} direct from the chart.

This cycle, called the Rankine cycle, is of great importance, as it has been adopted as the basis of comparison for any real steam engine. Unlike the Carnot cycle, it allows the use of a condenser apart from the working cylinder. Although the Rankine cycle is reversible, the efficiency is less than that of the Carnot cycle.

This can be seen roughly by comparing areas in the $T\phi$ diagrams of Figs. 167 and 169. It will be seen that, although a larger amount of work per pound of steam is obtained, yet the extra amount of heat taken in is in a greater proportion. The reason for this lower efficiency is because all the heat is not taken in at the highest temperature, as in the Carnot cycle, the water taking in heat between the two temperatures T_1 and T_2.

It is interesting to calculate the Rankine efficiency for roughly normal conditions. Let the boiler pressure be 300 lb. per sq. in. absolute and the condenser pressure be 2 lb. per sq. in. absolute.

From the $H\phi$ chart we find the total heat of dry saturated steam at 300 lb. per sq. in. to be 674 C.H.U. per lb. Drawing a vertical straight line from this point on the chart until it cuts the 2 lb. per sq. in. pressure line, we find the total heat is 491 C.H.U. per lb. The work done in the Rankine cycle between these two pressures is therefore $674 - 491 = 183$ C.H.U. per lb.

The total heat of water at the saturation temperature corresponding to a pressure of 2 lb. per sq. in. ($52 \cdot 27° C.$) $= 52 \cdot 16$ C.H.U. per lb. This heat in the water is theoretically available for use again in the boiler.

Hence the Rankine efficiency $= \dfrac{674 - 491}{674 - 52 \cdot 16} = 29 \cdot 4$ per cent.

With an ideal steam plant therefore only $29 \cdot 4$ per cent of the available energy would be converted into work under these conditions. In practice, as will be seen later, even this efficiency could not be reached for the given pressures. The ratio of the actual work done in a steam engine compared to the work done in the corresponding Rankine cycle is termed the efficiency ratio.

Thus efficiency ratio of a steam engine $= \dfrac{\text{Heat converted into work}}{\text{Adiabatic heat drop}}$

The ideal steam consumption for any two pressures is easily obtained from the adiabatic heat drop.

Work done per pound of steam in Rankine cycle

$$= \text{Adiabatic heat drop.}$$

and 1 h.p. hour $= \dfrac{33{,}000 \times 60}{J}$ heat units.

$\therefore$ Steam per horse-power hour $= \dfrac{33{,}000 \times 60}{J \text{ (adiabatic heat drop)}}$ lb.

Suppose an actual engine working with dry steam between the pressures of 300 lb. per sq. in. and 2 lb. had an efficiency ratio of 62 per cent. We have just seen that the Rankine efficiency is $29 \cdot 4$ per cent for these pressures. The actual thermal efficiency of the engine would therefore be $29 \cdot 4 \times 0 \cdot 62 = 18 \cdot 23$ per cent.

The ideal steam consumption would be

$$\frac{33,000 \times 60}{1400 \times 183} = 7 \cdot 73 \text{ lb. per H.P. hour.}$$

The actual steam consumption would be

$$\frac{7 \cdot 73}{0 \cdot 62} = 12 \cdot 47 \text{ lb. per H.P. hour.}$$

Effect of Condensing on the Rankine Cycle. This will be explained best by an actual example. Suppose instead of condensing down to 2 lb. per sq. in. pressure as in the last paragraph, the exhaust is at atmospheric pressure, namely, 14·7 lb. per sq. in. From the $H\phi$ chart we see that the total heat after expanding adiabatically from 300 lb. per sq. in. to 14·7 lb. per sq in. is 550 C.H.U. per lb.

The work done in the Rankine cycle in this case is therefore $674 - 550 = 124$ C.H.U.

The total heat of water at 14·7 lb. per sq. in. $= 100$ C.H.U. per lb.

Hence the Rankine efficiency $= \dfrac{124}{674 - 100} = 21 \cdot 6$ per cent.

The ideal steam consumption $= \dfrac{33,000 \times 60}{1400 \times 124} = 11 \cdot 4$ lb. per H P. hour.

As there is no condenser, the total heat of the water, namely, 100 C.H.U. per lb., would not be directly available in practice for using again in the boiler. In modern plants, however, the heat in the exhaust steam of a non-condensing engine is utilized for feedwater heating, or to do work in a low pressure turbine.

Efficiency of the Rankine Cycle in Terms of Temperature.

Case I. Steam saturated on admission to cylinder.

Let the steam on admission to the cylinder be of dryness fraction x_1. Its state will then be represented by the point B, Fig. 172, and $\dfrac{AB}{AB'} = x_1$ The cycle will be $ABCE$. Let the temperature of the steam on admission be T_1 and on condensation be T_2

Then $AB =$ Gain in entropy when water at T_1 is converted into steam of dryness fraction $x_1 = \dfrac{x_1 L_1}{T_1}$

$EC =$ Gain in entropy when water at T_2 is converted into steam of dryness fraction $x_2 = \dfrac{x_2 L}{T_2}$

Fig. 172

$ED =$ Gain of entropy when water at T_2 is converted into water at T_1

$$= \log_\epsilon \frac{T_1}{T_2}, \text{ approximately.}$$

Area $EAGF =$ Heat taken in when water is heated from T_2 to T_1

$$= T_1 - T_2 \text{ approximately.}$$

Then work done in cycle $=$ Area $ABCE = FEABK - ECKF$

$$= (EAGF + ABKG) - (EDGF + DCKG)$$

$$= (T_1 - T_2) + (AB \times BK) - (DG \times ED) - (AB \times DG)$$

$$= T_1 - T_2 + \frac{x_1 L_1}{T_1} \times T_1 - T_2 \log_\epsilon \frac{T_1}{T_2} - \frac{x_1 L_1}{T_1} T_2$$

$$= (T_1 - T_2)\left(1 + \frac{x_1 L_1}{T_1}\right) - T_2 \log_\epsilon \frac{T_1}{T_2}$$

The heat taken in

$$= \text{Area } EABKF = EAGF + ABKG$$

$$= (T_1 - T_2) + \frac{x_1 L_1}{T_1} \times T_1 = T_1 - T_2 + x_1 L_1$$

Efficiency of cycle $= \dfrac{\text{Work done}}{\text{Heat taken in}}$

$$= \frac{(T_1 - T_2)\left(1 + \frac{x_1 L_1}{T_1}\right) - T_2 \log_\epsilon \frac{T_1}{T_2}}{T_1 - T_2 + x_1 L_1}$$

The dryness fraction after expansion

$$= x_2 = \frac{EC}{ER} = \frac{ED + DC}{ER} = \frac{\log_\epsilon \frac{T_1}{T_2} + \frac{x_1 L_1}{T_1}}{\frac{L_2}{T_2}}$$

If the steam is dry saturated on admission to the cylinder, then the cycle is $AB'C'E$ and $x_1 = 1$ in the above expressions.

Case II. Steam superheated on admission to cylinder.

Let the steam be superheated to a temperature T_{su} at a constant pressure corresponding to the saturation temperature T_1. Then the state of the steam before adiabatic expansion will be represented by B, Fig. 173, on the continuation in the superheat region of the constant pressure line corresponding to T_1. The cycle will then be $ASBCE$.

SW = Increase in entropy when dry saturated steam at T_1 is superheated to T_{su} at constant pressure

$$= C_p \log_\varepsilon \frac{T_{su}}{T_1} \text{ (approx.) where } C_p = \text{Mean specific heat}$$

of steam between T_1 and T_{su}

Area $SBKX$ = Heat taken in when dry saturated steam at T_1 is superheated at constant pressure to T_{su}
$$= C_p(T_{su}-T_1) \text{ (approx.)}$$
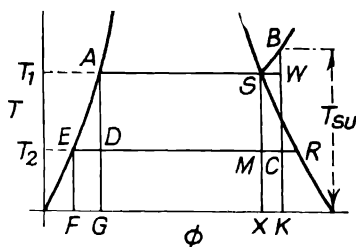
FIG. 173

The work in cycle = Area $ASBCE = EASBKF - ECKF$

$= (EAGF + ASXG + SBKX) - (EDGF + DMXG + MCKX)$

$= (T_1 - T_2) + AS \times SX + C_p(T_{su} - T_1) - DG \times ED - DM \times MX$
$$- CK \times MC$$

$= T_1 - T_2 + \dfrac{L_1}{T_1} \times T_1 + C_p(T_{su} - T_1) - T_2 \log_\varepsilon \dfrac{T_1}{T_2} - \dfrac{L_1}{T_1} T_2$
$$- T_2 C_p \log_\varepsilon \frac{T_{su}}{T_1}$$

$= (T_1 - T_2)\left(1 + \dfrac{L_1}{T_1}\right) + C_p(T_{su} - T_1) - T_2\left(\log_\varepsilon \dfrac{T_1}{T_2} + C_p \log_\varepsilon \dfrac{T_{su}}{T_1}\right)$

The heat taken in = Area $EASBKF = EAGF + ASXG + SBKX$

$= T_1 - T_2 + L_1 + C_p(T_{su} - T_1)$

∴ Efficiency of cycle

$$= \frac{(T_1 - T_2)\left(1 + \dfrac{L_1}{T_1}\right) + C_p(T_{su} - T_1) - T_2\left(\log_\varepsilon \dfrac{T_1}{T_2} + C_p \log_\varepsilon \dfrac{T_{su}}{T_1}\right)}{T_1 - T_2 + L_1 + C_p(T_{su} - T_1)}$$

If a comparison is made of the efficiency of a Rankine cycle using in one case dry saturated steam and in the other superheated steam between the same pressures, it will be found that there is very little gain in thermodynamic efficiency due to the superheat.

The chief value of superheating is due to practical considerations resulting from the fact that as the steam in transmission loses heat, condensation does not take place.

Example 2. Sketch the pressure volume and temperature entropy diagrams of the Rankin cycle for a steam engine. Determine the Rankine efficiency of a steam engine working between pressure limits of 100 and 5 lb. per sq. in., and supplied with dry saturated steam. By how much per cent is this efficiency increased by superheating the steam 100° C. ? What effect would superheating produce on the efficiency ratio of an actual steam engine, and what are the reasons for this effect ?　　　　(*U.L., B.Sc. (Eng.), 1924.*)

(1) Total heat of dry steam at 100 lb. per sq. in. absolute $= H_B$
$= 661 \cdot 83$ lb. calories (from tables or $H\phi$ chart).

Dryness fraction after adiabatic expansion to 5 lb. per sq. in. abs.

$$= \begin{pmatrix} \text{Entropy of} & \text{Entropy of} & \text{Latent heat at} \\ \text{water at} & \text{water at} & \dfrac{100 \text{ lb. per sq.in.}}{\text{Saturation tem-}} \\ 100 \text{ lb.} & 5 \text{ lb.} & + \text{Saturation tem-} \\ \text{per sq. in.} & \text{per sq. in.} & \text{perature at} \\ & & 100 \text{ lb. per sq. in.} \end{pmatrix} \times \dfrac{\text{Saturation tempera-}}{\text{Latent heat at}}$$
$$\text{ture at 5 lb. per sq. in.}$$
$$5 \text{ lb. per sq. in.}$$

$$= \left(\phi_{w1} - \phi_{w2} + \frac{L_1}{T_1} \right) \frac{T_2}{L_2}$$

From the tables $\phi_{w1} - \phi_{w2} = 0 \cdot 4739 - 0 \cdot 2346$
$$= 0 \cdot 2393$$

In the absence of tables an approximate value can be calculated

from $2 \cdot 3026 \times \log_{10} \dfrac{T_1}{T_2} = 2 \cdot 3026 \times \log_{10} \dfrac{437 \cdot 38}{345 \cdot 48} = 0 \cdot 236$

Taking the first value—

Dryness fraction $= \left(0 \cdot 2393 + \dfrac{496 \cdot 12}{437 \cdot 38} \right) \times \dfrac{345 \cdot 48}{555 \cdot 38} = 0 \cdot 855$

The dryness fraction can be found directly from the $H\phi$ chart. Taking a vertical line down from the intersection of the 100 lb. per sq. in. line with the saturation line, we see that the 5 lb. per sq. in. line is cut at a dryness fraction of $0 \cdot 855$.

Total heat of steam after adiabatic expansion to 5 lb. per sq. in.
$$= H_c = h + xL$$
$$= 72 \cdot 26 + 0 \cdot 855 \times 555 \cdot 38 = 547 \cdot 11 \text{ lb. calories.}$$

The value of H_c can also be obtained from the $H\phi$ diagram, the point on the 5 lb. per sq. in. line of dryness fraction $0 \cdot 855$ having a total heat of $547 \cdot 5$ lb. calories.

The total heat of water at 5 lb. per sq. in. $= h = 72 \cdot 26$ (from tables).

$\therefore$ Rankine efficiency $= \dfrac{\text{Adiabatic heat drop}}{\text{Heat put in}} = \dfrac{H_B - H_c}{H_B - h}$

$$= \frac{661 \cdot 83 - 547 \cdot 11}{661 \cdot 83 - 72 \cdot 26} \times 100 = 19 \cdot 4 \text{ per cent.}$$

(2) The saturation temperature at 100 lb. per sq. in. $= 164\cdot28°$ C.

$\therefore$ Temperature of steam at 100 lb. per sq. in. superheated 100° C.
$$= 264\cdot28° \text{ C.}$$

From the point on the $H\phi$ chart, where the 100 lb. per sq. in. line is cut by the constant temperature line corresponding to $264\cdot28°$ C., we find that total heat before expansion $= 715$ lb. calories.

Passing vertically down from this point until we cut the 5 lb. per sq. in. line, the point so obtained gives total heat after expansion $= 584$ lb. calories.

Hence, Rankine efficiency with superheated steam

$$= \frac{715 - 584}{715 - 72} \times 100 = 20\cdot4 \text{ per cent.}$$

$\therefore$ Gain in efficiency due to superheating $= 20\cdot4 - 19\cdot4 = 1$ per cent.
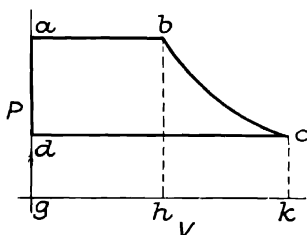
Rankine Cycle in Terms of Pressure and Volume. The work done in the Rankine cycle may be expressed approximately in terms of pressure and volume. The result, which will be in foot-lb. per lb. of steam, will be approximate, for the assumption is made that, in the adiabatic expansion law of steam, namely, $pv^n = $ constant, the value of n is constant throughout the expansion.

Let $abcd$, Fig. 174, represent the Rankine cycle for 1 lb. of steam (the feed-pump work being neglected).

The work done $=$ area $abcd = abhg + bckh - dckg$

$$= p_b v_b + \frac{p_b v_b - p_c v_c}{n - 1} - p_c v_c$$

$$= \frac{n}{n - 1}(p_b v_b - p_c v_c)$$

$$= \frac{n}{n - 1} p_b v_b \left(1 - \frac{p_c v_c}{p_b v_b}\right)$$

But $p_b v_b{}^n = p_c v_c{}^n$ $\therefore \dfrac{v_c}{v_b} = \left(\dfrac{p_b}{p_c}\right)^{\frac{1}{n}}$

$\therefore$ Work done $= \dfrac{n}{n - 1} p_b v_b \left\{ 1 - \dfrac{p_c}{p_b} \times \left(\dfrac{p_b}{p_c}\right)^{\frac{1}{n}} \right\}$

$$= \frac{n}{n - 1} p_b v_b \left\{ 1 - \left(\frac{p_b}{p_c}\right)^{\frac{1}{n} - \frac{n}{n}} \right\}$$

Hence if 1 lb. of steam of specific volume v_1 before the adiabatic expansion be caused to pass through a Rankine cycle between the

pressures p_1 and p_2, the work done is given approximately by the expression

$$\frac{n}{n-1} p_1 v_1 \left\{ 1 - \left(\frac{p_1}{p_2}\right)^{\frac{1-n}{n}} \right\} = \frac{n}{n-1} p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \right\}$$

Another method of obtaining this expression is as follows—

The area $abcd$ is measured by $\displaystyle\int_{P_c}^{P_b} V \, dP$

$$= \int \left(\frac{K}{P}\right)^{\frac{1}{n}} dP \quad \text{where} \quad PV^n = K$$

$$= K^{\frac{1}{n}} \int_{P_c}^{P_b} P^{-\frac{1}{n}} dP = K^{\frac{1}{n}} \times \left(\frac{P^{1-\frac{1}{n}}}{1-\frac{1}{n}} \right)_{P_c}^{P_b}$$

$$= K^{\frac{1}{n}} \times \frac{n}{n-1} \left(P_b^{\frac{n-1}{n}} - P_c^{\frac{n-1}{n}} \right)$$

But $K^{\frac{1}{n}} = P_b^{\frac{1}{n}} V_b$ $\therefore$ $abcd = \dfrac{n}{n-1} P_b^{\frac{1}{n}} V_b \left\{ P_b^{\frac{n-1}{n}} - P_c^{\frac{n-1}{n}} \right\}$

$$= \frac{n}{n-1} P_b V_b \left\{ 1 - \left(\frac{P_c}{P_b}\right)^{\frac{n-1}{n}} \right\}$$

Incomplete Expansion. In the Rankine cycle the steam is expanded right down to the pressure at which condensation takes
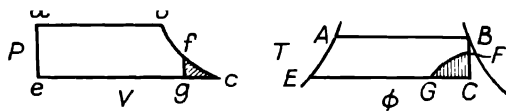
Fig. 175

place, as at C, Fig. 175. If the steam is released from the cylinder at a higher pressure than that of the condenser, as at f, there will be a sudden drop of pressure fg at constant volume, the amount of work fcg being lost. The heat taken in being the same as for complete expansion, there is a corresponding loss in efficiency of the cycle. In order to show the effect of incomplete expansion on the $T\phi$ diagram, we note that the drop of pressure fg on release takes place at constant volume, that is, instantaneously while the piston in the ideal engine is at the end of its expansion stroke. In the ideal Rankine cycle, where the whole operation takes place within the cylinder, the same effect as the above will be produced if a certain amount of the steam is condensed by removing heat while the piston is stationary at the end of the expansion stroke. The state of the steam in the cylinder would pass along the constant

volume line FG in the $T\phi$ diagram.   The cycle would therefore be
represented by $ABFGE$, the loss of available work due to incomplete
expansion being represented by the area FCG, this loss of work
being equivalent to the extra amount of heat given up to the
condenser.

The Theoretical Indicator Diagram.   In practice, the expansion in
a steam engine is not complete.   The volume of the cylinder neces-
sary would be excessive, as the specific volume of steam is large at
low pressures compared to that at higher pressures.   In practical
cases, therefore, the pressure at release is greater than the pressure
at the exhaust pipe and approximates to the cycle *abfge*, Fig. 175.

Now in practice there are, among others, two important modifica-
tions to be considered.   In the first place, when the piston is at
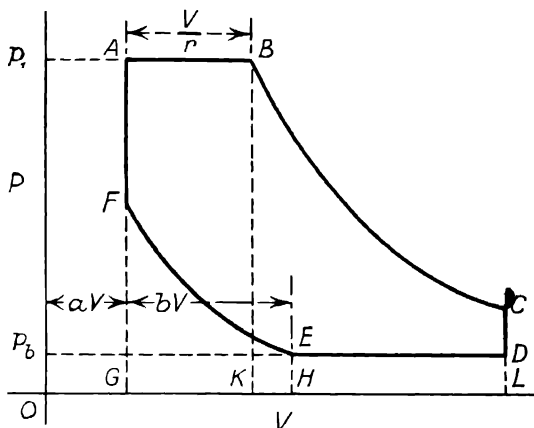
Fig. 176

either end of the stroke there is the clearance volume, that is, the
volume of working substance contained between the piston and the
cylinder end, and in the ports leading from the cylinder to the
valve chest.   In the second place, the working substance is not
pushed out to exhaust for the whole of the return stroke, but a
certain amount is compressed into the clearance space.   The
theoretical indicator diagram so modified is shown in Fig. 176.

In the diagram, OG represents the clearance volume, AB shows
the admission of steam to the cylinder, BC the expansion which is
assumed hyperbolic for this purpose, CD is the pressure drop at
release, while DE is the exhaust and EF the compression portions
of the return stroke.   After compression the pressure is represented
by GF.   Admission of fresh steam at F then raises the pressure
to the initial·pressure GA.

Mean Effective Pressure from the Theoretical Diagram.   Let the
stroke·volume $GL = V$;   clearance volume $OG = aV$;   compression

volume $GH = bV$; admission volume, $AB = \dfrac{V}{r}$; initial pressure,

$AG = p_1$; back pressure $DL = p_b$.

Then, as the area $ABCDEF$ represents the work done during the cycle, the mean effective pressure

$$p_m = \frac{\text{Area } ABCDEF}{V}, \text{ (the area being measured in } p \text{ and } v \text{ units)}$$

$$= \frac{ABKG + BCLK - EDLH - FEHG}{V}$$

$$= \frac{(BK \times AB) + \left(BK \times OK \log_\varepsilon \dfrac{OL}{OK}\right)}{V} -$$

$$\frac{(DL \times HL) + EH \times OH \log_\varepsilon \dfrac{OH}{OG}}{V}$$

$$= \frac{p_1 \dfrac{V}{r} + p_1 \left(aV + \dfrac{V}{r}\right) \log_\varepsilon \dfrac{V + aV}{aV + \dfrac{V}{r}}}{V} -$$

$$\frac{p_b(V - bV) + p_b(aV + bV) \log_\varepsilon \dfrac{aV + bV}{aV}}{V}$$

$$= p_1 \left(\frac{1}{r} + \left(a + \frac{1}{r}\right) \log_\varepsilon \frac{1 + a}{a + \dfrac{1}{r}}\right) - p_b \left(1 - b + (a + b) \log_\varepsilon \frac{a + b}{a}\right)$$

If there is no compression, then $b = 0$.

and $p_m = p_1 \left(\dfrac{1}{r} + \left(a + \dfrac{1}{r}\right) \log_\varepsilon \dfrac{1 + a}{a + \dfrac{1}{r}}\right) - p_b$

If there is no compression and no clearance, then $a = 0$ and $b = 0$

and $p_m = \dfrac{p_1}{r} (1 + \log_\varepsilon r) - p_b$.

Steam Consumption from the Theoretical Diagram (the Willans Line). At the point of cut-off B the volume of steam in the cylinder is $\left(aV + \dfrac{V}{r}\right)$. If W_1 is the weight of a cubic foot of steam at pressure p_1, then the weight of steam in the cylinder at cut-off is $\left(a + \dfrac{1}{r}\right) V W_1$

At the point of compression E, the volume of steam in a cylinder is

$(aV + bV)$. If W_b is the weight of a cubic foot of steam at the pressure p_b, then the weight of steam in the cylinder at the beginning of compression is $(a + b)VW_b$.

The weight of steam used per stroke is therefore

$$\left(a + \frac{1}{r}\right)VW_1 - (a + b)VW_b$$

Assuming for a double-acting engine that the diagrams for each end of the cylinder are the same, then, if the engine is running at N r.p.m., the steam consumption in pounds per hour is given by

$$W = 2N \times 60 \times V\left(\left(a + \frac{1}{r}\right)W_1 - (a + b)W_b\right)$$

If the assumption is made that steam is dry at the points B and E, the values of W_1 and W_b can be obtained from the steam tables.

Now a trial calculation will show that, for ordinary ranges of pressure, the density of steam is a linear function of the pressure, that is, $W = C_1 + C_2 p$, where C_1 and C_2 are constants.

Hence the steam consumption W may be written

$$W = 2N \times 60 \times V\left\{\left(a + \frac{1}{r}\right)(C_1 + C_2 p_1) - (a + b)(C_1 + C_2 p_b)\right\}$$

If the engine is governed by throttling, that is, by varying p_1 while the speed N and the cut-off are kept constant, then

$$W = C_3 p_1 + C_4, \text{ where } C_3 \text{ and } C_4 \text{ are constants.}$$

With the same assumption, the mean effective pressure may be written $p_m = C_5 p_1 - C_6$ where C_5 and C_6 are constants.

But I.H.P. $= \dfrac{p_m LA 2N}{33,000}$ $\therefore$ I.H.P. $= C_7 p_m = C_8 p_1 - C_9$

$$\therefore p_1 = \frac{\text{I.H.P.} + C_9}{C_8}$$

Hence $W = C_3 p_1 + C_4 = \dfrac{C_3(\text{I.H.P.} + C_9)}{C_8} + C_4$

i.e. $W = (A \times \text{I.H.P.}) + B$, where A and B are constants.

That is, the steam consumption is a linear function of the indicated horse-power.

The Effect of Governing on the Theoretical Indicator Diagram.

A reciprocating steam engine may be governed by means of a throttle valve, or by alteration of the cut-off. It is instructive to consider the effect of these two methods, by use of the equations given in the last two paragraphs. The matter will best be made clear by means of a numerical example. Assume the following conditions for a double-acting engine—

Piston speed $= 2LN = 500$ ft. per min. ; area of piston $A = 80$ sq. in. ; stroke $L = 15$ in.

$$\therefore \text{Speed } N = \frac{500 \times 12}{2 \times 15} = 200 \text{ r.p.m.}$$

$$\text{Stroke volume } V = LA = \frac{15}{12} \times \frac{80}{144} = 0 \cdot 695 \text{ cub. ft.}$$

Initial pressure, $p_1 = 100$ lb. per sq. in. abs. ; back pressure $p_b = 15$ lb. per sq. in. abs. ; ratio of expansion $r = 3$; clearance volume $= 0 \cdot 05 \times$ stroke volume, where $a = 0 \cdot 05$. Zero compression assumed.

Mean effective pressure

$$p_m = 100 \left\{ 0 \cdot 333 + (0 \cdot 05 + 0 \cdot 333) \, 2 \cdot 3 \log_{10} \frac{1 \cdot 05}{0 \cdot 333 + 0 \cdot 05} \right\} - 15$$

$$= 56 \cdot 9 \text{ lb. per sq. in. absolute}$$

$$\text{I.H.P.} = \frac{p_m \, LA \, 2N}{33,000} = \frac{56 \cdot 9 \times 80 \times 500}{33,000} = 68 \cdot 9$$

$$\text{Steam consumption} = W = 2N \times 60V \left\{ \left(a + \frac{1}{r} \right) w_1 - a w_b \right\}$$

$$= 2 \times 200 \times 60 \times 0 \cdot 695 \left(\frac{0 \cdot 05 + 0 \cdot 333}{4 \cdot 451} - \frac{0 \cdot 05}{26 \cdot 27} \right)$$

$$= 1400 \text{ lb. per hour.}$$

Case I. Throttle Governing. Values of p_m, I.H.P., and W may be calculated for various values of the initial pressure p_1, all other conditions being kept constant. In this way the following figures will be obtained—

p_1	p_m	I.H.P.	W (lb. per hr)
100	56 9	68 9	1400
80	42 5	51·4	1135
70	35 3	42·7	1000
60	28·1	34	852
40	13 8	16·7	584

Case II. Cut-off Governing. Values of p_m, I.H.P. and W may be calculated for various values of the expansion ratio r with the constant value $p_1 = 100$ and all other conditions constant.

In this way the following figures are obtained—

$\frac{1}{r}$	p_m	I.H.P.	W (lb. per hr.)
0·333	56·9	68·9	1400
0·25	47·6	57·5	1100
0·20	40·8	49·4	903
0·15	33·1	40	716
0·1	24·2	29·3	517

Corresponding values of W and I.H.P. can be plotted as in Fig. 177, which clearly indicates the theoretical advantage of cut-off governing over throttle governing.

The Actual Indicator Diagram.   We will now consider how the actual indicator diagram varies from the ideal.   If the boiler pressure is represented by the height of the line AB, Fig. 178, the pressure in the cylinder at the beginning of the stroke will be somewhat less, as at F, because a certain difference of pressure is necessary to produce a flow of steam from the boiler to the engine cylinder.   As the stroke proceeds there will generally be a slight fall of pressure, as shown at FG, owing to wire-drawing through the
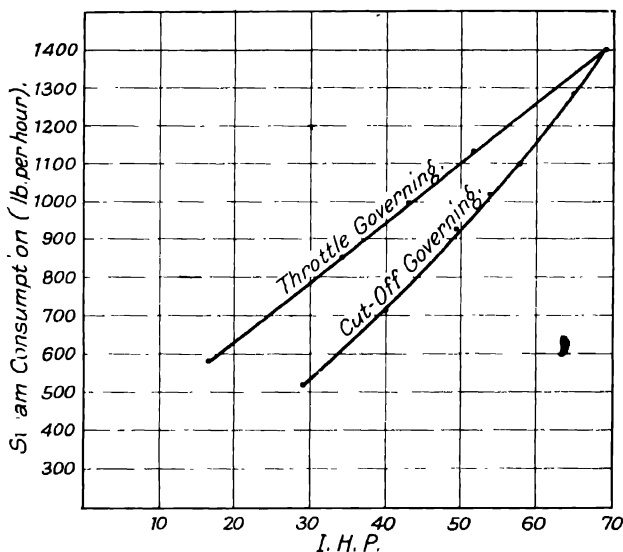
FIG. 177

steam ports.   The point of cut-off will not be sharp but rounded off as at G.   This wire-drawing at cut-off will be more marked in the case of a slide valve, where cut-off is not instantaneous, than in the case of a Corliss or a drop valve provided with a trip mechanism, where cut-off is performed much more quickly.   Release must occur at H before the end of the stroke, owing to wire-drawing through the exhaust ports, thus causing a rounding off of the toe of the diagram.   During the exhaust stroke the pressure in the cylinder will be somewhat higher than that of the condenser in the case of a condensing engine, and somewhat higher than atmospheric pressure in the case of a non-condensing engine.   The exhaust valve closes at some point K, before the end of the exhaust stroke.   The reason for this is twofold.   In the first case the steam left in the cylinder forms a cushion, which gradually brings the piston to rest

and thus obviates the shock on the bearings which would otherwise be produced. In the second case, the resulting increase of pressure reduces the amount of live steam necessary to bring the pressure at the beginning of the working stroke up to the admission pressure. Owing to wire-drawing effects, the live steam is admitted just before the end of the exhaust stroke at *L*, the pressure produced by cushioning up to this point being thus raised to the admission pressure by the time the piston has arrived at the end of the exhaust stroke.

The effect of wire-drawing is obviously to decrease the area of the diagram, that is, the work done by the engine is decreased. This,
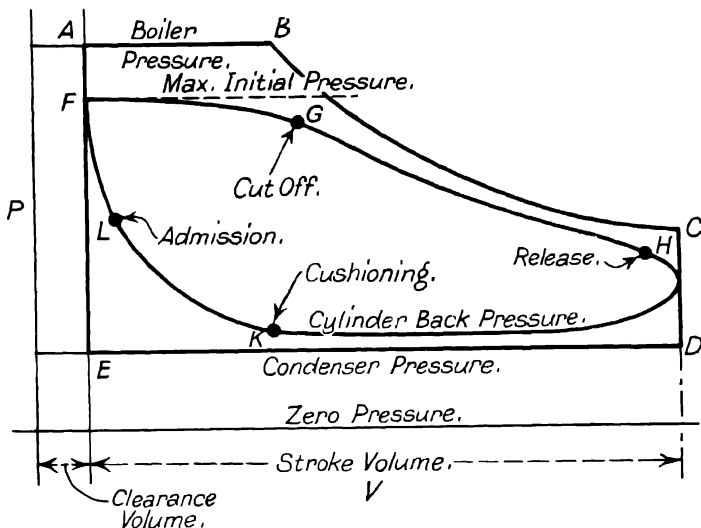
Fig 178

however, is to some extent compensated for by the fact that wire-drawing or throttling dries the steam slightly.

The effect of clearance would, at first sight, appear to be simply to increase the steam consumption, as before the steam at admission can do work it must raise the pressure from that at the point *L* to that at the point *F*, but from p. 453 we can see, that clearance increases the mean effective pressure and thus increases the work done. The net effect of clearance, however, is to decrease the efficiency. The increase in steam consumption due to clearance can be reduced by making the point of cushioning earlier, and thus increasing the pressure obtained at *L* when the live steam is admitted to the cylinder Earlier cushioning, however, obviously decreases the area of the diagram, that is, the work done is lessened.

We will now discuss the admission and expansion curve *FGH*.

During the exhaust stroke the cylinder and piston have been in contact with steam at a low pressure and a corresponding lower saturation temperature than the temperature of the live steam. If the steam on admission is dry saturated it must give up heat to the walls of the clearance space, and hence a certain amount of initial condensation takes place. This initial condensation will, of course, be greater the larger the amount of clearance surface exposed to the incoming steam. The condensation of the steam continues for a short time after cut-off, due to the fact that energy is being ' taken out of the steam as work is done. As the expansion proceeds the pressure drops and hence the saturation temperature falls, eventually below the temperature of the cylinder walls, thus causing re-evaporation to take place during the latter portion of the working stroke. This re-evaporation is increased at release, due to the sudden pressure drop and the consequent drop in saturation temperature at the end of the stroke. This re-evaporation may be enough to make the steam left in the cylinder during the exhaust stroke practically dry. If the steam were admitted to the cylinder in a superheated condition this effect of condensation and re-evaporation would be modified, for the cooling effect of the clearance surface would be to lower the temperature of the steam and not to condense it until the saturation temperature was reached.

The Diagram Factor k. From the previous paragraph it is obvious that the ideal mean effective pressure obtained from the expression

$$p_m = p_1 \left(\frac{1}{r} + \left(a + \frac{1}{r} \right) \log_e \frac{1 + a}{\frac{1}{r} + a} \right) - p_b \left\{ 1 - b + (a + b) \log_e \frac{a + b}{a} \right\}$$

will in general be greater than the actual mean effective pressure obtained in an engine. In order to obtain the probable actual mean effective pressure p_a, we must multiply the ideal value p_m by a factor k. Thus $p_a = k p_m$.

The value of the diagram factor to be used in any particular case depends, as we have seen in the last paragraph, on a number of considerations. The particular formula used for calculating p_m will affect the value of k. Again, the value will depend on the condition of the steam, the initial pressure, the back pressure, the speed of the engine, the type of engine, the type of valves, whether the cylinder is jacketed or not, etc.

An average value for k for a simple slide valve condensing engine is about 0·8; for a compound condensing engine 0·7, for a compound condensing engine using superheated steam 0·65.

Example 3. A double-acting steam engine has a single cylinder of diameter 33¼ in. by 3·25 ft. stroke, and develops 600 I.H.P. at 100 r.p.m. Assuming a diagram factor of 0·82, find the ratio of expansion if the initial steam pressure is 155 lb. per sq. in. absolute and the back pressure 2 lb. per sq. in. absolute.

In this case a rough estimate may be made by assuming no clearance and no compression.

The actual M.E.P., or p_a, is given by

$$\text{Indicated horse-power} = \frac{p_a l A 2 N}{33,000},$$

where $N =$ the revolutions per minute for a double-acting engine.

$$\therefore p_a = \frac{600 \times 33,000}{3 \cdot 25 \times 200 \times 961 \cdot 8} = 35 \cdot 3$$

Also, $\qquad p_a = k \left\{ \frac{p_1}{r} (1 + \log \varepsilon\ r) - p_b \right\},$

where k is the diagram factor.

$$\therefore \quad \frac{35 \cdot 3}{0 \cdot 82} \quad 43 = \frac{155}{r} (1 + \log_\varepsilon r) - 2$$

$$\therefore \quad 155 + 155 \log_\varepsilon r = 45 r$$

$$31 + 31 \times 2 \cdot 3 \log_{10} r = 9 r$$

and $\qquad\qquad \log_{10} r = \frac{9r - 31}{71 \cdot 3} = x.$

Take different values of r and plot values of $\log r$ and x, and so find $r = 12$, which makes $\log r = x$. Therefore ratio of expansion $r = 12$.

Example 4. A non-condensing steam engine with a single cylinder has to develop 75 B.H.P. Determine from the data given below the diameter of the cylinder and the stroke of the piston.

(1) Boiler pressure, 80 lb. by gauge.
(2) Cut-off steam occurs at 28 per cent of piston stroke.
(3) Clearance volume, 6·5 per cent of volume swept by piston.
(4) Piston speed, 750 ft. per min.
(5) Diagram factor, 0·88.

You are to assume hyperbolic expansion and that the stroke of the piston is 1·5 the diameter. (*U.L., B.Sc. (Eng.), 1917.*)

Let atmospheric pressure be 15 lb. per sq. in. absolute.

Then boiler pressure $p_1 = 80 + 15 = 95$ lb. per sq. in. absolute.

$$\text{Mean effective pressure} = p_m = p_1 \left\{ \frac{1}{r} + \left(a + \frac{1}{r} \right) \log_\varepsilon \frac{1 + a}{\frac{1}{r} + a} \right\} - p_b$$

$$= 95 \left\{ 0 \cdot 28 + (0 \cdot 065 + 0 \cdot 28) 2 \cdot 3 \log_{10} \frac{1 \cdot 065}{28 + 0 \cdot 065} \right\} - 15$$

$$= 48 \cdot 5 \text{ lb. per sq. in.}$$

$\therefore$ Actual M.E.P. $= p_a = 0 \cdot 88 \times 48 \cdot 5 = 42 \cdot 7$ lb. per sq. in.

Piston speed $= 2 l N = 750$ ft. per min. (for a double-acting engine).

If we assume a mechanical efficiency of 90 per cent, then

$$\text{I.H.P.} = \frac{\text{B.H.P.}}{\text{M.E.}} = \frac{75}{0\cdot9} = 83, \text{ say.}$$

Now $\text{I.H.P.} = \dfrac{p_a LA2N}{33,000}$; $\therefore A = \dfrac{33,000 \times 83}{42\cdot7 \times 750} = 85\cdot5$ sq. in.

$\therefore$ Diameter $= \sqrt{\dfrac{4 \times 85\cdot5}{3\cdot14}} = 10\cdot4$ in.

$\therefore$ Stroke $= 1\cdot5 \times 10\cdot4 = 15\cdot6$ in.

Application of the Saturation Curve to the Indicator Diagram (the Missing Quantity). The weight of working substance (steam
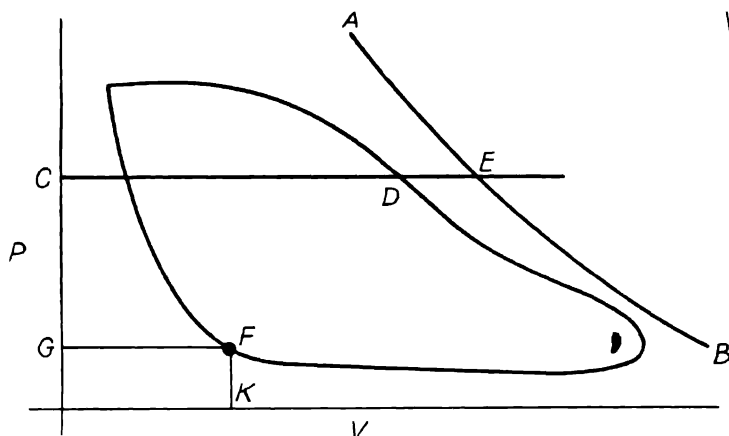
FIG. 179

and water) in the cylinder at any point between cut-off and release is made up of the clearance steam and the stuff which is admitted to the cylinder between admission and cut-off, that is :

Weight of stuff = weight of clearance steam + cylinder feed,

where the cylinder feed is the weight of stuff passing through the cylinder per stroke.

If a pressure volume curve for this weight of dry saturated steam were plotted from the steam tables on to the indicator diagram, some such curve as AB, Fig. 179, would be obtained, the reason being that the stuff in the cylinder would not in general be dry saturated.

The volume of the water is negligible compared to that of the steam, and hence CD represents the volume of steam in the cylinder at the point D. Were all the stuff in the cylinder at this point dry saturated steam, its volume would be equal to CE. Hence the dryness fraction of the working substance in the cylinder at the

point D is measured by $\dfrac{CD}{CE}$ It will thus be evident that the indicator diagram, while accounting for that part of the working substance which is steam, gives no indication of how much water is present, unless the saturation curve is drawn on it.   The volume of steam represented by DE is called the missing quantity.   To apply the saturation curve we must know the weight of the clearance steam and the weight of the cylinder feed.

The weight of the clearance steam can be found from the indicator diagram as follows :   It has been explained that just after the beginning of cushioning the steam left in the cylinder is practically dry.   Let F, Fig. 179, be a point just after cushioning.   Then GF is the volume of the clearance steam at a pressure represented by FK.   Hence if W = weight of 1 cub. ft. of dry steam at the pressure represented by FK, then

Weight of clearance steam = (Volume represented by GF) $\times$ W

Steam tables may not always give values of W, but they generally give values of V, the volume of 1 lb. of dry saturated steam at various pressures.   Obviously then, weight of clearance steam

$$= \frac{\text{Volume } (GF)}{V}.$$

If the cylinder feed is now known, the saturation curve AB for a weight of dry saturated steam equal to weight of (clearance steam + cylinder feed) can be drawn on the indicator diagram.

If all the steam that passed into the steam chest of the engine also passed through the cylinder, and none leaked past the valve or the piston, then the cylinder feed could be measured by weighing the condensate in the case of a condensing engine.   Experimental evidence, however, goes to show that there is a certain amount of leakage past the valve and the piston, and hence, in this case, if the saturation line AB be drawn from the weight of condensate, then the missing quantity DE will not only be due to condensation but also to leakage.   In this case, $\dfrac{CD}{CE}$ will no longer accurately represent the dryness fraction in the cylinder.

Example 5.   The following results were obtained by measurements taken on an indicator card from a double-acting steam engine.

(1) Immediately after cut-off -
      Volume = 3·37 cub. ft. ;   pressure = 178 lb. per sq. in. absolute.

(2) Immediately after compression has begun—
      Volume = 1·03 cub. ft. ;   pressure = 57 lb. per sq. in. absolute.

The speed of the engine was 105 r.p.m. and the measured steam supply per minute was 405·3 lb.   Find the missing quantity in pounds per minute.

(*U.L., B.Sc. (Eng.*), 1917.)

Volume of 1 lb. of dry steam at 57 lb. per sq. in.
        = 7·539 cub. ft. (from tables)

$$\text{weight of cushion steam} = \frac{1\cdot03}{7\cdot539}$$

$$= 0\cdot137 \; lb. \; (\text{assuming steam is dry})$$

Weight of steam passing through cylinder per stroke

$$= \frac{405\cdot3}{2 \times 105} = 1\cdot93 \; lb.$$

∴ weight of stuff in cylinder just after cut-off
$$= 1\cdot93 + 0\cdot137 = 2\cdot067 \; lb.$$

Volume of 1 lb. of dry steam at 178 lb. per sq. in.
$$= 2\cdot59 \; cub. \; ft. \; (\text{from tables})$$

∴ Indicated weight of steam just after cut-off $= \dfrac{3\cdot37}{2\cdot59} = 1\cdot301 \; 1$

∴ missing quantity $= 2\cdot067 - 1\cdot301 = 0\cdot766 \; lb. \; \text{per stroke}$
$$= 0\cdot766 \times 2 \times 105 = 161 \; lb. \; \text{per min.}$$

It should be noted that this is the missing quantity at the
beginning of the expansion. Owing to re-evaporation during expan-
sion the missing quantity at
the end of expansion would
generally become less. This
quantity could be calculated
if the indicated pressure and
volume just before release were
known.

Fig. 180
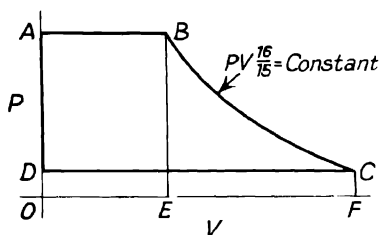
The Steam Jacket. We have
seen in a previous paragraph
that the steam in a steam
engine cylinder is wet during
expansion. This condensation can be reduced to some extent by
steam jacketing the cylinder. It must be remembered, however,
that although by this means the actual expansion curve on the
indicator diagram can be brought nearer to the saturation curve,
steam must be used to pass through the jacket.

Let us first consider the theoretical case where dry saturated
steam is supplied to the engine and this steam is kept dry throughout
the cycle by means of a steam jacket. The law of the expansion
curve can be taken as $PV^{\frac{16}{15}} = \text{constant}$.

Let $ABCD$, Fig. 180, be the cycle, there being no clearance
Then, work done during the cycle $=$ Area $ABCD$

$$= ABEO + EBCF - DCFO = P_{\text{B}}V_{\text{B}} \frac{P_{\text{B}}V_{\text{B}} - P_{\text{c}}V_{\text{c}}}{\frac{16}{15} - 1} - P_{\text{c}}V_{\text{c}}$$

$$= 16(P_B V_B - P_C V_C) \text{ work units} = \frac{16}{J}(P_B V_B - P_C V_C) \text{ heat units.}$$

The heat taken from the boiler $= $ Total heat at $B = H_B$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (measured from 0° C.)
The heat taken in from jacket $= Q$
The total heat rejected $\qquad = $ Total heat at $C = H_C$
$\therefore$ net heat taken in $\qquad = H_B + Q - H_C$

$$\therefore H_B + Q - H_C = \frac{16(P_B V_B - P_C V_C)}{J}$$

$\therefore$ heat taken in from jacket during cycle is

$$Q = \frac{16(P_B V_B - P_C V_C)}{J} + H_C - H_B$$

Let $Q' = $ heat taken in from jacket during expansion. Then work
done during expansion $= \dfrac{P_B V_B - P_C V_C}{\dfrac{16}{15}} = 15(P_B V_B - P_C V_C)$

Heat in steam at $B = H_B$; and heat in steam at $C = H_C$

$$\therefore Q' + H_B = \frac{15(P_B V_B - P_C V_C)}{J} + H_C$$

$$\therefore \qquad Q' = \frac{15(P_B V_B - P_C V_C)}{J} + H_C - H_B$$

In the above equations the values must be calculated for the total amount of steam in the cylinder. In this case, as there is no clearance, this will be the weight of cylinder feed.

In practice, even if steam were dry on entering the cylinder, it would be wet at cut-off owing to initial condensation. The clearance steam can be found from the indicator diagram, and if the cylinder feed is known, then the total weight of stuff in the cylinder can be obtained, and hence the dryness fraction just after cut-off and just before release can be calculated. The values of pressure and volume just after cut-off and just before release being found from the indicator diagram, the value of n in the expansion law $PV^n = $ constant can be calculated. Values of total heat of the stuff at cut-off and release are calculated from the equation $H = h + xL$, and hence the amount of heat supplied by the jacket during expansion can be found.

Example 6. The pressure indicated at cut-off in the cylinder of a jacketed steam engine is 75 lb. per sq. in. absolute, the volume is 0·45 cub. ft. and the dryness fraction of the steam is 0·73. If the values of pressure and volume at release are 34·2 lb. per sq. in. and 1·05 cub. ft., find the heat which passes through the cylinder walls during expansion. Assume $PV^n = $ constant.

Volume of 1 lb. of dry steam at 75 lb. per sq. in. absolute is 5·83 cub. ft.

∴ indicated weight of steam at cut-off is

$$\frac{\text{Indicated volume at cut-off}}{5\cdot83} = \frac{0\cdot45}{5\cdot83} \quad 0\cdot077 \text{ lb.}$$

∴ weight of stuff in cylinder during expansion is

$$\frac{\text{Indicated weight of steam}}{\text{Dryness fraction}} = \frac{0\cdot077}{0\cdot73} = 0\cdot105 \text{ lb.}$$

Volume of 1 lb. of dry steam at 34·2 lb. per sq. in. absolute is 12·16 cub. ft.

∴ volume of stuff in cylinder at release if all were dry is

$$12\cdot16 \times 0\cdot105 = 1\cdot277 \text{ lb.}$$

Dryness fraction at release is

$$\frac{\text{Indicated volume of steam}}{\text{Volume of stuff if it were all dry}} \quad \frac{1\cdot05}{1\cdot277} = 0\cdot823$$

From the steam tables the following values are obtained–

p (lb. per sq. in. absolute)				h (C.H.U.)	L (C.H.U.)	(dryness fraction)
75	,,	,,	,,	154·07	504·38	0·73
34·2	,,	,,	,,	125·82	523·26	0·823

∴ Heat in steam just after cut-off

$$= 0\cdot105(154\cdot07 + 0\cdot73 \times 504\cdot38) = 54\cdot84 \text{ lb. calories.}$$

Heat in steam just before release

$$= 0\cdot105(125\cdot82 + 0\cdot823 \times 523\cdot26) = 58\cdot43 \text{ lb. calories.}$$

Now the value of n in the law for expansion $PV^n = \text{constant}$ is found from

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} = \frac{\log 75 - \log 34\cdot2}{\log 1\cdot05 - \log 0\cdot45} = 0\cdot927$$

∴ Work done during expansion

$$= \frac{144(75 \times 0\cdot45 - 1\cdot05 \times 34\cdot2)}{1400(0\cdot927 - 1)} = 3\cdot04 \text{ lb. calories.}$$

Heat in steam just after cut-off + Heat supplied from steam jacket = Heat in steam just before release + Heat equivalent of work done.

∴ Heat supplied from steam jacket is

$$58\cdot43 + 3\cdot04 - 54\cdot84 = 6\cdot6 \text{ lb. calories.}$$

The Heat Balance in a Steam Engine. The equation of steady flow, namely,

$$H_1 - H_2 + \frac{U_1{}^2 - U_2{}^2}{2g} = W + Q$$

holds for any heat appliance, and hence can be applied to the steam engine.

The velocities U_1 and U_2 in the admission and exhaust pipes of the steam engine are both comparatively small enough to be neglected, and hence we have for the steam engine the relation $H_1 - H_2 = W + Q$, that is, the difference between the total heat H_1 in the admission pipe and the total heat H_2 in the exhaust pipe is equivalent to the work done W in the engine, together with the heat lost Q to the surroundings. It must be noted that Q is the whole amount of heat lost between the two points where H_1 and H_2 are measured. It can be reduced in practice by lagging the steam cylinder. If the loss of heat Q could be reduced to zero then $H_1 - H_2 = W$, that is, the work done in the engine is equal to the actual total heat drop in the steam in passing from the admission pipe to the exhaust pipe.

This actual heat drop in a practical case is not the same as the adiabatic heat drop for the admission and exhaust pressures. For this to be the case, two conditions must be fulfilled. In the first place no heat must pass from or to the surroundings, and secondly all processes through which the steam passes must be reversible. Now in a practical case, although we may consider the heat lost Q as negligible, the process through which the steam passes in the cylinder is by no means reversible. As we have seen, owing to initial condensation and re-evaporation, there is a constant interchange of heat between the steam and the cylinder walls. This takes place while the steam is at a different temperature to that of the walls, and hence, although there may not be any loss of heat to the surroundings, yet the action is an irreversible one. In the next place, throttling or wire-drawing taking place in the steam and exhaust ports involves internal friction, which also produces an irreversible action. Both these effects, therefore, reduce the actual heat drop to a value less than the corresponding adiabatic heat drop.

If, therefore, we assume that no heat is lost to the surroundings in a practical case, the efficiency ratio of the engine, which is measured by $\dfrac{\text{Work done}}{\text{Adiabatic heat drop}}$ may be expressed by $\dfrac{\text{Actual heat drop}}{\text{Adiabatic heat drop}}$.

If there is a quantity of heat Q lost to the surroundings, then work done $W = H_1 - H_2 - Q$,

and the efficiency ratio $= \dfrac{\text{Actual heat drop} - Q}{\text{Adiabatic heat drop}}$.

Application of the Indicator Diagram to the Temperature Entropy Chart. If a temperature entropy chart is provided with constant volume and constant pressure lines it is a comparatively simple matter to transfer the indicator diagram of a steam engine to the $T\phi$ chart. It must be remembered that, while the indicator diagram represents the actual weight of the steam in the cylinder between cut-off and release, the $T\phi$ chart is drawn for 1 lb of stuff.

The expansion line can be transferred direct from one chart to another if the saturation line is drawn on the indicator diagram.[c] Thus, in Fig. 181, the dryness fraction for any point b on the PV
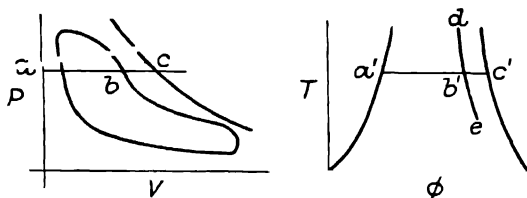
FIG. 181

diagram between cut-off and release is measured by $\dfrac{ab}{ac}$. Hence, if $a'c'$ and ac are at corresponding pressures, and $\dfrac{a'b'}{a'c'}$ is made equal to $\dfrac{ab}{ac}$, the point b' on the $T\phi$ chart will correspond to the point b on the PV chart. In this way the whole expansion line may be drawn. For any other point the weight of stuff in the cylinder during expansion (including clearance) must be calculated as shown in a previous paragraph. If this weight be W, then, volume to be plotted on $T\phi$ chart $= \dfrac{\text{Volume from } PV \text{ chart}}{W}$. Having in this way corrected the volumes from the PV chart to volumes corresponding to 1 lb. of working substance, each pair of values of pressure and volume can be plotted on the $T\phi$ chart, by the aid of the constant pressure and constant volume lines.

Boulvin's Method of Drawing the $T\phi$ Diagram. In this method four scales for entropy, temperature, pressure, and specific volume are drawn from a common centre C, Fig. 182.

The following curves are now drawn in the four sections so obtained—

Section I. The curves connecting temperature and entropy for water (A) and for dry saturated steam (B).

Section II. The curve connecting temperature and pressure for dry saturated steam (C).

Section III. The curve connecting pressure and specific volume for dry saturated steam (*D*).

The weight of stuff in the cylinder during expansion must now be calculated as shown in a previous paragraph. Knowing this weight *W*, the volumes on the indicator diagram can now be reduced to volumes corresponding to 1 lb. of stuff, and the indicator diagram so reduced can be drawn in Section III of the chart.

The indicator diagram so obtained for 1 lb. of stuff can be transferred to Section 1 as follows. In Section 1 draw any constant
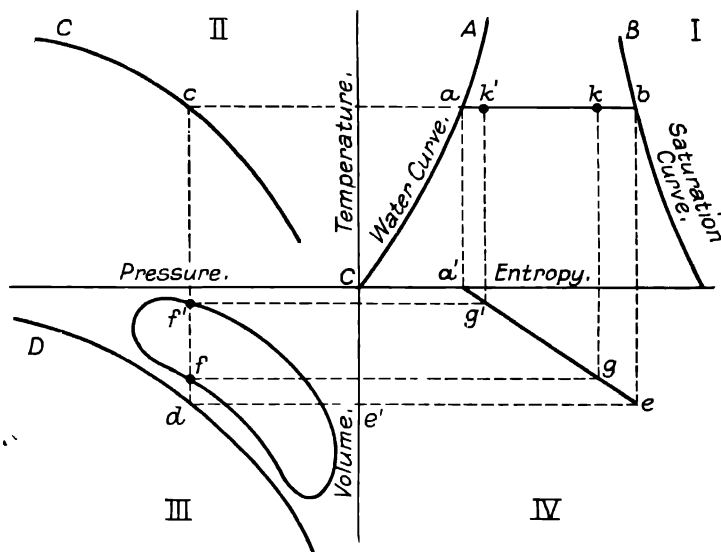
Fig. 182

pressure line *ab* and produce it across to cut the pressure temperature curve *C* in *c*. From this point *c* drop a vertical line to cut the pressure volume curve *D* in *d*. Draw a horizontal line through *d* and a vertical line down through *b* to cut one another in *e*. Project *a* vertically down to cut the entropy scale at *a'*.

A little thought will show that the point *e* on the volume scale measures the volume of 1 lb. of dry steam of a temperature and entropy given for the point *b*. The volume of 1 lb. of stuff corresponding to *a* can be taken as zero, as it is there all water. Hence, if we join *ea'* we obtain a line *ea'* which gives the volume of 1 lb. of stuff at a pressure and temperature corresponding to the point *b* for all conditions from dry steam to water. The two points on the indicator diagram corresponding to this pressure are *f* and *f'*. Hence, if we project these two points *f* and *f'* across to *g* and *g'*, and then up to *k* and *k'*, we obtain the points *k* and *k'* on the *Tϕ*

chart, corresponding to the points f and f' on the PV chart. By proceeding thus for various pressures, the whole $T\phi$ diagram may be obtained.

Compound Engines. The tendency in modern practice is to use higher and higher boiler pressures. This makes it necessary to have a large ratio of expansion which, if carried out in a single cylinder, would be uneconomically and practically disadvantageous. In the first place the large ratio of expansion would necessitate a very early cut-off and a very large and heavy cylinder, because not only would it be necessary for the cylinder to accommodate the large volume of steam at the low exhaust pressure, but the walls would have to be thick enough to withstand the high initial pressure. The initial stress on the moving parts at the beginning of the stroke would also be excessive, and heavy moving parts would be necessary. Again, with a single cylinder engine the turning moment on the crankshaft is very variable, and a heavy flywheel would be required to even out the variation of torque during the cycle. Another disadvantage would be the amount of initial condensation, which, of course, increases with the temperature variation of the cylinder walls, which increases with the difference of pressure of incoming and exhaust steam.

By dividing the pressure drop between two or more cylinders, the cut-off in any one of the cylinders is reduced, the low pressure cylinders need not have such thick walls, while, as the range of stress on any one piston is reduced, the moving parts may be of smaller dimensions. By arranging the cranks at 90° to one another in the case of two cylinders, or at 120° in the case of three cylinders, the torque on the crankshaft will be more uniform during the cycle. This, however, is not the case if the cranks are in line or at 180° to one another.

The reduction of initial condensation is due in the first case to the smaller pressure difference in any one cylinder, and secondly to the fact that, as evaporation occurs during the exhaust stroke, the steam leaving one cylinder is practically dry again as it enters the succeeding cylinder.

Compound engines have two, three, or four cylinders. Two-cylinder compound engines may have the two cranks at 0°, 180°, or 90° to one another. In the first case there need be only one crank, the two pistons being in tandem on the same piston rod. In this case, and in the case where the cranks are 180° apart, the two pistons are at the end of their respective strokes at the same time, and hence the steam can be exhausted from the high pressure cylinder directly into the low pressure cylinder, the two cylinders being connected throughout the stroke. In this case there will be continuous expansion throughout the stroke in the low pressure cylinder and no independent cut-off. When an independent cut-off in the low pressure cylinder is required, or when the cranks are 90°

apart, a receiver must be provided between the two cylinders
This receiver takes the steam from the high pressure cylinder and
holds it until the low pressure cylinder is ready to draw from it.
The pressure in the receiver will obviously vary throughout the
stroke, according to the supply of steam from the high pressure
cylinder and the demand of the low pressure cylinder. In modern
engines the receiver volume is generally amply provided for by the
connecting pipe and low pressure steam chest between the cylinders.
Receiver space is also necessary for triple expansion engines where
the three cranks are 120° apart.
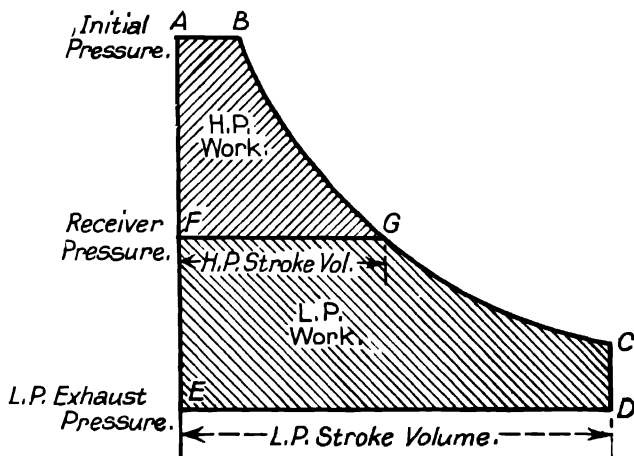
Fig. 183

Two-cylinder Compound Expansion. In Fig. 183, let $ABCDE$
represent the hypothetical pressure volume diagram for the com-
plete expansion in a two cylinder compound engine, neglecting
clearance and compression, and let the pressure at the point F be
the mean receiver pressure. It has already been pointed out that
the receiver pressure will vary throughout the stroke. If the
volume of the receiver space is large compared to that of the
cylinders we may assume that the receiver pressure remains constant
throughout the stroke. In this case $ABGF$ will represent the PV
diagram for the high pressure cylinder and $FGCDE$ will represent
the PV diagram for the low pressure cylinder. The stroke volume
of the low pressure cylinder is independent of the value of the
mean receiver pressure, that is, it is not affected by the position of
the line FG which divides the diagram up between the two cylinders.
In other words, the stroke volume of the low pressure cylinder is
such as if the total number of expansions $\dfrac{ED}{AB}$ takes place in the low

pressure cylinder alone, and it is not affected by the ratio of the cylinder stroke volumes $\frac{ED}{FG}$. It will be noticed that in this case, where the volume of steam in the low pressure cylinder at cut-off, namely FG, is the same as the stroke volume in the high pressure cylinder, there is complete expansion (BG) in the high pressure cylinder. In practice, however, for reasons which will appear later, there is generally a drop of pressure at release in the high pressure cylinder, as shown at HK, Fig. 184.

For the purposes of calculation it is generally assumed that the expansions BH and GC in the two cylinders lie on the same curve,
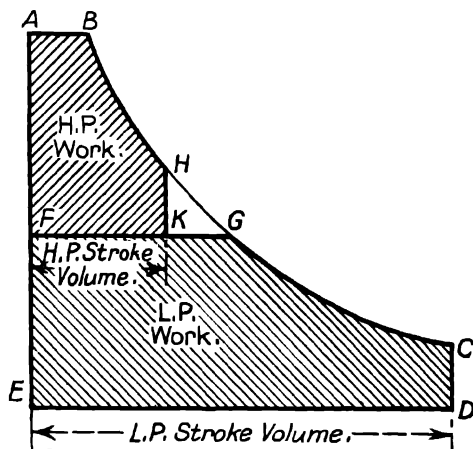
Fig. 184

and the law assumed for this curve is that for hyperbolic expansion, namely, $PV =$ constant.

Ratio of the Cylinder Volumes (R). The total number of expansions in the engine is represented by the ratio $\frac{ED}{AB}$, Fig. 184. The number of expansions in the high pressure cylinder is represented by $\frac{FK}{AB}$, or the cut-off in the high pressure cylinder is represented by $\frac{AB}{FK}$.

Hence the cylinder volume ratio $= R = \frac{ED}{FK} = \frac{ED}{AB} \times \frac{AB}{FK}$

$\qquad$ = Total number of expansions × Cut-off in high pressure

or, Total number of expansions, $r = \dfrac{R}{\text{Cut-off in high pressure}}$.

Distribution of the Pressure Drop Between the Two Cylinders. Let p_1 = initial pressure in high pressure cylinder ; p_r = receiver pressure ; p_b = exhaust pressure.

In determining the receiver pressure, two conditions are desirable. In the first place, the initial load on the piston exerted by the steam at the commencement of the stroke should be approximately the same for the two cylinders. That is—

$$AF \times \text{Area of high pressure} = FE \times \text{Area of low pressure (Fig. 184)},$$

i.e.
$$(p_1 - p_r) = (p_r - p_b) \frac{\text{Area of low pressure}}{\text{Area of high pressure}}.$$

If the stroke is the same for the two cylinders, the ratio of the areas will be the same as the ratio of the cylinder volumes R.

Hence, in order that the initial loads on the two pistons should be equal $(p_1 - p_r) = (p_r - p_b) R$.

The second condition is that the work done in the engine should be equally divided between the two cylinders.

Now work in a cylinder per stroke

$$= \text{Mean effective pressure} \times \text{Stroke volume.}$$

Hence, for equal distribution of work, and assuming the simplest formula for the mean effective pressure, we have

$$\frac{p_1}{r_1} (1 + \log_e r_1) - p_r = \left\{ \frac{p_r}{r_2} (1 + \log_e r_2) - p_b \right\} R$$

If clearance and compression are known, the more complicated formulæ for the mean effective pressure may be used.

The two conditions just discussed cannot generally be satisfied when the expansion curve is continuous, as in Fig. 183, and hence a compromise is made by having a pressure drop at release in the high pressure cylinder as at HK, Fig. 184.

Example 6. Estimate the cylinder dimensions of a compound steam engine to develop 500 I.H.P. at 120 r.p.m. Initial pressure 120 lb. per sq. in. absolute, back pressure 4 lb. per sq. in. absolute, allowable piston speed 500 ft. per min., cylinder ratio 3·5, diagram factor 0·85, cut-off in high pressure cylinder at 0·4 stroke. If the point of cut-off in the low pressure cylinder is at 0·53 stroke, determine the approximate low pressure receiver pressure and compare the initial loads on the two pistons. Assume no clearance.

$$\text{Total number of expansions} = \frac{\text{Cylinder volume ratio } (R)}{\text{Cut-off in high pressure cylinder}}$$

$$= \frac{3 \cdot 5}{0 \cdot 4} = 8 \cdot 75$$

Mean effective pressure for whole engine $= \dfrac{p_1}{r} (1 + \log_e r) - p_b$

$$= \frac{120}{8 \cdot 75} (1 + 2 \cdot 3 \log_{10} 8 \cdot 75) - 4 = 39 \cdot 5 \text{ lb. per sq. in.}$$

∴ actual mean effective pressure $= K \times 39 \cdot 5 = 0 \cdot 85 \times 39 \cdot 5$

$$= 33 \cdot 6 \text{ lb. per sq. in.}$$

I.H.P. $= \dfrac{PLA2N}{33{,}000}$, where $2LN = 500$ ft. per min.

$$\therefore \quad A = \frac{33{,}000 \times \text{I.H.P.}}{P \times 2LN} = \frac{33{,}000 \times 500}{33 \cdot 6 \times 500} = 982 \text{ sq. in.}$$

∴ Diameter of low pressure cylinder $= \sqrt{\dfrac{982 \times 4}{3 \cdot 14}} = 35 \cdot 4$ in.

∴ Diameter of high pressure cylinder $= \dfrac{35 \cdot 4}{\sqrt{R}} = \dfrac{35 \cdot 4}{\sqrt{3 \cdot 5}} = 18 \cdot 93$ in.
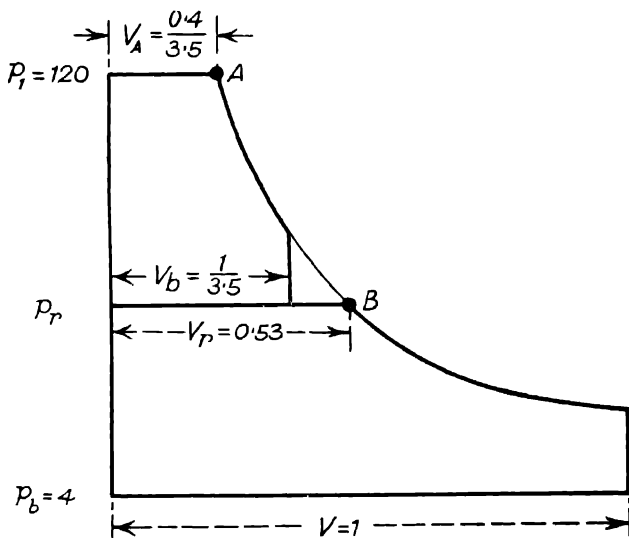
Fig. 185

Hence, the diameter of high pressure cylinder is 19 in. ; diameter of low pressure cylinder, $35\frac{1}{2}$ in. ; and the stroke, 25 in.

Fig. 185 shows the hypothetical indicator diagram for the engine taking the stroke volume of the low pressure cylinder as unity.

To find the value of p_r we have, for the point A,

$$p_a v_a = 120 \times \frac{0 \cdot 4}{3 \cdot 5}$$

∴ pressure at $B = p_r = \dfrac{120}{0 \cdot 53} \times \dfrac{0 \cdot 4}{3 \cdot 5} = 25 \cdot 9$ lb. per sq. in.

$$\frac{\text{Load on high pressure piston}}{\text{Load on low pressure piston}} \quad \frac{(p_1 - p_r)}{(p_r - p_b) R} \quad \frac{(120 - 25 \cdot 9)}{(25 \cdot 9 - 4)3 \cdot 5}$$

$$\frac{1 \cdot 22}{1}$$

Governing of Compound Engines. We will consider three cases, namely, governing by throttle, by variation of cut-off in the high pressure cylinder, and by variation of cut-off in the low pressure cylinder. For simplicity, consider an engine with a cylinder volume ratio of 5 : 2, and with cut-off in the high pressure cylinder at full
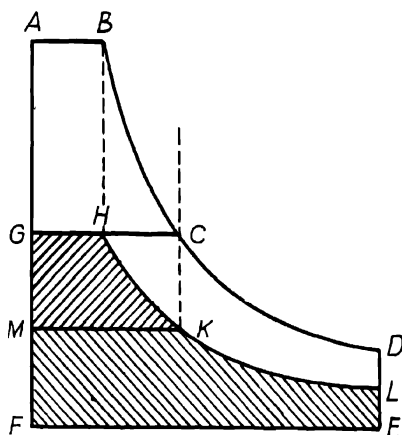
FIG. 186A. THROTTLE GOVERNING   FIG. 186B. H.P. CUT-OFF GOVERNING
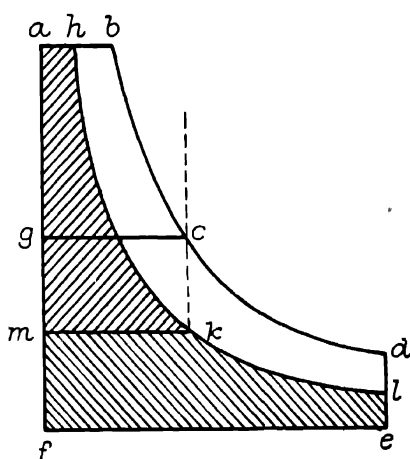
load of 1/2. Let the back pressure in the low pressure cylinder be zero.

Fig. 186 illustrates the three cases, the full load diagrams being *ABDEF*, *abdef*, *a'b'd'e'f'* respectively, the dividing line between the high pressure and low pressure diagrams being *GC*, *gc*, and *g'c'* respectively.

Case I. Throttle Governing. Suppose the full load pressure *FA* reduced to half by throttling, that is, to *FG*, the cut-off in each cylinder remaining the same. The light load diagram will become *GHLEF*. The high pressure work will be reduced from *ABCG* to *GHKM* and the low pressure work will be reduced from *GCDEF* to *MKLEF*.

Case II. High Pressure Cut-off Governing. Suppose for the light load the cut-off in high pressure is reduced from 1/2 to 1/4, the initial pressure and the cut-off in low pressure remaining constant. The light load diagram will become *ahlef*, the high pressure work becoming *ahkm*, which is not very different from *abcg*, while the

low pressure work is reduced from *gcdef* to *mklef*. Governing by varying the cut-off in the high pressure cylinder will therefore reduce the proportion of the work done in the low pressure cylinder on light loads. With condensing engines at very light loads this may cause the average pressure in the low pressure cylinder to fall below that necessary to overcome the back pressure and the frictional resistance, thus reducing the efficiency of the engine.

A comparison of the diagrams Figs. 186A and 186B shows that the high pressure cut-off governing is more economical than throttle governing. The light load diagrams for the low pressure cylinder, namely, *MKLEF* and *mklef*, are the same in each case, the release pressure *LE* or *le* also being the same. Hence the same volume of steam at the same release pressure is exhausted from the engine in each case, whereas work done with throttle governing is represented by *GHLEF*, while the work done with high pressure cut-off governing is represented by the larger area *ahlef.*

Case III. Low Pressure Cut-off Governing. Suppose the high pressure cut-off is kept constant at $\frac{1}{2}$, while the low pressure cut-off is changed from 2/5 to 3/5. By this change of cut-off the high pressure work is increased from *a'b'c'g'* to *a'b'c'k'm'*, while the low pressure work is decreased from *g'c'd'e'f'* to *m'l'd'e'f'*. Thus, by making the cut-off in the low pressure cylinder earlier, the total work done by the engine is only slightly affected, but the proportion of the total work done in the low pressure cylinder is reduced, while the work done in the high pressure cylinder is increased.

Fig. 186c. Cut-Off Governing

Example 7. Obtain the ratio of cylinder areas for a compound steam engine receiving steam at 100 lb per sq. in. absolute and exhausting at 3 lb. per sq. in. absolute. Cut-off in high pressure at 0·4 stroke and low pressure at 0·66 of stroke. Take the clearance in each case as 1/10 of the volume swept through by piston. Assume the cylinder feed to be 0·6 lb. per stroke and the weight shut in the high pressure and low pressure cylinders to be 0·05 and 0·03 respectively. Draw the probable combined diagram, putting in the intermediate pressure line so that the work shall be approximately equally divided between the cylinders. Estimate the horse-power of the combined engine and the steam used per horse-power hour. Assume a dryness fraction of 0·75 in the high pressure cylinder for initial condensation.

The diagram, Fig. 187, is obtained in the following manner—

Weight of stuff in high pressure cylinder at cut-off (B)

$$= \text{cylinder feed} + \text{clearance} = 0\cdot6 + 0\cdot05 = 0\cdot65 \text{ lb.}$$

The dryness fraction at $B = 0\cdot75$.

Volume of 1 lb. of dry steam at 100 lb. per sq. in. $= 4\cdot451$ cub. ft

$\therefore$ indicated volume at $B = 4\cdot451 \times 0\cdot75 \times 0\cdot65 = 2\cdot17$ cub. ft.

Now the volume $AB = 0\cdot4 \times$ stroke volume of high pressure.

$\therefore$ Stroke volume of high pressure $= \dfrac{AB}{0\cdot4}$.

FIG. 187

Also, $AB +$ Clearance of high pressure $=$ Volume at $B = 2\cdot17$

i.e. $\quad AB + \dfrac{\text{Stroke volume of high pressure}}{10} = 2\cdot17$

$\therefore \quad AB + \dfrac{AB}{0\cdot4 \times 10} = 2\cdot17$. $\quad \therefore$ volume $AB = 1\cdot736$ cub. ft.

$\bullet \ \therefore$ stroke volume of high pressure $\dfrac{1\cdot736}{0\cdot4} = 4\cdot34$ cub. ft.

Clearance volume of high pressure is $0\cdot434$ cub. ft., and indicated volume at $E = 4\cdot34 + 0\cdot434 = 4\cdot774$ cub. ft.

For the work to be equally distributed between the cylinders we have

$$p_1\left(\frac{1}{r_1} + \left(a + \frac{1}{r_1}\right)\log_\varepsilon \frac{1+a}{\frac{1}{r_1}+a}\right) - p_r$$

$$= \left\{ p_r\left(\frac{1}{r_2} + \left(a + \frac{1}{r_2}\right)\log_\varepsilon \frac{1+a}{\frac{1}{r_2}+a}\right) - p_b \right\} \mathcal{R}$$

where, from the question, $p_1 = 100$; $p_b = 3$; $\dfrac{1}{r_1} = 0.4$; $\dfrac{1}{r_2} = 0.66$;

$a = 0.1$; $R = $ cylinder volume ratio.

$$\therefore 100 \left\{ 0.4 + (0.1 + 0.4)2.3 \log_{10} \frac{1.1}{0.5} \right\} - p_r$$

$$= \left\{ p_r \left(0.66 + (0.1 + 0.66)2.3 \log_{10} \frac{1.1}{0.76} \right) - 3 \right\} R$$

i.e. $\qquad 79.4 - p_r = (0.941\, p_r - 3)R$.   .   .   .   .   (1)

To find R. Let V be the stroke volume of low pressure ; then volume $FC = 0.66V$.

$\therefore$ indicated volume at $C = V_c = $ volume $FC + $ clearance volume of high pressure $= 0.66\ V + 0.1 V = 0.76\ V$. $\quad \therefore V = \dfrac{V_c}{0.76}$.

Now $P_B V_B = 100 \times 2.17$, and $P_c V_c = P_B V_B$. $\therefore V_c = \dfrac{217}{P_c} = \dfrac{217}{p_r}$

$\therefore$ stroke volume of low pressure $= V = \dfrac{V_c}{0.76} = \dfrac{217}{0.76\, p_r}$

$\therefore$ cylinder volume ratio $R = \dfrac{\text{Volume of low pressure}}{\text{Volume of high pressure}}$

$$\frac{217}{0.76} \times \frac{1}{4.34} = \frac{65.8}{p_r}.$$

Substituting this value of R in equation (1), we have

$$79.4 - p_r = (0.941\, p_r - 3) \frac{65.8}{p_r}$$

whence $p_r = 25.3$ lb. per sq. in. ; $\therefore R = \dfrac{65.8}{25.3} = 2.6$.

Stroke volume of low pressure $= 2.6 \times$ stroke volume of high pressure
$$= 2.6 \times 4.34 = 11.284 \text{ cub. ft.}$$

$\therefore$ Volume of point $D = 11.284 + 1.128 = 12.41$ cub. ft.

Volume of point $C = \dfrac{217}{p_r} = \dfrac{217}{25.3} = 8.577$ cub. ft.

Pressure at $D = \dfrac{217}{12.41} = 17.5$ lb. per sq. in.

The volumes at H and L can be obtained if we assume that the steam at these points is dry.

Then, volume at $H = $ weight of clearance steam in high pressure multiplied by specific volume of steam at 25 lb. per sq. in.
$$= 0.05 \times 16.29 = 0.815 \text{ cub. ft.}$$

Volume at L = weight of clearance steam in low pressure multiplied by specific volume of steam at 3 lb. per sq. in.

$$= 0.03 \times 118.6 = 3.56 \text{ cub. ft.}$$

Assuming PV = constant for the compression curves, the pressures at M and K can be found.

Horse-power. Assume speed at N r.p.m.

In the high pressure cylinder, mean effective pressure $= 79.4 - 25.3$ from Equation (1) $= 54.1$ lb. per sq. in.

$$\therefore \text{Horse-power} = \frac{PLA2N}{33,000} \text{ (}A\text{ being in square inches)}$$

$$= \frac{54.1 \times 4.34 \times 144}{33,000} \times 2N = 2.05N.$$

In low pressure cylinder mean effective pressure

$$= 0.941 \times 25.3 - 3 = 20.8 \text{ lb. per sq. in.}$$

$$\therefore \text{Horse-power per stroke} = \frac{20.8 \times 11.28 \times 144}{33,000} \times 2N = 2.05N$$

$\therefore$ Total horse-power $= 4.1N$, and

$$\text{steam consumption} = \frac{\text{Cylinder feed} \times 2N \times 60}{\text{Horse power}}$$

$$\frac{0.6 \times 2N \times 60}{4.1N} = 18 \text{ lb. per H.P. hour.}$$

<center>EXAMPLES X</center>

1. An engine takes in dry saturated steam at 80 lb. per sq. in. absolute, and exhausts at 15 lb. per sq. in. absolute. Determine the work done per lb. of steam and the efficiency when working on (*a*) the Carnot cycle and (*b*) the Rankine cycle. (*I.Mech.E.*, 1925.)

2. Compare the Rankine and Carnot cycles for a vapour like steam, illustrating your answers with sketches of the pressure-volume, and temperature-entropy diagrams. Obtain an expression for the work available per lb. of steam by an engine working on the Rankine cycle. A steam engine takes in steam at 100 lb. per sq. in. absolute, and exhausts it at 5 lb. per sq. in. absolute. Assume that its efficiency ratio using the Rankine cycle standard is 62 per cent, and estimate its steam consumption per I.H.P. hour. (*I.Mech.E.*, 1923.)

3. Explain why the increase in efficiency obtained by using superheated steam in reciprocating engines is more than the gain which would be expected from consideration of the increase in efficiency of the corresponding Rankine cycle.

A steam engine, working over a constant pressure range of 90 to 14.7 lb. per sq. in. absolute, has a constant ratio of expansion, and is supplied with

steam at various degrees of superheat. The steam consumption was observed to be as follows—

			dry	202°	245°	290°
Temp. of steam supply ° C.	.	.	dry	202°	245°	290°
Steam per I.H.P. hour, lb.	.	.	42·0	37·4	32·3	27·1

Plot curves showing the variation with degree of superheat of the actual efficiency and the Rankine efficiency. (*U.L., B.Sc. (Eng.)*, 1923.)

4. Explain why the Rankine cycle rather than the Carnot cycle is employed as the standard with which to compare the performance of actual steam engines.

In an efficient steam engine plant the boiler supplies the engine with dry saturated steam at a pressure of 180 lb. per sq. in. absolute. If the condenser pressure is 3 lb. per sq. in. absolute, calculate the Rankine efficiency of the engine and selecting your own values for the boiler efficiency, the mechanical efficiency, and the efficiency ratio, estimate the probable overall efficiency of the plant from coal to brake. (*U.L., B.Sc. (Eng.)*, 1926.)

5. Steam reaches a turbine at a pressure of 180 lb. per sq. in. absolute, and 2 per cent wet, but before entering the nozzle chest it is throttled to a pressure of 120 lb. per sq. in. absolute. In the turbine it expands to 20 lb. per sq. in. absolute, with an efficiency ratio of 55 per cent. The steam flow amounts to 5·2 lb. per sec. Determine (*a*) the condition of the steam at exhaust, and (*b*) the power developed by the turbine. (*U.L., B Sc. (Eng)*, 1921.)

6. State what is meant by the " Willans Line." A steam engine governed by throttling, uses 1,180 lb. of steam per hour when developing 55 I.H.P., and 4,800 lb. per hour when developing 280 I H.P. Find the approximate thermal efficiency of the engine when developing 200 I H.P., assuming the steam supplied to be dry and saturated at 180 lb. per sq. in. absolute, and the back pressure to be 2 lb per sq. in absolute. (*I.Mech.E.*, 1926.)

7. Explain what is meant by the " Willans Line." The test results from a turbine governed by throttling can be represented by a line whose equation is

$$G = 0·0021 N + 1·15,$$

where G is lb. of steam per second, and N is horse-power. The maximum power is 5,000 H.P., and the conditions of steam supply at this load are 200 lb. per sq. in. absolute, and 100° F. superheat. Throttling is at constant heat and the condenser pressure is 1·0 lb. per sq. in. absolute. Assuming that the steam pressure varies directly as the steam flow, calculate the efficiency ratio of the turbine at one-third power. (*I.Mech.E.*, 1924.)

8. Calculate the cylinder diameter and stroke of a double acting steam engine that will develop 120 I.H.P. under the following conditions : Initial pressure 80 lb. per sq. in. absolute ; back pressure, 16 lb per sq. in. absolute ; revolutions per minute, 100 ; average piston speed, 500 ft. per min. ; cut-off at 0·4 stroke. Take the diagram factor as 0·85. (*I.Mech.E.*, 1925.)

9. A single cylinder double acting steam engine has a piston diameter of 11 in., a stroke of 18 in., and the clearance volume is 8·5 per cent of the swept volume. The steam supply is at 70 lb. per sq. in. gauge pressure, and the exhaust at atmospheric pressure (30 in. of mercury). Cut-off is at 40 per cent of the stroke and compression at 80 per cent. Assuming that the actual indicator card has an area 90 per cent of that of the theoretical card having hyperbolic expansion and compression, find the indicated horse-power of the engine when running at 185 revolutions per min. (*U.L., B.Sc. (Eng.)*, 1925.)

10. Show how the " missing quantity " in a steam engine cylinder may be determined for any point of the stroke between cut-off and release.

Determine the " missing quantity " at cut-off from the following particulars : Cylinder diameter, 12 in. ; stroke, 30 m ; clearance 8 per cent of volume swept by piston, consumption at 104 R.P.M. ; 2,490 lb. of steam per hour.

Event	Fraction of stroke	Pres
Cut-off .	0·26	112 lb. per sq. in. abs.
Compression	0·82	24 lb. per sq. in. abs.

(I.Mech.E., 1924.)

11. The effective volume of the cylinder of a steam engine is 0·92 cubic ft., and the clearance volume is 8·5 per cent of this volume. The weight of steam in the cylinder at half stroke is 0·063 lb., and the pressure is 42 lb. per sq. in. absolute. The steam expands hyperbolically to 0 95 stroke. Calculate the final dryness of the steam, and determine the number of heat units that pass through the walls into the cylinder during expansion.

(U.L., B.Sc. (Eng.), 1922.)

12. Calculate the cylinder diameter and stroke of a compound engine to develop 400 I.H.P., under the following conditions : Admission pressure, 165 lb. per sq. in absolute ; back pressure, 2 lb. per sq. in. absolute ; revolutions per minute, 120 ; average piston speed, 720 ft. per min ; number of expansions, 15 ; diagram factor, 0·85 ; cut-off in H.P. at 0·27 stroke. How would you fix the point of cut-off in the L P. cylinder ?

(I Mech.E., 1925.)

13. A compound steam engine is to develop 125 horse-power at 110 revolutions per minute. The steam supply is at 105 lb. per sq. in absolute, and the condenser pressure is 3 lb per sq. in. absolute. Assuming hyperbolic expansion, an expansion ratio of 15, a diagram factor of 0 7, and neglecting clearance volume and receiver losses, determine the diameters of the cylinders so that they may develop equal powers. The stroke of each piston may be taken equal to the low pressure cylinder diameter.

(U.L., B.Sc. (Eng.), 1922.)

14. The two cylinders of a compound steam engine have the same stroke, and the ratio of piston diameters is 1·8. Assuming hyperbolic expansion, and neglecting clearance, find the point of cut-off in the low pressure cylinder, which will theoretically give maximum efficiency if the cut-off in the high pressure cylinder is at half stroke.

A series of trials on this engine made at approximately constant speed, and in which the point of low pressure cut-off was varied gave result as follows---

Steam supply dry and saturated at 92 lb. per sq in. absolute.

Condenser pressure 2·75 lb. per sq. in. absolute.

L.P. cut-off . .	0·225	0·254	0·339	0 350	0·508	0·67
B.H.P. . . .	49·0	57·0	56 5	56·1	54·4	47·6
Steam per hour, lb. .	1060	1214	1180	1165	1190	1090

Plot on a low pressure cut-off base a curve of thermal efficiency (B.H.P. basis). *(U.L., B.Sc. (Eng.), 1924.)*

Velocity after Expansion Through a Nozzle. If steam is allowed to expand through a suitably shaped nozzle, part of the heat energy in the steam may be converted into kinetic energy. In order to investigate the process let us consider the equation of steady flow, the heat appliance in our imaginary enclosure being a nozzle. The ideal nozzle, being an appliance to convert heat energy of the steam into kinetic energy, should neither gain heat from nor lose it to the surroundings, and no external work should be done by the expansion. Therefore, in the equation of steady flow (p. 43),

$$H_1 - H_2 + A \frac{U_1^2 - U_2^2}{2g} = AW + Q$$

both W and Q are zero.

Hence
$$\frac{U_2^2 - U_1^2}{2g} = J(H_1 - H_2)$$

This equation shows that the gain in kinetic energy is equal to the change in total heat of the fluid, J being introduced, as both sides of the equation must be expressed in the same units.

If, for the sake of simplicity, we assume that the initial velocity U_1 is negligible compared to the final velocity U_2, then

$$U_2 = \sqrt{2gJ(H_1 - H_2)}.$$

If, in addition to the assumption that there is no heat transfer from or to the enclosure and surroundings, we also assume absence of friction, then at first sight it would appear that $H_1 - H_2$ is the adiabatic heat drop for the difference in pressure considered. It will be shown, later on, that the ordinary type of adiabatic expansion so far considered does not, in general, happen in the practical case of expansion through a nozzle ; but it will be convenient to assume, in the first instance, that this type of expansion does take place.

Hence the velocity of steam leaving a nozzle is

$$U = \sqrt{2gJ} \text{ (Adiabatic heat drop in the nozzle)}.$$

If we, therefore, wish to find the velocity obtained by expansion through a nozzle from a pressure P_1 to a pressure P_2, then the adiabatic heat drop between these two pressures must be either calculated by first finding the dryness fraction after expansion, and hence the value of the total heat after expansion, or it may be read off the $H\phi$ chart direct.

Example 1. Dry saturated steam, at a pressure of 200 lb. per sq. in. absolute, expands adiabatically through a suitably shaped nozzle to a final pressure of 2 lb. per sq. in. absolute. What is the final velocity of the steam ?

Initial total heat $H_1 = 669.69$ C.H.U., at 200 lb. per sq. in. (from Table I). Dryness fraction after expansion

$$x = \left(\log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1}\right)\frac{T_2}{L_2}$$

$$= \left(2.3 \times \log_{10} \frac{467.46}{325.37} + \frac{472.2}{467.46}\right)\frac{325.37}{566.51} = 0.788$$

∴ Total heat after expansion

$$= H_2 = h + xL, \text{ at 2 lb. per sq. in. (from Table 1)}$$

$$= 52.16 + (0.788 \times 566.51) = 498.6 \text{ C.H.U.}$$

∴ Adiabatic heat drop $H_1 - H_2 = 669.69 - 498.6 = 171$ C.H.U.

This heat drop can be measured directly on the $H\phi$ chart. The initial state point is given by the intersection of the 200 lb. per sq. in. pressure line with the saturation line. A vertical line through this point will cut the 2 lb. per sq. in. pressure line at the final state point. The vertical distance between these two points, measured on the total heat scale, gives the heat drop.

Then velocity $U = \sqrt{2gJ \text{ (heat drop)}}$

$$= \sqrt{2 \times 32.2 \times 1400 \times 171} = 3926 \text{ ft. per sec.}$$

The velocity may also be expressed in terms of pressure and volume. It has already been shown that if a fluid expands according to the law $PV^n = $ constant, the value of the heat drop $(H_1 - H_2)$ for an expansion from pressure P_1 to pressure P_2 is given by

$$\int_{P_2}^{P_1} VdP = \frac{n}{n-1}(P_1V_1 - P_2V_2).$$

Hence $U_2 = \sqrt{2g \dfrac{n}{n-1}(P_1V_1 - P_2V_2)}$

$$= \sqrt{2g \cdot \frac{n}{n-1} \cdot P_1V_1\left(1 - \frac{P_2V_2}{P_1V_1}\right)}$$

But $\dfrac{V_2}{V_1} = \left(\dfrac{P_1}{P_2}\right)^{\frac{1}{n}}$, and substituting for $\dfrac{V_2}{V_1}$

$$U_2 = \sqrt{2g \frac{n}{n-1} P_1V_1\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

This formula should give the same value for U_2 as that calculated from $U_2 = \sqrt{2gJ \text{ (adiabatic heat drop)}}.$

The two values will not be quite the same for, as we have already seen, the value to be taken for n when steam expands adiabatically is not definite. If the initial state of the steam is taken as dry saturated then the value $n = 1.135$ may be taken.

Area of Cross-section of Nozzle. Let the area of cross-section of the nozzle at the point where the pressure is P_2 be A_2 sq. ft. If the weight of steam passing per second be M lb., then the total volume of steam passing the point considered is $M V_2$ cub. ft. per sec., where V_2 is the specific volume of the steam at that point in cubic feet per pound. The total volume of the steam passing per second is obviously also given by $A_2 U_2$ cub. ft.

$$\therefore A_2 U_2 = M V_2, \text{ or } \frac{M}{A_2} = \frac{U_2}{V_2}$$

But as $P_2 V_2{}^n = P_1 V_1{}^n$, $\dfrac{1}{V_2} = \dfrac{1}{V_1}\left(\dfrac{P_2}{P_1}\right)^{\frac{1}{n}}$.

Hence

$$\frac{M}{A_2} = \frac{1}{V_1}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} U_2$$

$$= \frac{1}{V_1}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} \sqrt{2g\,\frac{n}{n-1}\,P_1 V_1\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

or

$$\frac{M}{A_2} = \sqrt{2g\,\frac{n}{n-1}\,\frac{P_1}{V_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right\}}$$

From this equation, if M is known, the area of the nozzle at any point where the pressure is P_2 may be calculated.

Another method for finding the nozzle area is as follows. We have, as before,

$$A_2 U_2 = M V_2$$

and

$$U_2 = \sqrt{2gJ(H_1 - H_2)}$$

If the dryness fraction x after expansion to the pressure P_2 is found, then

$$V_2 = x V_{s2}$$

where V_{s2} is the specific volume of dry saturated steam at the pressure P_2.

Hence,

$$\frac{M}{A_2} = \frac{U_2}{V_2} = \frac{\sqrt{2gJ(H_1 - H_2)}}{x V_{s2}}$$

Example 2. Find the shape of a nozzle of circular cross-section to expand dry saturated steam from a pressure of 200 lb. per sq. in. absolute to a back pressure of 2 lb. per sq. in. absolute. Assume frictionless adiabatic expansion, with the pressure falling uniformly along the nozzle.

The figures are shown in the table below. The values of the dryness fraction were obtained from the $H\phi$ chart. The values of

heat drop from an initial pressure of 200 lb. per sq. in. down to the various back pressures were also obtained from the $H\phi$ chart.

The values of V_s for dry saturated steam were obtained from the tables.
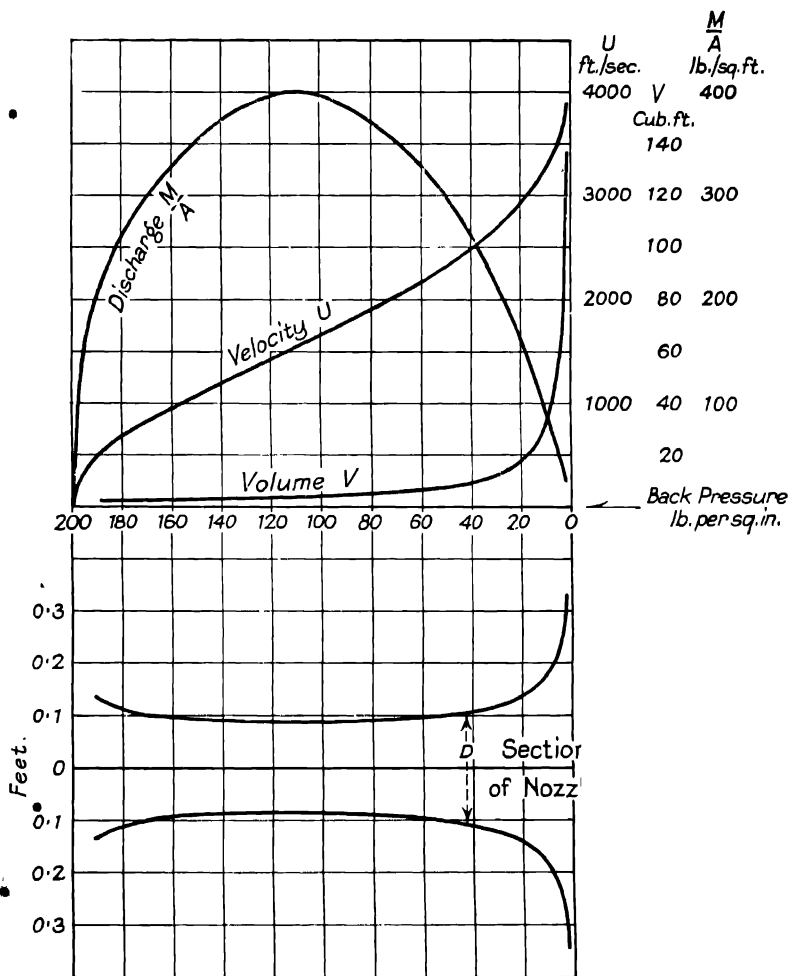
Fig. 188

In Fig. 188 the values of specific volume, velocity and discharge per unit area are shown plotted with the axis of the nozzle as the horizontal co-ordinate. The section of the nozzle obtained by plotting the various values of D along the axis of the nozzle is also given.

NOZZLE FOR FRICTIONLESS ADIABATIC FLOW OF STEAM

Pressure P lb./sq. in.	Dryness fraction x	Heat Drop $H_1 - H_2$ C.H.U.	Velocity U ft./sec.	Vs cub. ft /lb.	Specific Volume $V = xVs$ cub. ft./lb.	Discharge per unit area $\frac{M}{A} = \frac{U}{V}$ lb./sq. ft.	Area A sq. ft.	Diameter D ft.
200	1	—	—	—	—	.	—.	—
190	0·995	2 9	511	2 435	2·423	211	0 0475	0 246
180	0 992	5·0	703	2·562	2 64	266	0 0376	0·219
150	0 98	12 5	1061	3 041	2·98	356	0·0281	0·189
100	0·954	31·5	1085	4·451	4 25	397	0 0252	0·179
50	0 915	60·5	2335	8·520	7·80	300	0·0334	0·206
40	0·904	69	2494	10·50	9 49	263	0·0381	0·220
30	0·89	80·5,	2695	13 74	12 23	220	0·0455	0·241
20	0 873	95	2926	20·08	17 53	167	0·0600	0·277
10	0·847	119	3275	38·39	32·52	101	0·0990	0·355
6	0·828	135·5	3494	61·91	51 25	68	0·1470	0·433
2	0·794	168	3891	173·50	137 76	28·5	0·3520	0·670

From these curves it will be seen that at the higher pressures the specific volume V increases at first slowly as the pressure drops, while the velocity U increases at a greater rate. As the expansion proceeds the increase of the specific volume V becomes greater than the increase of velocity U. Hence, as we proceed from the high pressure end to the low pressure end of the nozzle, the area A, which is proportional to $\frac{V}{U}$, decreases to a minimum and then increases. In other words, the nozzle is of a convergent divergent section. The point of the nozzle where the area is a minimum is called the throat. From the curve it will be seen that the pressure at the throat is about 115 lb. per sq. in. The ratio of the pressure at the throat to the initial pressure is $\frac{115}{200} = 0.57$, approximately. The conditions at the throat will now be considered more fully.

Conditions at the Throat. At the throat, where the area is a minimum, the mass of steam passing per second per unit of area is a maximum.

Hence, $\dfrac{M}{A} = \sqrt{2g \dfrac{n}{n-1} \dfrac{P_1}{V_1} \left\{ \left(\dfrac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\dfrac{P_2}{P_1}\right)^{\frac{n+1}{n}} \right\}}$ must be a maximum.

For any given initial conditions the variable quantity in this equation is $\left(\dfrac{P_2}{P_1}\right)$. Differentiating with respect to $\left(\dfrac{P_2}{P_1}\right)$ and equating to zero, we have

$$\frac{2}{n}\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}-1} - \frac{n+1}{n}\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}-1} = 0$$

This particular value of P_2 is the pressure at the throat. If we call this pressure P_t we have

$$\frac{P_t}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

From this equation it follows that the throat pressure is dependent solely on the initial pressure and the value of n.

If this value of $\left(\dfrac{P_t}{P_1}\right)$ is substituted in the general equation for velocity, namely,

$$U_2 = \sqrt{2g\,\frac{n}{n-1}\,P_1 V_1 \left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

we have for the value of the velocity at the throat

$$U_t = \sqrt{2g\,\frac{n}{n-1}\,P_1 V_1 \left\{1 - \left[\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right]^{\frac{n-1}{n}}\right\}}$$

$$= \sqrt{2g\,\frac{n}{n+1}\,P_1 V_1}.$$

The specific volume of the steam at the throat

$$V_t = V_1 \left(\frac{P_1}{P_t}\right)^{\frac{1}{n}}$$

$$\therefore \quad \frac{1}{V_t} = \frac{1}{V_1}\left(\frac{P_t}{P_1}\right)^{\frac{1}{n}} = \frac{1}{V_1}\left[\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right]^{\frac{1}{n}} = \frac{1}{V_1}\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}.$$

The mass of steam passing per second per unit area at the throat

$$\frac{M}{A_t} = \frac{U_t}{V_t} = \frac{1}{V_1}\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\sqrt{2g\,\frac{n}{n+1}\,P_1 V_1}$$

$$= \sqrt{gn\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\frac{P_1}{V_1}}.$$

From this equation it is obvious that, provided the final pressure is not greater than $P_t = P_1\left(\dfrac{2}{n+1}\right)^{\frac{n}{n-1}}$, the mass of steam passing per second through a convergent divergent nozzle is dependent solely on the initial conditions of the steam and the throat area, and is independent of the final pressure at the end of the nozzle. This pressure is called the *critical pressure*. Thus, the addition of the divergent part of the nozzle after the throat does not affect the discharge of the nozzle, but it does affect the final velocity of the steam. This final velocity will be determined from the equation

$$U_2 = \sqrt{2g\,\frac{n}{n-1}\,P_1 V_1 \left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

where P_2 is the pressure at the end of the nozzle, the value of P_2 being determined by the final area of the nozzle, provided the back pressure outside the end of the nozzle is not greater than this value

of P_2 so determined. If the final back pressure is lower than the critical pressure the nozzle will be convergent divergent. If the back pressure is equal to or higher than the critical pressure, then the nozzle should be convergent only and the mass flow through such a nozzle will be determined by the final mouth area of the nozzle.

The Value of n. The Zeuner equation for n in the adiabatic expansion of steam is $n = 1.035 + 0.1x$, and gives a value of 1.135 for n for originally dry saturated steam. If this value of n is substituted in the equation

$$P_t = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} \times P_1$$

we obtain for the critical pressure $P_t = 0.577 P_1$.

If the same value of n is substituted in the equation

$$\frac{M}{A_t} = \sqrt{ gn \left(\frac{2}{n+1} \right)^{\frac{n+1}{n-1}} \frac{P_1}{V_1} }$$

we obtain $M = 3.604 \, A_t \sqrt{\dfrac{P_1}{V_1}}$; A_t being in square feet,

P_1 in pounds per square foot, and M in pounds per second.

If p is in pounds per square inch and A_t in square inches, this reduces to $M = 0.3003 \, A_t \sqrt{\dfrac{p_1}{V_1}}$.

Now, for dry saturated steam

$$p_1 V_1^{\frac{16}{15}} = 490, \text{ where } p \text{ is in pounds per square inch.}$$

Substituting this value of V_1 in the above equation,

$$M = 0.0165 \, A_t \, p_1^{\frac{31}{32}}$$

an equation connecting the discharge M in pounds per second and the throat area in square inches for a convergent divergent nozzle with steam initially dry and at a pressure p_1 lb. per sq. in.

As will be seen later, the values of M determined from these equations are not confirmed by practical measurement, showing that ordinary adiabatic expansion does not take place.

We must now consider the case where superheated steam is expanded.

The value of n for the adiabatic expansion of superheated steam is given by Callendar as 1.3. If this value is substituted in the

equation $\qquad P_t = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} \times P_1$

we obtain for the critical pressure $P_t = 0.5457 \, P_1$.

If the same value of n is substituted in the equation,

$$M = \sqrt{gn\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\frac{P_1}{V_1}}$$

we obtain $M = 3\cdot786\, A_t \dfrac{P_1}{V_1}$, A_t being in square feet; P_1 in pounds per square foot ; M in pounds per second.

The Effect of Friction in a Nozzle. If there were no friction during expansion through a nozzle, then, assuming the expansion were adiabatic, the expansion line would be AB and ab, Fig. 189. The heat energy transformed into kinetic energy would be represented by the area $ABDE$. This area is $\displaystyle\int V\, dP = H_A - H_B$; where H_A and H_B are the total heats at A and B respectively.
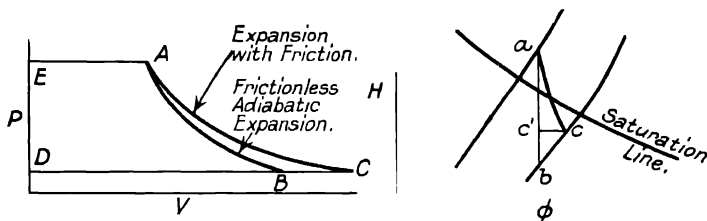
Fig. 189

In a practical case, however, there is friction between the fluid and the sides of the nozzle, as well as dissipation of energy due to eddies being formed in the fluid itself. At any point in the expansion, some of the energy which, in the absence of friction would be in the form of kinetic energy, has been dissipated in overcoming friction and reappears in the form of heat. At this point, therefore, there is less kinetic energy and more heat energy than would be the case in frictionless flow. The specific volume of the fluid will be greater than in the case of frictionless flow, and the actual expansion curve would be some curve AC on the PV chart, Fig. 189. Owing to the loss by nozzle friction the expansion is not reversible, and hence there will be a progressive increase in entropy ; the actual expansion on the $H\phi$ chart follows some such curve as ac, the final state point c being on the constant pressure line through b. From the $H\phi$ chart we see that, owing to friction, the actual heat drop is reduced from ab to ac', and hence the final kinetic energy generated will be less than in the case of frictionless adiabatic flow. The $H\phi$ chart also shows that the effect of friction is to re-heat the fluid, the final state of the steam being drier in the case of wet steam, and at a higher temperature in the case of superheated steam, than would be the case in the absence of friction.

Considering the PV diagram, Fig. 189, the amount of heat energy converted into kinetic energy with frictionless flow is represented by the area $ABDE = \int_{P_B}^{P_A} V\, dP$

With frictionally resisted flow the heat energy converted into kinetic energy is not represented by the area $ACDE$. To illustrate this, consider the cycle ABC on the PV diagram, Fig. 189. Starting at the state point A, and allowing the steam to expand with frictional resistance along AC, there will be a certain amount of kinetic energy generated due to the expansion. Let this be K. In order to pass from the state point C to the state point B, heat must be extracted from the fluid at constant pressure. This amount will be $H_C - H_B$. On compressing the steam back to its original condition at A along BA, the energy required will be the same as the kinetic energy obtained by expanding from A to B without friction, namely, $H_A - H_B$. Since the fluid has passed round a closed cycle and returned to its original condition,

Energy got out = energy put in ;

i.e. $K + H_C - H_B = H_A - H_B$

i.e. $K = (H_A - H_B) - (H_C - H_B).$

Now, $H_A - H_B$ is represented by the area $ABDE = \int_{P_B}^{P_A} V\, dP.$

The kinetic energy produced with friction, therefore, is less than that produced without friction by the amount $(H_C - H_B)$. This amount is represented on the $H\phi$ chart by $c'b$. It also follows that $K = H_A - H_C$ is the actual heat drop, represented on the $H\phi$ chart by ac'.

Example 3. Find the throat and exit diameters of a convergent divergent nozzle which will discharge 120 lb. of steam per hour from a pressure of 120 lb. per sq. in. absolute into a chamber having a pressure of 30 lb. per sq. in. absolute. Assume the steam is originally dry saturated and that the flow is adiabatic (*a*) without friction, (*b*) with the friction loss in the divergent part of the nozzle equal to one-tenth of the total heat drop. What will be the final velocity of the steam in each case ?

(*a*) *Frictionless Adiabatic Flow.* To find the throat area we may use the formula

$$\frac{M}{A_t} = 0.3003 \sqrt{\frac{p_1}{V_1}}$$

where $M = \dfrac{120}{60 \times 60} = \dfrac{1}{30}$ lb. per sec., and $V_1 = 3.751$ cub. ft. per lb. (from Table I).

$\therefore$ Area of throat $A_t = \dfrac{1}{30 \times 0.3003} \times \sqrt{\dfrac{3.751}{120}} = 0.0196$ sq. in.

$\therefore$ Diameter of throat $D_t = \sqrt{\dfrac{4 \times 0.0196}{3.14}} = 0.158$ in.

As the steam is originally dry saturated, the throat area may also be calculated from the equation

$$M = 0.0165 \, A_t p_1^{\frac{31}{32}}$$

$\therefore \qquad A_t = \dfrac{120}{60 \times 60 \times 0.0165} \times \dfrac{1}{120^{\frac{31}{32}}} = 0.01955$ sq. in.
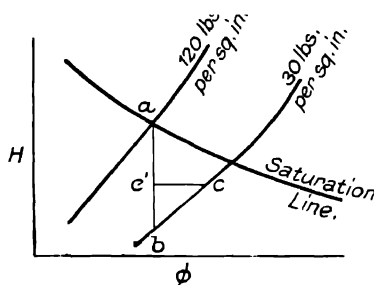
Fig. 190

To find the *exit area* we may use the equations

$$\frac{M}{A_2} = \sqrt{2g \, \frac{n}{n-1} \, \frac{P_1}{V_1} \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}} \right\}}$$

$$\frac{M}{A_t} = 3.604 \sqrt{\frac{P_1}{V_1}}$$

whence $\qquad \dfrac{A_2}{A_t} = \dfrac{3.604 \sqrt{\dfrac{P_1}{V_1}}}{\sqrt{2g \, \dfrac{n}{n-1} \, \dfrac{P_1}{V_1} \left\{ \left(\dfrac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\dfrac{P_2}{P_1}\right)^{\frac{n+1}{n}} \right\}}}$

$$= \frac{3.604}{\sqrt{\dfrac{2 \times 32.2 \times 1.135}{0.135} \left\{ \left(\dfrac{30}{120}\right)^{\frac{2}{1.135}} - \left(\dfrac{30}{120}\right)^{\frac{2.135}{1.135}} \right\}}} = 1.34$$

$\therefore$ Area of outlet $A_2 = 1.34 \times 0.0196 = 0.0263$ sq. in.

whence diameter of outlet $= 0.183$ in.

From the $H\phi$ chart the final state of the steam is given by the point b, Fig. 190, showing a dryness fraction of 0.92.

Heat drop for expansion $ab = 60 \cdot 1$ C.H.U. (from $H\phi$ chart).

$\therefore$ Velocity at exit $U_2 = \sqrt{2gJ \text{ (heat drop)}}$

$$= \sqrt{2 \times 32 \cdot 2 \times 1400 \times 60 \cdot 1} = 2328 \text{ ft. per sec.}$$

It should be noted that the exit area could be calculated from this value of U_2, using the equation

$$U_2 A_2 = M \times V_s$$

where V_s is the specific heat of dry saturated steam at 30 lb. per sq. in.

$$\therefore A_2 = \frac{M \times V_s}{U_2} = \frac{120}{60 \times 60} \times \frac{0 \cdot 92 \times 13 \cdot 74}{2328} \times 144 = 0 \cdot 0261 \text{ sq. in.}$$

(b) *Frictionally Resisted Flow.* As the friction is only in the divergent part of the nozzle, the throat area is the same as in the first case. To find the exit area we cannot use the formula containing n as we do not know its value. The actual heat drop with friction

$$= 0 \cdot 9 \times 60 \cdot 1 = 54 \cdot 09 \text{ C H.U.}$$

$\therefore$ the final velocity

$$U_2 = \sqrt{2 \times 32 \cdot 2 \times 1400 \times 54 \cdot 09} = 2208 \text{ ft. per sec.}$$

The dryness fraction at exit may be obtained from the $H\phi$ chart as follows : Find the point C' (Fig. 190) such that $c'b = \frac{1}{10} ab$. Draw a horizontal line through c' to cut the 30 lb. per sq. in. pressure line in C. This point will show the final state of the steam to be in this case $0 \cdot 932$.

$$\therefore \text{ Exit area} = \frac{M \times V_s}{U_2} = \frac{120}{60 \times 60} \times \frac{0 \cdot 932 \times 13 \cdot 74}{2208} \times 144$$

$$= 0 \cdot 0278 \text{ sq. in., whence diameter of exit} = 0 \cdot 188 \text{ in.}$$

Supersaturated Expansion in Nozzles. As already pointed out, if the value of n for dry saturated steam is taken as $1 \cdot 135$ the value of the discharge calculated from the equation

$$M = 3 \cdot 604 \, A_t \sqrt{\frac{P_1}{V_1}}$$

is less than the discharge actually measured by experiment, although the calculated discharge from this theoretical equation, which assumes the absence of friction, should obviously be greater than the discharge obtained in practice where friction is present. This fact seems to show that the usual type of adiabatic expansion, where there is at any point during the expansion an equilibrium mixture of vapour and liquid, does not take place. It is now generally assumed that supersaturated expansion takes place. This type of expansion has already been touched on in a previous paragraph (page 429), but we must now go farther into the

matter. We have already seen how this type of expansion can be shown on the $T\phi$ chart, and we will now indicate it on the $H\phi$ chart.

Let AB and CD, Fig. 191, be two constant pressure lines at P_1 and P_2 lb. per sq. in. respectively. It will be remembered that these lines are curved in the superheated region and straight in the wet region. The ordinary type of adiabatic expansion will be represented for a pressure drop from P_1 to P_2 by the vertical line EF. Supersaturated expansion will be represented by EG, where G is the point on the continuation of the curved portion CC^1 of the constant pressure line CD into what is normally the wet region. In this case of supersaturated expansion we must imagine a state of affairs in which the saturation line, which normally divides the superheated region from the wet region, has no existence. Under these circumstances, at the point G, after supersaturated expansion, the fluid will be a homogeneous gas, whereas, had the ordinary adiabatic expansion taken place, the state of the fluid given by the point F would have been that of a wet mixture. It should be remembered that the temperature in the state indicated by the point G will be lower than the saturation temperature corresponding to the pressure P_2.
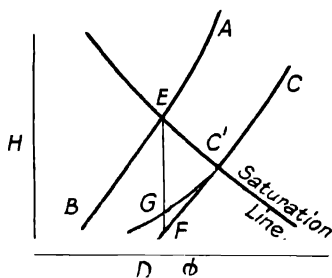
FIG. 191

Supersaturated expansion does not, however, proceed indefinitely. As we have already seen, the density of supersaturated steam at any point in the expansion is greater than it would be at the same point had the expansion proceeded under ordinary adiabatic conditions. The limit of supersaturation is attained when the density of the supersaturated vapour is about eight times that of the ordinary saturated vapour at the corresponding pressure. As soon as this limit is reached, the vapour will begin to condense and will eventually become an equilibrium mixture of vapour and liquid. The time taken for this condensation is a debatable point and is too complicated for discussion here, but for the present purpose we may assume that, when the supersaturation limit is reached, condensation to the proper equilibrium mixtures takes place instantaneously.

The limit of supersaturated expansion was first shown by the experiments of C. T. R. Wilson in 1897, and the subsequent work of H. M. Martin has enabled a curve, termed by him the *Wilson line*, to be applied to the $H\phi$ chart. If expansion is carried below this line it can be here assumed for practical purposes that instantaneous condensation takes place when the Wilson line is reached. Such an expansion is shown in Fig. 192.

Suppose expansion takes place from a pressure P_1 to a pressure P_3. The first stage of the expansion AB is supersaturated, the point B being the intersection of the curved extension of that constant pressure line P_2 which cuts the Wilson line vertically below A. At this point condensation takes place. If this is instantaneous it will take place at constant pressure, and as there is no heat interchange, the total heat will remain constant, and the condensation is represented by the horizontal line BC. When equilibrium is attained by condensation there will be a quantity of heat available due to the condensation. This heat will cause the steam to be drier at C than at E, the point which would have been reached had ordinary adiabatic expansion taken place down to the pressure P_2. The remainder of the expansion is indicated by CD, this being ordinary adiabatic expansion. The actual state of affairs is more complicated than this, but with the present state of knowledge, for practical purposes, the steam may be considered to attain the equilibrium condition as soon as the Wilson line is reached.
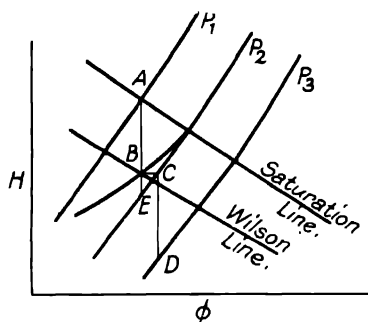
FIG. 192

As we have already seen, during supersaturated expansion the same equations held as in the case of adiabatic expansion of superheated steam, namely,

$$P(V-b)^{1\cdot3} = \text{constant} \;\; ; \;\; \frac{P}{T^{\frac{13}{3}}} = \text{constant} \;\; ; \;\; T^{\frac{10}{3}}(V-b) = \text{constant}.$$

It will be remembered that the temperature of supersaturated steam at any particular pressure during the expansion is lower than the saturation temperature corresponding to this pressure. The degree of undercooling at a pressure P_2 due to supersaturation can therefore be obtained by first finding the temperature T'_2 from the equation

$$\frac{P_1}{T_1^{\frac{13}{3}}} = \frac{P_2}{T_2^{\frac{13}{3}}}$$

and then subtracting this temperature from the saturation temperature corresponding to the pressure P_2.

In order to calculate the total heat of supersaturated steam we may use the Callendar equation (p. 414)

$$H = \frac{p(V - 0\cdot0123)}{2\cdot2436} + 464 \text{ calories}$$

p being in pounds per square inch, and V in cubic feet.

The critical pressure and discharge of a nozzle with supersaturated steam will be the same as in the case of superheated steam, namely,

$$P_t = 0.5457\, P_1, \text{ and } M = 3.786\, A_t \sqrt{\frac{P_1}{V_1}}$$

The value of the discharge calculated from this equation will be 4 or 5 per cent greater than the discharge actually obtained by experiment.

Heat Drop in Supersaturated Expansion. The heat drop between two pressures P_1 and P_2 can be calculated from the equation for total heat just given, the value of V for the lower pressure being calculated from the equation

$$P_1(V_1 - b)^{1.3} = P_2(V_2 - b)^{1.3}$$

The value of $b = 0.01602$ cub. ft. can be neglected for elementary purposes.

From previous work we know that the heat drop

$$H_1 - H_2 = \int_{P_2}^{P_1} \frac{V\, dP}{J} \qquad \frac{n}{n-1} \frac{(P_1 V_1 - P_2 V_2)}{J}$$

where $n = 1.3$.

This gives another method of calculation.

A third equation in terms of pressures can be obtained as follows,

$$P(V - b)^{1.3} = P_1(V_1 - b)^{1.3}$$

$$\therefore \qquad V = \left(\frac{P_1}{P}\right)^{\frac{10}{13}} (V_1 - b) + b$$

$\therefore$ Heat drop,

$$H_1 - H_2 = \int_{P_2}^{P_1} \frac{V\, dP}{J} = \frac{1}{J} \int_{P_2}^{P_1} \left\{ \left(\frac{P_1}{P}\right)^{\frac{10}{13}} (V_1 - b) + b \right\} dP$$

$$\frac{P_1^{\frac{10}{13}}(V_1 - b)}{J} \int_{P_2}^{P_1} \frac{dP}{P^{\frac{10}{13}}} + \frac{b}{J} \int_{P_2}^{P_1} dP$$

$$= \frac{13}{3} \frac{P_1^{\frac{10}{13}}(V_1 - b)}{J} \left[P_1^{\frac{3}{13}} - P_2^{\frac{3}{13}} \right] + \frac{b}{J}(P_1 - P_2)$$

Example 4. Explain what is meant by the supersaturated expansion of steam and give some idea of the limits within which this condition is possible. Steam is expanded from a pressure of 60 lb. per sq. in. absolute and 170° C. to a pressure of 20 lb. per sq. in. absolute. If the expansion is supersaturated and occurs with a friction loss of 5 per cent, determine the actual and isentropic drops and degree of undercooling.

For the supersaturated state you may use the approximate Callendar equations—

$$V = \frac{2.2436\,(H - 464)}{P}; \frac{P}{T^{\frac{14}{5}}} = \text{constant}; \text{ and } PV^{1.3} = \text{constant}$$

Take the specific heat as 0.52. Volume of 1 lb. of steam at 60 lb. per sq. in. and 170° C. is 7.685 cub. ft. (*U.L.*, *B.Sc. (Eng.)*, 1921.)

Supersaturated Flow. $PV^{1.3} = K$

∴ specific volume after expansion $V_2 = \left(\dfrac{P_1}{P_2}\right)^{\frac{1}{1.3}} V_1$

$$\left(\frac{60}{20}\right)^{\frac{1}{1.3}} \times 7.685 = 17.89 \text{ cub. ft}$$

But $V = \dfrac{2.436\,(H - 464)}{P}$

∴ total heat before expansion,

$$H_1 = \frac{VP}{2.2436} + 464 = \frac{7.685 \times 60}{2.2436} + 464 = 669.5 \text{ C.H.I}$$

∴ total heat after expansion

$$H_2 = \frac{17.89 \times 20}{2.2436} + 464 = 623.5 \text{ C.H.U.}$$

∴ heat drop (without friction) = 669.5 – 623.5 = 46 C.H.U.

∴ actual heat drop = 0.95 × 46 = 43.7 C.H.U.

The heat drop (without friction) can also be calculated from the equation

$$H_1 - H_2 = \frac{n}{n - 1} \left(\frac{P_1 V_1 - P_2 V_2}{J}\right)$$

$$= \frac{1.3}{0.3} \left(\frac{60 \times 7.685 - 20 \times 17.89}{1400}\right) \times 144 = 46 \text{ C.H.U.}$$

Isentropic Flow.

Total heat before expansion, $H_1 = 669.5$ C.H.U

Dryness fraction after expansion

$$x = \left(\log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} + C_p \log_\varepsilon \frac{T_{su}}{T_1}\right) \frac{T_2}{L_2}$$

$$= \left(2.3 \log_{10} \frac{417.89}{381.97} + \frac{510.22}{417.89} + 0.52 \times 2.3 \log_{10} \frac{443}{417.89}\right) \frac{381.97}{533.87}$$

$$= 0.959$$

∴ Total heat after expansion

$$H_2 = h_2 + xL_2 = 108.95 + 0.959 \times 533.87 = 621.14 \text{ C.H.U.}$$

∴ isentropic heat drop (with friction)

$$= 0.95(H_1 - H_2) = 0.95\,(669.5 - 621.1) = 46 \text{ C.H.U.}$$

Degree of Undercooling. $\quad \dfrac{P}{T^{\frac{13}{3}}} = \text{constant.}$

∴ Temperature after supersaturated expansion

$$T_2 = \left(\frac{P_1}{P_2}\right)^{\frac{3}{13}} T_{su} = \left(\frac{20}{60}\right)^{\frac{3}{13}} \times 443 = 343.8° \text{ C. (abs.)}$$

But saturation temp. at 20 lb. per sq. in. (abs.) $= 381\cdot97°$ C. (abs.).

∴ undercooling $= 381\cdot97 - 343\cdot8 = 38\cdot2°$ C.

Example 5. A nozzle is supplied with steam at 100 lb. per sq. in. absolute pressure and 275° C. Find the temperature and velocity of the steam at the throat of the nozzle.

If the diverging portion of the nozzle is 2 in. long and the throat diameter ¼ in., determine the angle of the cone so that the steam may leave the nozzle at 15 lb. per sq. in. absolute. Assume a friction loss of 15 per cent. of the heat drop used in this diverging part. (*U.L., B.Sc. (Eng.), 1924.*)

Saturation temperature for 100 lb. per sq. in. $= 164\cdot28°$ C.

∴ amount of superheat $= 275 - 164\cdot28 = 110\cdot72°$ C.

First find the state of the steam after ordinary adiabatic expansion to 15 lb. per sq. in. from the $H\phi$ chart. A vertical line drawn from the intersection of the 275° C. temperature line with the 100 lb. per sq. in. pressure line (or of the 110·72 superheat line with the 100 lb. per sq. in. pressure line) will cut the 15 lb. per sq. in. pressure line at a point where the dryness fraction is 0·982. An $H\phi$ chart on which the Wilson line is drawn will show that this point is above this line. Hence the whole expansion may be taken as super-saturated.

The critical pressure, $P_t = 0\cdot5457\,P_1 = 0\cdot5457 \times 100 = 54\cdot57$.

Hence, temperature at throat,

$$T_t = \left(\frac{P_t}{P_1}\right)^{\frac{1}{13}} T_1 = \left(\frac{54\cdot57}{100}\right)^{\frac{3}{13}} \times (275 + 273) = 476\cdot5°\ \text{C. (abs.)}.$$

∴ temperature at throat, $t_t = 476\cdot5 - 273 = 203\cdot5°$ C.

The temperature of saturation at 54·57 lb. per sq. in. $= 141\cdot4°$ C.

∴ degree of superheat at throat $= 203\cdot5 - 141\cdot4 = 62\cdot1°$ C.

(This temperature can be read direct from the $H\phi$ chart, from the point of intersection of the vertical expansion line with the 54·57 lb. per sq. in. pressure line.)

To obtain the velocity at the throat, the heat drop from 100 lb. per sq. in. to 54·57 lb. per sq. in. is required.

Total heat of superheated steam at 100 lb. per sq. in. and 275° C. (i.e. 110·72° of superheat),

$H_1 = 720\cdot42$ C.H.U. (by interpolation from tables).

Total heat of superheated steam at 54·57 lb. per sq. in. and 203·5° C. (62·1° of superheat),

$H_2 = 686\cdot92$ C.H.U.

∴ Heat drop $H_1 - H_2 = 720\cdot42 - 686\cdot92 = 33\cdot50$ C.H.U.

(This value can be read direct from the $H\phi$ chart.)

∴ Velocity at throat $= \sqrt{2gJ}$ (heat drop)

$$= \sqrt{2 \times 32\cdot2 \times 1400 \times 33\cdot5} = 1738\ \text{ft. per sec.}$$

Now $V = \dfrac{2 \cdot 2436 \, (H - 464)}{p} + 0 \cdot 0123$

$\therefore$ specific volume at throat

$$= \frac{2 \cdot 2436 \, (686 \cdot 92 - 464)}{54 \cdot 57} + 0 \cdot 0123 = 9 \cdot 18 \text{ cub. ft.}$$

$\therefore$ Using suffixes t and e for throat and exit respectively (neg-lecting friction),

$$P_t V^{1 \cdot 3} = P_e V_e^{1 \cdot 3}, \text{ and } V_e = \left(\frac{P_t}{P_e} \right)^{\frac{1}{1 \cdot 3}} V_t$$

$$= \left(\frac{54 \cdot 57}{15} \right)^{\frac{1}{1 \cdot 3}} \times 9 \cdot 18 = 24 \cdot 77 \text{ cub. ft.}$$

$\therefore$ Total heat at exit, H_e (neglecting friction)

$$= \frac{p_e \, (V_e - 0 \cdot 0123)}{2 \cdot 2436} + 464$$

$$= \frac{15(24 \cdot 77 - 0 \cdot 0123)}{2 \cdot 2436} + 464 = 629 \cdot 52 \text{ C.H.U.}$$

$\therefore$ Heat drop between throat and exit (no friction)

$$= 686 \cdot 92 - 629 \cdot 52 = 57 \cdot 4 \text{ C.H.U.}$$

$\therefore$ actual heat drop in diverging portion

$$= 0 \cdot 85 \times 57 \cdot 4 = 48 \cdot 8 \text{ C.H.U.}$$

But heat drop in converging portion $= 33 \cdot 5$ C.H.U.

$\therefore$ total heat drop along nozzle $= 33 \cdot 5 + 48 \cdot 8 = 82 \cdot 3$ C.H.U.

$\therefore$ final velocity $= \sqrt{2gJ \text{ heat drop}}$

$$= \sqrt{2 \times 32 \cdot 2 \times 1400 \times 82 \cdot 3} = 2724 \text{ ft. per sec.}$$

Now $MV = UA$; $\therefore \dfrac{A_e}{A_t} = \dfrac{V_e}{U_e} \times \dfrac{U_t}{V_t}$

or $\dfrac{\text{Diameter at exit}}{\text{Diameter at throat}} = \sqrt{\dfrac{A_e}{A_t}} = \sqrt{\dfrac{V_e}{U_e} \times \dfrac{U_t}{V_t}}$

$\therefore$ Diameter at exit $= \dfrac{1}{4} \sqrt{\dfrac{24 \cdot 77}{2724} \times \dfrac{1738}{9 \cdot 18}} = 0 \cdot 328$ in.

Hence if $\phi =$ angle of cone, $\tan \phi = \dfrac{0 \cdot 328 - 0 \cdot 25}{2 \times 2}$

$$= 0 \cdot 0195,$$

and $\therefore \phi = 1° 6'$.

EXAMPLES XI

1. Steam flows through a correctly designed nozzle with a pressure drop from 200 lb. to 2 lb. per in. absolute. Assuming adiabatic frictionless flow,

find the dryness fraction and the velocity of the steam as it leaves the nozzle when the steam entering the nozzle is (a) dry saturated, (b) superheated 50° C. above the saturation temperature.

2. Find the velocity of the steam, in Question (1), assuming the flow is supersaturated throughout the expansion.

3. Find the correct throat and exit diameters of a convergent-divergent nozzle which will discharge 120 lb. of steam per hour from a pressure of 120 lb. per sq. in., superheated to 220° C., into a chamber having a pressure of 30 lb. per sq. in. absolute. The friction loss in the divergent part of the nozzle may be taken as one-tenth of the total heat drop.

(*U.L., B.Sc. (Eng.), 1923.*)

4. A convergent divergent nozzle is to be designed to discharge 0·15 lb. of steam per sec. into a vessel in which the pressure is 20 lb per sq. in. absolute, when the nozzle is supplied with steam at 100 lb. per sq. in. absolute, superheated to 200° C. Find the throat and exit diameters of the nozzle on the assumption that the friction loss in the diverging part is 10 per cent of the total heat drop (*U.L., B.Sc. (Eng.), 1925*)

5. A nozzle discharges steam dry saturated at 160 lb. per sq. in. absolute pressure into a reservoir where the pressure is 1 lb. per sq. in. absolute. The diameter of the nozzle at the throat is ⅜ in. If the flow is frictionless adiabatic, what weight of steam will pass through the nozzle per minute ? If 10 per cent of the energy goes in friction, what weight of steam is delivered per minute ? (*U.L., B.Sc. (Eng.).*)

6. Dry and saturated steam at 25 lb. per sq. in. absolute pressure is allowed to discharge through a long convergent nozzle into the atmosphere (15 lb. per sq. in.). Assuming frictionless adiabatic flow, find the dryness of the steam at discharge and calculate the weight of steam which should be discharged per second if the exit diameter of the nozzle is 0·5 in. If the weight of steam actually discharged be 94 per cent of this calculated quantity, estimate the percentage of the heat drop which is wasted in friction.

(*U.L., B.Sc (Eng.), 1923.*)

7. A divergent nozzle for a steam turbine has to deliver 800 lb. of steam per hour under a supply condition of 160 lb. per sq. in absolute, with 200° F. (111° C.) of superheat, and a stage pressure of 1·5 lb. per sq. in. absolute. Assuming a coefficient of discharge of 0·97 and a frictional loss of 12 per cent, determine the throat and outlet areas. (*I.Mech.E., 1926.*)

● 8. The throat and exit diameters of a nozzle are 0·300 and 0·375 inches respectively. If dry and saturated steam at 100 lb. per sq. in absolute is allowed to expand through the nozzle into a chamber in which the pressure is 15 lb. per sq. in. absolute, find the weight of steam passing per second. If 10 per cent of the heat drop is wasted in friction, what is the probable pressure of the steam at its exit from the nozzle ? (*U.L., B.Sc. (Eng.), 1922.*)

CHAPTER XII

STEAM TURBINES

The Velocity Diagram. We have seen in the last chapter that when steam expands through a suitably shaped passage or nozzle, some of the heat energy in the steam may be converted into kinetic energy, the steam thus attaining a high velocity. If this steam is now allowed to pass through blades situated in a wheel so arranged that it may revolve freely about its axis (Fig. 193), motion will be imparted to the wheel and mechanical work can be thus obtained.
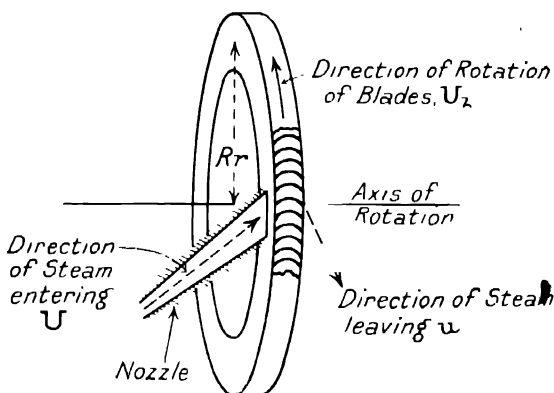
Fig. 193

As the blades are situated radially round the wheel, the linear velocity of the outside edges will be greater than that of the inside edges. Let the linear velocity of the blades at the mean radius R_r, Fig. 193, be U_b. We can now draw velocity triangles for the steam on entering and leaving the blading.

In Fig. 194, let CD be a moving blade.

At Entry. Let $AB = U$ = Velocity of steam leaving nozzle and entering moving blade (feet per sec.)

$CB = U_b$ = Mean velocity of moving blade (feet per sec.)

$\therefore AC = R$ = Velocity of steam relative to moving blade at entry (feet per sec.)

In order that the steam should enter the moving blade without shock, there must be no component velocity of the steam relative to the blade at right angles to the profile of the blade at entry. In

498

other words, the entry angle of the moving blade must be the same as the angle β made by the direction of the steam AC at entry relative to the direction of motion CB of the moving blade.

At Exit. $FD = U_b$ = Mean velocity of moving blade (ft. per sec.)

$\qquad DE = r$ = Velocity of steam relative to moving blade at exit (feet per sec.)

$\qquad \therefore FE = u$ = Actual velocity of steam at exit (feet per sec.)

It is obvious that the direction of the relative velocity at exit DE will be at an angle γ to the direction of motion of the moving blade, the angle γ being the exit angle of the blade.

These two velocity triangles can be combined for convenience in one diagram (Fig. 195).

From this diagram it will be seen that the change in velocity produced in the steam by its passage through the moving blade is ED. If

Fig. 194

this change in velocity is resolved parallel and at right angles to the direction of motion of the moving blade, then CD is the component change in velocity produced in the steam parallel to the direction of motion of the moving blade. Let this velocity be U_c.

Fig. 195

Work Done in Blading. According to Newton's second law of motion, force is measured by the rate of change of momentum.
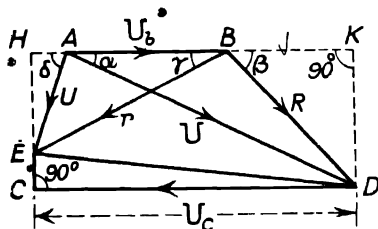
Hence, if W lb. of steam pass through the blading per second, the force exerted in the direction of motion by the steam on the moving blades $= \dfrac{W}{g} \times$ change in velocity of the steam in the direction of motion (in feet per second).

$$= \frac{W}{g} \times U_c \text{ lb.} \quad \text{(Fig. 195)}$$

∴ work done per second

$$= \text{Force} \times \text{Distance moved per second}$$

$$= \frac{W}{g} \times U_c \times U_b \text{ foot-lbs. per second.}$$

∴ horse-power of blading $= \dfrac{W}{g} \times \dfrac{U_c U_b}{550}$.

It should be noted that if Fig. 195 is drawn to scale, the values U_c and U_b can be read from the diagram.

Another expression for the work done can be found as follows—
From Fig. 195 we have

$$AD^2 = DB^2 + AB^2 + 2DB \times AB \cos \beta$$

$$\therefore \quad U^2 = R^2 + U_b{}^2 + 2RU_b \cos \beta$$

also $AE^2 = EB^2 + AB^2 - 2EB \times AB \cos \gamma$

$$\therefore \quad u^2 = r^2 + U_b{}^2 - 2rU_b \cos \gamma.$$

Subtracting these two equations,

$$2U_b(R \cos \beta + r \cos \gamma) = U^2 - u^2 - R^2 + r^2.$$

But from Fig. 195 we see that

$$U_c = HA + AK = HB + BK$$

$$= u \cos \delta + U \cos \alpha - r \cos \gamma + R \cos \beta$$

But $2U_b(R \cos \beta + r \cos \gamma) = U^2 - u^2 - R^2 + r^2$

$$\therefore \quad 2U_b U_c = U^2 - u^2 - R^2 + r^2.$$

∴ Work done per second

$$= \frac{W}{g} \cdot U_c U_b = W\left(\frac{U^2 - u^2 - R^2 + r^2}{2g} \right)$$

This result can be obtained direct from a consideration of kinetic energy. The kinetic energy of the steam on entry to the moving blades is $\dfrac{U^2}{2g}$ per lb. If r is greater than R, as in the reaction turbine, then this increase of velocity must have taken place by an expansion of steam in the moving blades, similar to the expansion in the nozzle. Hence the increase in kinetic energy due to expansion in the moving blades is $\left(\dfrac{r^2}{2g} - \dfrac{R^2}{2g} \right)$ per lb. The total kinetic energy supplied to the moving blades is therefore $\dfrac{U^2 + r^2 - R^2}{2g}$ per lb. of steam. The final kinetic energy of the steam on leaving the moving blades is $\dfrac{u^2}{2g}$ per lb. Hence the energy imparted to the moving blades is $\dfrac{U^2 + r^2 - R^2 - u^2}{2g}$ per lb.

Hence, work done per second $= \dfrac{W(U^2 - u^2 - R^2 + r^2)}{2g}$ ft.-lb.

$\therefore$ Horse-power $= \dfrac{W}{2g}\left(\dfrac{U^2 - u^2}{550}\dfrac{R^2 + r^2}{-}\right)$

It should be noted that if r is less than R, as in the case of an impulse turbine, then $\left(\dfrac{r^2}{2g} - \dfrac{R^2}{2g}\right)$ is negative and is the loss of energy due to friction as the steam passes through the moving blades.

The Impulse Turbine. In an impulse turbine expansion of the steam occurs practically only in the nozzles or fixed blades. The expansion converts heat energy into kinetic energy, as already explained, and the steam leaving the nozzles at a high velocity enters the moving blades, giving up some of its kinetic energy to them, and hence producing motion of the rotor. The state of affairs in a single stage simple impulse turbine is shown diagrammatically in Fig. 196, which should be compared with Fig. 193.

It is obvious that several nozzles, all supplied with steam from the same source, may be situated round the moving blade ring. The curves in Fig.
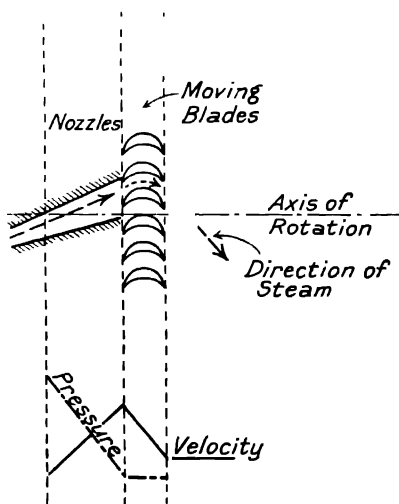
FIG. 196

196 indicate the variations of pressure and velocity throughout the stage.

The pressure will drop during the passage of the steam through the nozzles, and will then remain practically constant in the moving blades. The velocity will increase, due to the pressure drop in the nozzles, and will then decrease as kinetic energy is given to the moving blades.

Efficiency of Impulse Turbine Blading. As the pressure remains constant through the moving blades, any difference between R and r (Fig. 195) will be due to friction during the passage of the steam through the moving blades. Hence the value of r will be less than that of R.

Let $r = KR$. Then work done per second

$$= \frac{W(U^2 - u^2 - R^2 + r^2)}{2g} = \frac{W(U^2 - u^2)}{2g} - \frac{R^2}{2g}(1 - K^2).$$

Kinetic energy supplied per second $= \dfrac{WU^2}{2g}$

$\therefore$ Efficiency $= \dfrac{\text{Work done per second}}{\text{Kinetic energy supplied per second}}$

$$= \dfrac{\dfrac{W(U^2 - u^2 - R^2 + r^2)}{2g}}{\dfrac{WU^2}{2g}} = \dfrac{U^2 - u^2 - R^2 + r^2}{U^2}$$

$$= \dfrac{U^2 - u^2}{U^2} - \dfrac{R^2}{U^2}(1 - K^2).$$

If we assume that there are no losses in the moving blades, then $R = r$ and

Efficiency $= \dfrac{U^2 - u^2}{U^2}$.

The efficiency can also be expressed in other terms, as follows :
It has already been shown that

Work done per second $= \dfrac{W}{g} \times U_c \times U_b$.   (Fig. 195)

$\therefore$ Efficiency $= \dfrac{2\,U_c \times U_b}{U^2}$.

Maximum Efficiency of Impulse Turbine Blading (U and a given).
Work done per second $= \dfrac{W}{g} \times U_c \times U_b$

But $\qquad U_c = BK + BH$ (Fig. 195) $= R\cos\beta + r\cos\gamma$

$$= R\cos\beta + KR\cos\gamma = R\cos\beta\left(1 + K\dfrac{\cos\gamma}{\cos\beta}\right)$$

and $\quad R\cos\beta = AK - AB = U\cos a - U_b$

$$\therefore \quad U_c = (U\cos a - U_b)\left(1 + K\dfrac{\cos\gamma}{\cos\beta}\right)$$

$\therefore$ Work done per second $= \dfrac{W}{g} \times U_c \times U_b$

$$= \dfrac{W}{g}(U_b U\cos a - U_b{}^2)\left(1 + K\dfrac{\cos\gamma}{\cos\beta}\right)$$

$\therefore$ Efficiency $= \dfrac{\dfrac{W}{g}(U_b U\cos a - U_b{}^2)\left(1 + K\dfrac{\cos\gamma}{\cos\beta}\right)}{\dfrac{WU^2}{2g}}$

$$= 2\left(\dfrac{U_b}{U}\cos a - \dfrac{U_b{}^2}{U^2}\right)\left(1 + K\dfrac{\cos\gamma}{\cos\beta}\right)$$

If we assume that the exit blade angle γ is a constant proportion of the inlet blade angle β, then $\dfrac{\cos \gamma}{\cos \beta} = $ Constant.

$\therefore$ Efficiency $= 2\left(\dfrac{U_b}{U}\cos \alpha - \dfrac{U_b^2}{U_s}\right) C$, where C is constant.

$\therefore$ Differentiating and equating to zero we have, for maximum efficiency

$$\cos \alpha = \frac{2U_b}{U} \text{ or blade velocity } U_b = U^{\cos \alpha}$$

Substituting this value of U_b in the general equation for efficiency just obtained

$$\text{Maximum efficiency} = 2\left(\frac{\cos^2 \alpha}{2} - \frac{\cos^2 \alpha}{4}\right)\left(1 + K\frac{\cos \gamma}{\cos \beta}\right)$$

$$= \frac{\cos^2 \alpha}{2}\left(1 + K\frac{\cos \gamma}{\cos \beta}\right)$$
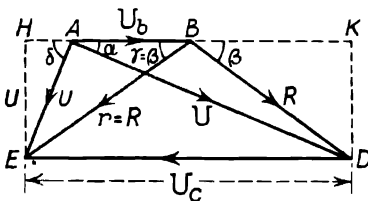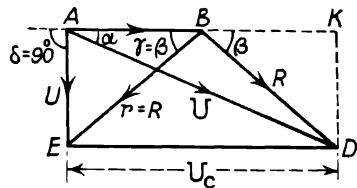
FIG. 197 (a)   FIG. 197 (b)

Maximum Efficiency of Impulse Turbine Blading (U and α given, and assuming symmetrical blading with no losses). If there are no losses, $R = r$, $\therefore K = 1$, and, with symmetrical blading, $\gamma = \beta$.

$\therefore$ Efficiency $= 2\left(\dfrac{U_b}{U}\cos \alpha - \dfrac{U_b^2}{U^2}\right)\left(1 + K\dfrac{\cos \gamma}{\cos \beta}\right)$

$$= 4\left(\frac{U_b}{U}\cos \alpha - \frac{U_b^2}{U^2}\right)$$

$\therefore$ Maximum efficiency $= \dfrac{\cos^2 \alpha}{2}\left(1 + K\dfrac{\cos \gamma}{\cos \beta}\right) = \cos^2 \alpha$.

These results can also be obtained by modifying Fig. 195. The diagram so modified is shown in Fig. 197 (a).

The triangle *DBE* becomes isosceles and the actual change of velocity *DE* (Fig. 195) becomes parallel to the direction of motion of the blades.

$$\therefore r \cos \gamma = R \cos \beta$$

$$\therefore U_c = 2R \cos \beta = 2(AK - AB) = 2(U \cos a - U_b)$$

$$\therefore \text{Work done per second} = \frac{W}{g} U_c U_b = \frac{W}{g} 2(U \cos a - U_b) U_b$$

$$= 2 \frac{W}{g} (U_b U \cos a - U_b^2)$$

$$\therefore \text{Efficiency} = \frac{2 \dfrac{W}{g} (U_b U \cos a - U_b^2)}{\dfrac{W U^2}{2g}} = 4 \left(\frac{U_b}{U} \cos a - \frac{U_b^2}{U^2} \right).$$

Differentiating and equating to zero,

$$\cos a = 2 \frac{U_b}{U} \quad \text{for maximum efficiency.} \qquad \backslash$$

This will be the case when the diagram is as Fig. 197 (b), where $\delta = 90°$; for here $\cos a = \dfrac{AK}{AD} = \dfrac{2AB}{AD} = \dfrac{2U_b}{U}$

Hence maximum efficiency

$$= \frac{\dfrac{W}{g} U_c U_b}{\dfrac{W}{2g} U^2} = 2 \times \frac{DE \times AB}{AD^2} \quad \text{(Fig. 197 (b).)}$$

$$= \frac{2 \times AK \times \dfrac{AK}{2}}{AD^2} = \cos^2 a.$$

The fact that for maximum efficiency the angle δ must be **90°** also follows from the expression for efficiency, namely, $\dfrac{U^2 - u^2}{U^2}$. For this to have a maximum value for any given value of U, the value of u must be a minimum. Fig. 197 shows that this will be the case where $\delta = 90°$, that is, when the direction of the steam on leaving the moving blades is at right angles to the direction of motion of the blades, the steam then having no velocity in the direction of motion of the blades.

Effect of Blade Speed on Efficiency of Impulse Turbine Blading. It is instructive to plot curves showing the variation of blade efficiency with the value of the ratio $\dfrac{\text{Blade speed}}{\text{Steam speed}} = \dfrac{U_b}{U}$. For simplicity we will assume symmetrical blades, i.e. $\gamma = \beta$. If then, in the general equation,

$$\text{Efficiency} = 2\left(\frac{U_b}{U}\cos\alpha - \frac{U_b^2}{U^2}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

$$= 2\frac{U_b}{U}\left(\cos\alpha - \frac{U_b}{U}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

we substitute constant values for U and K, we can calculate the values of the efficiency for various values of the ratio $\frac{U_b}{U}$. This has been done in Fig 198, curves A and B for a value of $\alpha = 20°$. Curve A is the efficiency curve for no losses, i.e. $R = r$, making $K = 1$. Curve B is the efficiency curve for a value of $K = 0\cdot8$.

$\frac{U_b}{U}$	$K=1$	$K=0\cdot8$
0	0	0
0·1	0·336	0·302
0·2	0·592	0·533
0·3	0·768	0·691
0·4	0·864	0·778
0·5	0·879	0·791
0·6	0·815	0·734
0·7	0·671	0·604
0·8	0·447	0·402
0·9	0·143	0·129
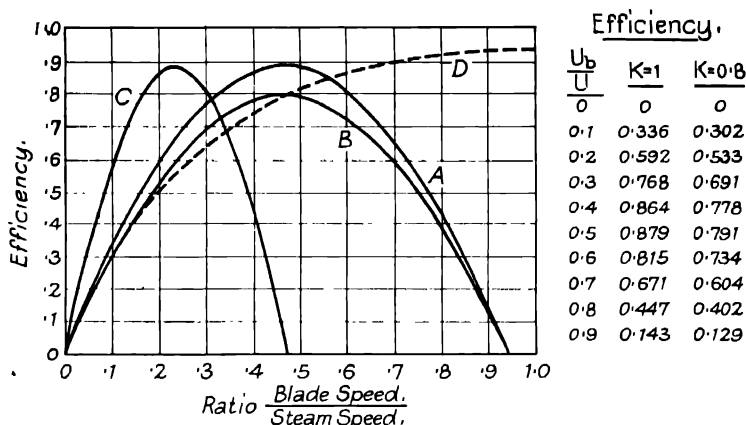
Fig. 198. Efficiency Curves

We see that the efficiency is zero when

$$\frac{U_b}{U} = 0, \text{ i.e. when } U_b = 0,$$

and when $\quad\dfrac{U_b}{U} = \cos 20° = 0\cdot9397$

The maximum efficiency is equal to $\cos^2 20° = 0\cdot883$ in the case where there are no losses. When $K = 0\cdot8$ the maximum efficiency is equal to $\dfrac{\cos^2 20°}{2}(1 + 0\cdot8) = 0\cdot795$. In both cases the maximum efficiency is obtained when

$$\frac{U_b}{U} = \frac{\cos 20°}{2} = 0\cdot4698$$

Example 1. A stage in an impulse turbine consists of converging nozzles and one ring of moving blades. The nozzle angles are 22° and the moving blades have both blade tip angles of 35°. If the velocity of the steam at

the exit from the nozzle is 1,500 ft. per sec., find the blade speed so that the steam shall pass on to the blade without shock, and find the stage efficiency, neglecting frictional losses, if the blades run at this speed.

If the relative velocity of steam to blade is reduced by 15 per cent in passing through the blade ring, find the actual efficiency and the end thrust on the shaft when the blade ring develops 50 H.P.

(U.L., B.Sc. (Eng.), 1924.)

(1) Let the mass of steam passing per second $= W = 1$ lb. Then Fig. 199 shows the velocity diagram, assuming no losses. Draw the angle $CAD = 22°$ and let the length AD represent 1,500 ft. per sec. to some convenient scale. As there are no losses and the blade angles are equal, the direction of the actual change of velocity of the steam will be parallel to the direction of motion of the blades. Therefore draw DF parallel to AC and make the angles DBC and ABE each equal to $35°$, and draw $BE = BD$. Then DE represents U_c, the change in velocity produced in the steam, and AB represents
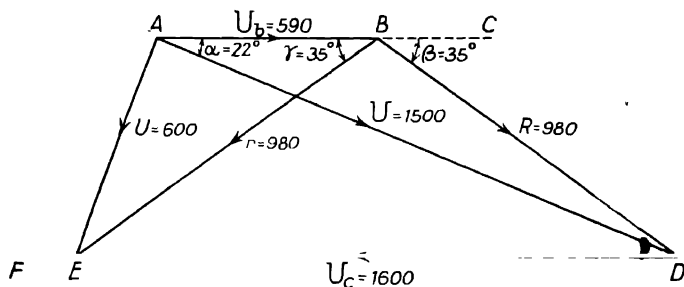
Fig. 199

U_b, the velocity of the blades. The values of the various vectors are shown on the diagram.

Blade speed for no shock and no losses $= U_b = 590$ ft. per sec.

Efficiency $= \dfrac{2\,U_c U_b}{U^2} = \dfrac{2 \times 1600 \times 590}{(1500)^2} = 83.9$ per cent

or   Efficiency $= \dfrac{U^2 - u^2}{U^2} = \dfrac{1500^2 - 600°}{1500^2} = 84$ per cent.

(2) With 15 per cent loss in passing through moving blade

$r = 0.85R$, i.e. $K = 0.85$.

Now, from Fig. 195, EC represents the change in velocity produced in the steam in a direction parallel to the axis of rotation. This change in velocity will produce a corresponding end thrust on the shaft.

From Fig. 195, $EC = DK - EH = U \sin \alpha - r \sin \gamma$

$= 1500 \times \sin 22° - 0.85\,R \sin 35°$

$$\therefore \qquad EC = 561 \cdot 9 - 0 \cdot 4876R \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

Also $DK = U \sin \alpha = R \sin \beta$

$$\therefore \quad R \sin \beta = 561 \cdot 9 \quad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

$$\text{Also} \quad U_c = r \cos \gamma + R \cos \beta$$
$$= 0 \cdot 85R \cos 35° + R \cos \beta$$
$$= 0 \cdot 6963R + R \cos \beta$$

$$\text{and} \qquad U_b = U \cos \alpha - R \cos \beta$$
$$= 1500 \times \cos 22° - R \cos \beta$$
$$= 1390 \cdot 8 - R \cos \beta$$

$$\text{But H.P.} = \frac{U_c \times U_b}{550 \, g}$$

$$\therefore \quad 50 = \frac{(0 \cdot 6963R + R \cos \beta) \, (1390 \cdot 8 - R \cos \beta)}{550 \times 32 \cdot 2} \qquad . \qquad . \qquad (3)$$

From the equations (1), (2), and (3), we can obtain solutions : $R = 968$ ft. per sec. ; $\beta = 35 \cdot 5°$; $U_b = 604$ ft. per sec. ; $U_c = 1461$ ft. per sec. ; EC (Fig. 195) = 90 ft. per sec.

$$\therefore \text{Efficiency} = \frac{2U_c U_b}{U^2} = \frac{2 \times 1461 \times 604}{(1500)^2} = 78 \cdot 44 \text{ per cent.}$$

$$\text{End thrust} \quad = \frac{90}{32 \cdot 2} = 2 \cdot 8 \text{ lb.}$$

Velocity Compounding. We have seen in the last chapter that when steam expands in a suitably shaped nozzle, a large velocity can be generated in the steam. Thus, if dry steam expands through a nozzle from 200 lb. per sq. in. (abs.) to 2 lb. per sq. in. (abs.), the final velocity of the steam is about 3,900 ft. per sec. We have just seen that, if this steam is passed through a single row of blades situated round the periphery of a wheel, the linear velocity of the blades must be somewhere about half that of the steam if the efficiency is to be a maximum. The speed of rotation of a rotor carrying a single row of blades would therefore be very high, and reduction gearing would be required to bring the speed down for most practical purposes. One method of reducing the speed of the turbine rotor is to provide the wheel with two rows of blades, between which there is a row of fixed blades attached to the stationary turbine casing. This row of fixed blades simply changes the direction of the steam leaving the first row of moving blades back to a direction suitable for entering the second row of moving blades. Fig. 200 shows diagrammatically a section of a velocity compounded wheel. Curves are also given showing the variation of pressure and velocity of the steam as it passes through the blading.

It will be noticed that the pressure drop takes place in the nozzles, there being no further pressure drop in either the fixed or

moving blades.   The high velocity produced in the steam is reduced
on passing through each row of moving blades.   As there is no
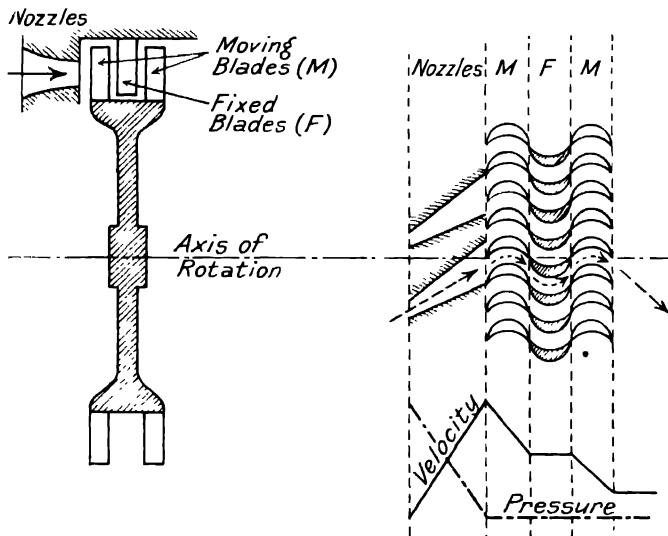
Fɪɢ. 200

pressure drop in the intermediate fixed row of blades, there is no
change in the value of the velocity but only in its direction.   In
practice, friction occurs and modifies matters to some extent.

Efficiency of a Velocity Compounded Stage.   The complete velocity
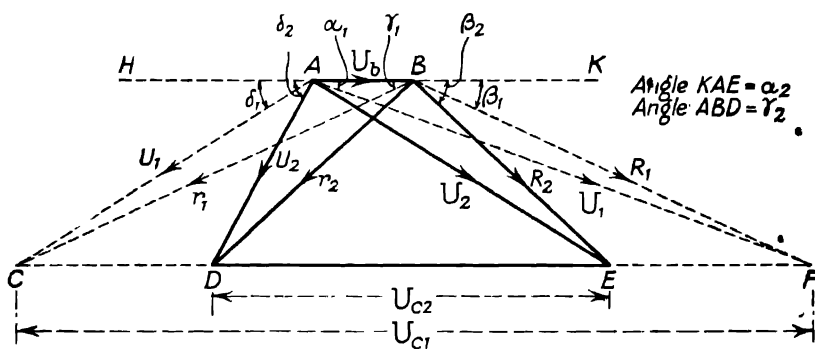
Fɪɢ. 201

diagram will consist of two diagrams, one for each set of moving
blades.   Let us assume for simplicity that the blading is sym-
metrical and that there is no friction.   Then Fig. 201 shows the
velocity diagram.

Let AF represent U_1 the velocity of the steam leaving the nozzles and entering the first row of moving blades. Then $ABFC$ is the velocity diagram for the first row of moving blades, the actual velocity of the steam on leaving this first row being represented by $AC = u_1$. The intermediate fixed blades must have an entry angle equal to the angle HAC if the steam is to enter without shock and an equal exit angle if the blade is symmetrical. Hence, if the angle KAE is made equal to the angle HAC and AE is made equal to AC, then AE will represent the velocity of the steam leaving the intermediate fixed blades, and this will be the velocity U_2 of the steam entering the second row of moving blades. It is obvious that, with the simple conditions assumed here, the point E will be on the line CF. The blade velocity U_b being the same for both rows of moving blades, the velocity diagram $ABED$ for the second row of moving blades may now be drawn, making $r_2 = R_2$ and angle $HBD =$ angle KBE. The point D will then also be on the line CF. The change in velocity produced in the steam by its passage through the first row of moving blades U_{c1} is represented by CF, while that produced by the passage through the second row of moving blades U_{c2} is represented by DE.

In Fig. 201 we have $U_2 = u_1$, $a_2 = \delta_1$, $r_2 = R_2$, $\gamma_2 = \beta_2$.

Then $CE = 2u_1 \cos \delta_1$
$$DE = 2R_2 \cos \beta_2 = 2(U_2 \cos a_2 - U_b) = 2(u_1 \cos \delta_1 - U_b)$$
$$\therefore CD = CE - DE = 2u_1 \cos \delta_1 - 2(u_1 \cos \delta_1 - U_b) = 2U_b$$
Also $CF = 2R_1 \cos \beta_1 = 2r_1 \cos \gamma_1 = 2(u_1 \cos \delta_1 + U_b)$
$$\therefore EF = CF - CE = 2(u_1 \cos \delta_1 + U_b) - 2u_1 \cos \delta_1 = 2U_b$$
$$\therefore DE = CF - (CD + EF)$$
$$\therefore U_{c2} = U_{c1} - 4U_b$$
$$\therefore \text{Total change of velocity } U_c = U_{c1} + U_{c2} = 2U_{c1} - 4U_b$$
But $U_{c1} = 2R_1 \cos \beta_1 = 2(U_1 \cos a_1 - U_b)$
$$\therefore U_c = 4(U_1 \cos a_1 - U_b) - 4U_b$$
$$= 4(U_1 \cos a_1 - 2U_b)$$
$\therefore$ If $W =$ mass of steam passing per second (in pounds)

Work done per second $= \dfrac{W}{g} U_c U_b$
$$= \dfrac{W}{g} 4(U_1 U_b \cos a_1 - 2U_b^2) \text{ (ft. lb.)}$$

Efficiency $= \dfrac{\dfrac{W}{g} 4(U_1 U_b \cos a_1 - 2U_b^2)}{\dfrac{WU_1^2}{2g}}$
$$= 8\left(\dfrac{U_b}{U_1} \cos a_1 - 2\left[\dfrac{U_b}{U_1}\right]^2\right)$$

This will be a maximum when $\cos \alpha_1 = 4 \dfrac{U_b}{U_1}$

$\therefore$ For maximum efficiency $\dfrac{U_b}{U_1} = \dfrac{\cos \alpha_1}{4}$

and Maximum Efficiency $= \cos^2 \alpha$

In a similar manner it can be shown that, with n rows of moving blades, maximum efficiency will be obtained when $\dfrac{U_b}{U_1} = \dfrac{\cos \alpha_1}{2n}$.
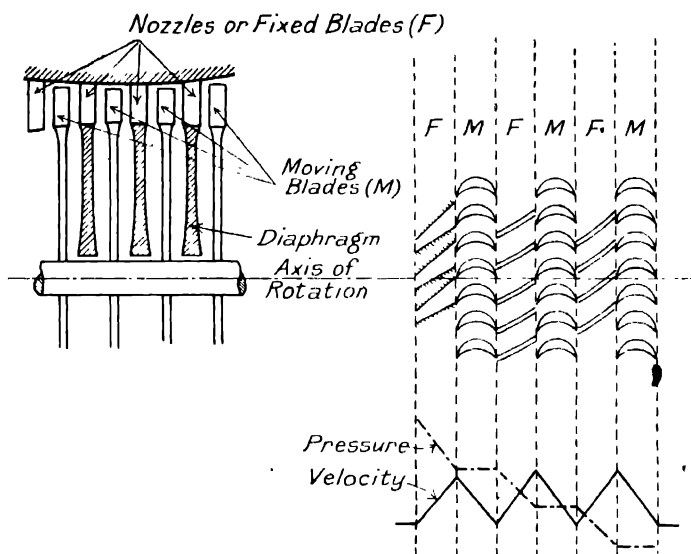
Fig. 202

Fig. 198, curve C, shows the variation of efficiency with the value of the ratio $\dfrac{\text{Blade speed}}{\text{Steam speed}}$ for a value of $\alpha_1 = 20°$.

It will be seen that the maximum efficiency equals $\cos^2 20° = 0{\cdot}883$ when the value of $\dfrac{\text{Blade speed}}{\text{Steam speed}} = \dfrac{\cos 20°}{4} = 0{\cdot}2349$ and the efficiency is zero when the value of $\dfrac{\text{Blade speed}}{\text{Steam speed}} = \dfrac{\cos 20°}{2} = 0{\cdot}4698$.

Pressure Compounding. Another method of obviating the difficulty of the high velocity produced by a large drop in pressure is that of pressure compounding. With this method the whole pressure drop does not take place in one set of nozzles, but it is split up

into a number of stages each consisting of a set of nozzles or fixed blades with a ring of moving blades, the various rings of moving blades all being fixed to the same rotor shaft. In this way the pressure drop in any one set of nozzles or fixed blades may be small and hence the velocity produced in the steam may be kept to any desirable value. The general arrangement is shown diagrammatically in Fig. 202.

The pressure drops take place only in the nozzles or fixed blades ; in fact a pressure compounded turbine is simply a number of simple turbines in series. A stage consists of one set of nozzles or fixed blades, together with a ring of moving blades. There are thus four stages shown in Fig. 202. By suitably adjusting the number of stages it can be arranged that the rotor runs at any required speed for any given pressure drop in the steam through the whole turbine. In the first one or two stages, where the pressure of the steam is high and its specific volume small, the nozzles or fixed blades do not always extend all round the periphery of the wheel. In sub-sequent stages, as the steam expands the nozzles cover the whole circumference of the wheel. In modern turbines working with steam at a high initial pressure, the first stage is sometimes velocity compounded. In this way a comparatively large pressure drop can take place in the first stage nozzles, thus reducing the pressure inside the main turbine casing.

Example 2. Steam is supplied to an impulse steam turbine at a pressure of 180 lb. per sq. in. absolute, and superheated to 250° C. The pressure in the wheel chamber is 80 lb per sq. in. absolute, and in the chamber there are two rings of moving blades separated by fixed blades. The tips of the moving blades are inclined 30° to the plane of motion. Assuming a 10 per cent friction loss in the nozzle and also a reduction of 8 per cent in the velocity of the steam relative to the blade due to frictional resistances in passing through a blade ring, determine the speed of the blades so that the final velocity of discharge shall be axial. Also state what should be the inclination of the nozzles to the plane of motion of the blades.

(*U.L.*, *B.Sc.* (*Eng.*), 1922.)

Since the pressure drop in the nozzles is from 180 lb. per sq. in. to 80 lb. per sq. in., the velocity of the steam on leaving the nozzles is given by

$$U_1 = \sqrt{2gJ \,(\text{Adiabatic heat drop from } 180 \text{ to } 80 \text{ lb. per sq. in.})}$$

$$= \sqrt{2 \times 32 \cdot 2 \times 1400 \times (703 - 667) \times 0 \cdot 9}$$

$$= 1709 \text{ ft. per sec.}$$

This is, therefore, the value of U_1, the velocity of the steam on entering the first ring of moving blades.

The velocity diagram (Fig. 203) can now be drawn, starting from the final velocity v_2 of the steam on leaving the second ring of moving blades.

Draw a triangle ABC, Fig. 203, with angle $BAC = 90°$ and angle $ABC = 30°$. Make angle $EBD = 30°$ and make $BD = \dfrac{CB}{0·92}$. Join AD. Then, as CB is 8 per cent less than BD, the figure $ABCD$ is the velocity diagram for the second ring of moving blades, drawn to some scale yet to be determined. Produce BC and let $AF = \dfrac{AD}{0·92}$ cut BC produced in F. Produce BD to H, making $BH = \dfrac{FB}{0·92}$. Join AH. Then the figure $ABHF$ will be the velocity diagram for the first ring of moving blades. AB will represent the velocity of the moving blades. AF will represent the velocity of the steam on leaving the first ring of moving blades. This velocity will be
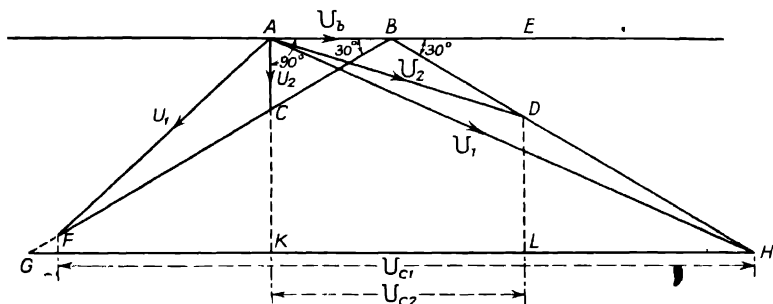
Fig 203

changed in direction and reduced by 8 per cent in magnitude by the passage through the fixed blades. Hence the velocity on leaving the fixed blades will be given by AD, which, therefore, also represents the velocity of the steam U_2 on entering the second ring of moving blades.

The velocity U_1 of the steam on leaving the nozzles and entering the first row of moving blades is represented by AH. This must be the value calculated at the beginning of the example, namely, 1709 ft. per sec. This value, compared to the length of AH, fixes the velocity scale of the diagram.

From the diagram : Speed of blades $= 392$ ft. per sec.

Nozzle angle $= BAH = 23·5°$.

Also, efficiency $= \dfrac{2(U_{c1} + U_{c2})U_b}{U_1{}^2} = \dfrac{2(2260 + 820) \times 392}{1709 \times 1709}$

$= 82·7$ per cent.

The Reaction Turbine. In the reaction turbine the steam expands in both the fixed and the moving blades, the pressure dropping

continuously right through the turbine. The velocity diagram for one reaction stage, that is, for one set of fixed and one set of moving blades, is similar to Fig. 195, with the modification that, as expansion takes place in the moving blades, r will be greater than R.

The work done per second $= \dfrac{W}{g} \times U_c \times U_b$.

$$\text{or} = \frac{W(U_1^2 - u^2 + r^2 - R^2)}{2g}.$$

The energy supplied to the stage per second

$$= W\left(\frac{U_1^2}{2g} \; + \; \begin{array}{l}\text{Kinetic energy generated, due to}\\ \text{expansion in the moving blades}\end{array}\right)$$

$$= W\left(\frac{U_1^2}{2g} + \frac{r^2 - R^2}{2g}\right)$$

$$\therefore \text{Efficiency} = \frac{U_1^2 - u^2 + r^2 - R^2}{U_1^2 + r^2 - R^2}.$$
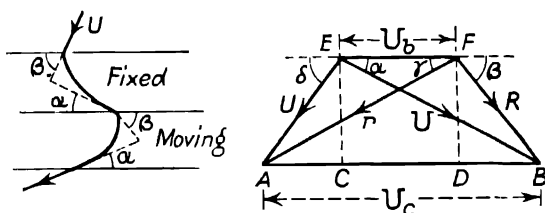
Fig. 204

The efficiency can obviously also be expressed as equal to

$$\frac{\dfrac{W}{g} \times U_c \times U_b}{\dfrac{W(U_1^2 + r^2 - R^2)}{2g}} = \frac{2 \times U_c \times U_b}{U^2 + r^2 - R^2}.$$

Efficiency of Reaction Blading with Fixed and Moving Blading Identical.

We will assume there are no losses due to friction. If the fixed and moving blades of a stage are identical, the velocity diagram reduces to Fig. 204.

In this case : $\gamma = a$, $\delta = \beta$, $U = r$, $u = R$

The fixed blade will receive steam from the preceding moving blade at a velocity u. The velocity of the steam after leaving the fixed blade is U. Hence—

Kinetic energy of supply to fixed blade $= \dfrac{Wu^2}{2g}$

Kinetic energy generated in fixed blad $\quad - \dfrac{W(U^2 - u^2)}{2g}$

Kinetic energy generated in moving blad $\quad \dfrac{W(r^2 - R^2)}{2g}$

$$= \dfrac{W(U^2 - u^2)}{2g}$$

$\therefore$ Total energy supplied $= 2\,\dfrac{W(U^2 - u^2)}{2g} + \dfrac{W u^2}{2g} = \dfrac{W}{2g}(2U^2 - u^2)$

But $\quad FB^2 = EB^2 + EF^2 - 2EB \times EF \cos FEB \quad$ (Fig. 204)

$\therefore \quad R^2 = U^2 + U_b{}^2 - 2UU_b \cos \alpha.$ But $u = R$

$\therefore$ Total energy supplied $= \dfrac{W}{2g}(2U^2 - u^2)$

$$= \dfrac{W}{2g}(2U^2 - [U^2 + U_b{}^2 - 2UU_b \cos \alpha])$$

$$= \dfrac{W}{2g}(U^2 - U_b{}^2 + 2UU_b \cos \alpha)$$

Work done per second $\quad = \dfrac{W}{g} U_c U_b$

But $\quad U_c \quad = AB = CB + AD - CD \quad$ (Fig. 204)

$$= U \cos \alpha + r \cos \gamma - U_b = 2U \cos \alpha - U_b$$

$\therefore$ Efficiency $= \dfrac{\dfrac{W}{g}(2U \cos \alpha - U_b)U_b}{\dfrac{W}{2g}(U^2 - U_b{}^2 + 2UU_b \cos \alpha)}$

$$= \dfrac{2\left(2\,\dfrac{U_b}{U}\cos \alpha - \dfrac{U_b{}^2}{U^2}\right)}{1 - \dfrac{U_b{}^2}{U^2} + 2\,\dfrac{U_b}{U}\cos \alpha}$$

$$= \dfrac{2(2\lambda \cos \alpha - \lambda^2)}{1 - \lambda^2 + 2\lambda \cos \alpha}$$

where $\lambda = \dfrac{\text{Blade speed}}{\text{Steam speed leaving fixed blading}}$.

To obtain the maximum efficiency, this expression must be differentiated and equated to zero. This gives, for maximum efficiency,

$$\lambda = \dfrac{U_b}{U} = \cos \alpha$$

whence maximum efficiency $= \dfrac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$

A curve D, shown in Fig. 198, indicates the variation of efficiency with the ratio $\dfrac{\text{Blade speed}}{\text{Steam speed}}$.

It must be remembered that the heat drop in the case of the reaction turbine takes place in two parts. In the impulse turbine the whole heat drop occurs in the nozzles or fixed blading, the final velocity on leaving the fixed blades being that corresponding to this heat drop. In the reaction turbine, part of the heat drop occurs in the fixed blading, thus increasing the velocity to U on leaving the fixed blades. There is then a further heat drop in the moving blades, thus increasing the relative velocity from R to r. In the simple case just discussed, as $\dfrac{U^2 - u^2}{2g} = \dfrac{r^2 - R^2}{2g}$, the total heat drop across the stage is split up into two equal parts. In other words, in the case of the impulse turbine there are two distinct operations in a stage. In the fixed blades some of the heat energy of the steam is converted into kinetic energy. In the moving blades part of the kinetic energy of the steam is transferred to the rotor. In the reaction turbine these two operations cannot be separated. Expansion of the steam takes place in both fixed and moving blades, and therefore there is a continuous conversion of heat energy into kinetic energy of the steam throughout the stage.

Height of Blading. (a) *Impulse Blading.* Let Fig. 205 (a) bo the section of two consecutive blades.

Let V = Volume of steam at point considered (cub. ft. per lb.)

h = Height of blade (in.) (Fig. 205 b)

r = Velocity of steam on leaving moving blade (ft. per sec.)

γ = Exit angle of moving blade

t = Thickness of edge of blade (in.) (Fig. 205 a)

n = Number of blades covered by nozzle

W = Mass of steam passing per second (lb.)

p = pitch (in.) (Fig. 205 a)

D = Mean diameter of blade ring (in.) (Fig. 205 b)

N = Number of revolutions per minute of blade ring.

If we consider the outlet end of the blades, we have the relation—

Leaving velocity of steam relative to blade $\times$ Area available to steam at right angles to this velocity = Mass of steam passing per second $\times$ Volume per pound of steam.

i.e. $r \times nh(p \sin \gamma - t) = WV \times 144$

(b) *Reaction Blading.* In a reaction turbine the admission of

steam is all round the rotor. The blades, as usually constructed, are also sharp, so that t may be neglected.

Hence, area available to steam per blade $= hp \sin \gamma$

$\therefore$ Total area $\quad = nhp \sin \gamma$. But $np = \pi D$

$\therefore$ Total area $\quad = \pi Dh \sin \gamma$

$\therefore r \times \pi Dh \sin \gamma = WV \times 144$

If the fixed and moving blades are similar, as in Fig. 204, then

$$r = U, \text{ and if } \frac{\text{Blade velocity}}{\text{Steam velocity}} = \frac{U_b}{U} = \lambda,$$

then $\quad r = U = \frac{U_b}{\lambda} = \frac{\pi DN}{12 \times 60 \, \lambda}$

$$\therefore \frac{\pi^2 D^2 Nh \sin \gamma}{720 \times 144 \, \lambda} = WV$$
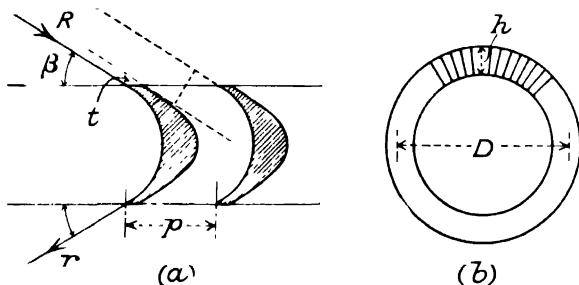
Fig. 205

Re-heating and the Condition Curve. Modern impulse turbines for any but small powers are multi-stage machines. The pressure drop in the steam is divided up into a comparatively small number of stages, as already described under pressure compounding. In some cases the first stage is velocity compounded. The reaction turbine consists of a large number of stages, the pressure drop being continuous right through the blading

We must now consider how the condition of the steam changes as it passes through the blading. Suppose the supply pressure to a turbine be P_1 and the discharge pressure P_2. For clearness, let us consider, say, five stages, the intermediate pressures being p_a, p_b, p_c, p_d. These pressure lines are indicated on the $H\phi$ chart, Fig. 206, which, it will be remembered, is drawn for 1 lb. of steam.

In the ideal machine the work done on the turbine rotor would be equal to the adiabatic heat drop aA. We have already seen, however, that, owing to friction losses, etc., the full adiabatic heat

drop is not converted into mechanical work on the rotor. Thus,
after expansion ab in the first stage, the state of the steam on leaving
that stage will not be represented by the point b, but by some
point a_1, the actual heat drop, and hence the work done on the rotor
being represented by ac and not by ab, which represents the adiabatic
heat drop for the stage. The total heat on leaving the first stage,
given by the point c, is greater than would be the case were the
expansion ideal. The steam is thus said to have been re-heated.
Considering the complete expansion through the five stages, the
state of the steam may be considered to change as represented by
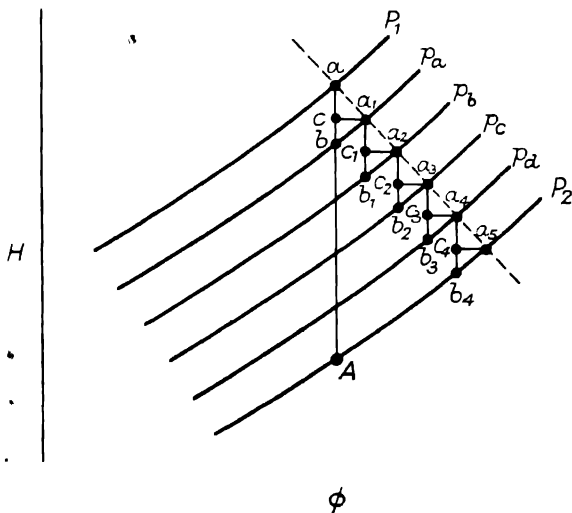
Fig. 206

the path, a, a_1, a_2, a_3, a_4, a_5 The sum of the separate adiabatic
heat drops, namely, Σab (called the cumulative heat drop) is,
obviously, greater than the adiabatic heat drop aA. The ratio
$\dfrac{\Sigma ab}{aA}$ is termed the re-heat factor. Let this be represented by R.
It will be noticed that, owing to this re-heating, there is an increase
of entropy and an additional amount of heat is lost to the turbine
exhaust.

Efficiency of a Compound Turbine. The stage efficiency ε_s will
be measured by the ratio of the work done on the rotor in that stage
to the stage adiabatic heat drop, i.e. $\varepsilon_s = ac/ab$.

This efficiency must not be confused with the blading efficiency,
which is a measure of the conversion of kinetic energy of the steam
into kinetic energy of the blades.

The internal efficiency of the turbine is measured by the ratio

of the total work done on the rotor (measured from the diagram) to the total adiabatic heat drop, i.e. $\varepsilon_i = \dfrac{\Sigma\, ac}{aA} = \dfrac{\Sigma\, \varepsilon_s \times ab}{aA}$.

If we assume that the stage efficiency ε_s is the same for all stages, then $\varepsilon_i = \varepsilon_s \dfrac{\Sigma\, ab}{aA} = \varepsilon_s R$.

The internal efficiency accounts for all the internal losses due to friction, etc. It obviously does not take account of outside losses due to friction at the bearings, etc. These are all accounted for by the " efficiency ratio," which is measured by the ratio of total work produced at the driving end of the shaft per pound of steam to the adiabatic heat drop across the turbine. The absolute thermal efficiency of the turbine is the ratio of the actual output of the turbine to the heat supplied to the turbine.

$U_h = 202$

$\delta = 35°$ $\alpha = 20°$ $\gamma = 20°$ $\beta = 35°$

$U_r = 265$ $r = 447$ $U = 447$ $R = 265$

D $U_c = 637$ C
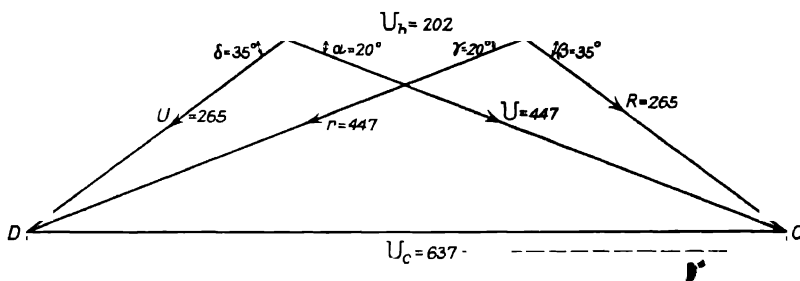
FIG. 207

Example 3. In a reaction steam turbine the fixed and moving blades have inclinations at the receiving and discharging tips of 35° and 20° respectively. If steam passes through the blades without shock, what would be the work developed in a ring of moving blades per pound of steam if the mean diameter of the blade ring circle is 5 ft. 6 in , and the speed of rotation 700 r.p.m. ?

Assuming an efficiency of 85 per cent for the one pair of fixed and moving blades, find the heat drop which takes place in the pair, and find the state of the steam when arriving at the fixed blades if the steam has a pressure of 20 lb. per sq. in. and dryness 0·92 when leaving the moving blades.

(*U.L., B.Sc. (Eng.)*, 1922.)

Blade speed at the mean periphery

$$= \frac{\pi \times 5\cdot 5 \times 700}{60} = 202 \text{ ft. per sec.}$$

Fig. 207 shows the velocity diagram. where AB represents this blade speed. Make the angle $EBC = 35°$ and the angle $EAC = 20°$. The diagram can now be completed. The values of the various velocities read off the diagram are given in the figure.

Work done per pound of steam

$$= \frac{U_b U_c}{g} = \frac{202 \times 637}{32\cdot 2} = 4000 \text{ ft.-lb. (approx.)}$$

The work done per pound of steam is also given by

$$\frac{U_1^2 - u^2}{2g} + \frac{r^2 - R^2}{2g} = 2\left(\frac{447^2 - 265^2}{2 \times 32 \cdot 2}\right) = 4024 \text{ ft.-lb.}$$

The discrepancy between the two results is, of course, due to slight inaccuracy in the drawing of the velocity diagram.

$$\text{Stage efficiency} = \frac{\text{Work done on rotor}}{\text{Adiabatic heat drop}}$$

$\therefore$ adiabatic heat drop $(H_1 - H_2) = \dfrac{4000}{0 \cdot 85 \times 1400} = 3 \cdot 36$ C.H.U.

Total heat after expansion

$$H_2 = h_2 + x_2 L_2 \text{ at 20 lb. per sq. in. (from tables)}$$
$$= 108 \cdot 95 + 0 \cdot 92 \times 533 \cdot 87 = 600 \cdot 11 \text{ C.H.U.}$$

$\therefore$ total heat before expansion $H_1 = 600 \cdot 11 + 3 \cdot 36 = 603 \cdot 47$ C.H.U.

The state point of the steam before expansion could now be obtained from the $H\phi$ chart. The heat drop is, however, so small that a large scale diagram would be required for accuracy. The state can be calculated as follows—

Entropy during expansion

$$= 2 \cdot 3 \log_{10} \frac{T_2}{273 \cdot 1} + \frac{x_2 L_2}{T_2} \text{ at 20 lb. per sq. in. (from tables)}$$

$$= 2 \cdot 3 \log_{10} \frac{381 \cdot 97}{273 \cdot 1} + \frac{0 \cdot 92 \times 533 \cdot 87}{381 \cdot 97} = 1 \cdot 62$$

$\therefore$ Entropy during expansion $= 1 \cdot 62 = 2 \cdot 3 \log_{10} \dfrac{T_1}{273 \cdot 1} + \dfrac{x_1 L_1}{T_1}$ (1)

Total heat before expansion

$$= 603 \cdot 47 = h_1 + x_1 L_1 = (T_1 - 273 \cdot 1) + x_1 L_1 \quad . \qquad . \quad (2)$$

$\therefore$ From (2), $x_1 L_1 = 603 \cdot 47 + 273 \cdot 1 - T_1 = 876 \cdot 57 - T_1$

Substituting in (1), $1 \cdot 62 = 2 \cdot 3 \log_{10} \dfrac{T_1}{273 \cdot 1} + \dfrac{876 \cdot 57 - T_1}{T_1}$.

Solving this equation, $T_1 = 384 \cdot 6°$ C. (abs.).

Pressure at this saturation temperature $= 22$ lb. per sq. in. approx.

$\therefore H_1 = h_1 + x_1 L_1$

$603 \cdot 47 = 111 \cdot 83 + x_1 \times 532 \cdot 09$ at 22 lb. per sq. in. (from tables)

$\therefore \quad x = 0 \cdot 924$

State of steam at entrance to fixed blades : Pressure $= 22$ lb. per sq. in. (abs.) ; dryness fraction $= 0 \cdot 924$.

Example 4. A reaction steam turbine is supplied with steam at 150 lb per sq. in. absolute pressure, superheated to 250° C., and exhausts at 1·5 lb.

34—(5434)

per sq. in. absolute. Assuming that the frictional loss in the blades is 25 per cent of the adiabatic heat drop, and taking a re-heat factor of 1·05, find the steam used per horse-power hour developed.

If the turbine develops 15,000 H.P. at 350 r.p.m., calculate the blade height and drum diameter at the point in the turbine where the pressure is 15 lb. per sq. in. absolute and the dryness factor 0·985. The discharge tip angle of the blades is 20°, and the blade velocity is 0·7 of the relative velocity of the discharging steam, and the blade height is one-twelfth of the drum diameter.   (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

Stage efficiency $= 0·75$

$\therefore$ internal efficiency of turbine $=$ Re-heat factor $\times$ Stage efficiency

$$= 1·05 \times 0·75 = 0·7875$$

Adiabatic heat drop from 150 lb. per sq. in. and 250° C. to 1·5 lb per sq. in. $= 179$ C.H.U. (from $H\phi$ chart).

$\therefore$ Neglecting outside losses : Work done on rotor per pound of steam

$$= 0·7875 \times 179 \times 1400 = 197,350 \text{ ft.-lb.}$$

$\therefore$ Weight of steam required per horse-power hour

$$= \frac{33,000 \times 60}{197,350} = 10 \text{ lb. (approx.)}$$

Steam used per second

$$= \frac{\text{H.P.} \times 10}{60 \times 60} = \frac{15,000 \times 10}{60 \times 60} = 41·67 \text{ lb. per sec.}$$

Specific volume of steam at 15 lb. per square inch

$= V = xV_s$ where $V_s =$ Specific volume of dry steam at 15 lb. per sq. in.

$$= 0·985 \times 26·27 = 25·88 \text{ cub ft. per lb.}$$

Blade height

$$h = \frac{WV \times 720 \times 144 \times \lambda}{\pi^2 D^2 N \sin \gamma} \left(\begin{array}{c} \text{where } D \text{ is mean diameter} \\ \text{of blade ring.} \end{array} \right)$$

$$= \frac{WV \times 720 \times 144 \times \lambda}{\pi^2 h^2 \times 144 \times N \sin \lambda} \left(\begin{array}{c} \text{neglecting difference between} \\ D \text{ and actual drum diameter.} \end{array} \right)$$

$$\therefore \quad h = \sqrt[3]{\frac{WV \times 720 \times \lambda}{\pi^2 \times N \sin \gamma}}$$

$$= \sqrt[3]{\frac{41·67 \times 25·88 \times 720 \times 0·7}{3·14 \times 3·14 \times 350 \times 0·342}} = 7^3/_4 \text{ in.}$$

$\therefore$ Mean drum diameter $= 7$ ft. 9 in.

EXAMPLES XII

1. The nozzles of a turbine stage deliver 9 lb. of steam per second at an outlet angle of 18° and a speed of 1,400 ft. per second. If the blading outlet angle is 22°, and the blading velocity coefficient is 0·76, determine the blade power developed and the inlet blade angle. Take the peripheral speed of the wheel as 550 ft. per second. (*I.Mech.E.*, 1924.)

2. The steam supplied to a single row impulse wheel turbine expands in the nozzles over such a range that the adiabatic heat drop is 38 B.Th.U. per lb. The nozzle efficiency is 93 per cent and the nozzle angle is 15°. If the blading speed is 575 ft. per second, the outlet blade angle 18°, and the velocity coefficient for the blading 0·82, determine a suitable inlet angle for the blade and the speed and direction of the steam after discharge from the blading.
(*I.Mech.E.*, 1923.)

3. At a stage of an impulse turbine the wheel diameter is 39 in. and the r.p.m. 3,000. The ratio, blade speed/steam speed is 0·34, the nozzle efficiency 0·93, disk and vane friction 3 per cent of stage heat drop ; blade entrance and exit angles 29° and 25° respectively, and velocity coefficient for blading 0·78. Draw the velocity diagram and calculate (*a*) the work done on blading per lb. of steam ; (*b*) the stage efficiency. (*I.Mech.E.*, 1927.)

4. Steam issues from a nozzle on to the blade ring of an impulse steam turbine with a velocity of 1,500 ft. per second. The receiving and discharging tips of the moving blades are inclined at 30° to the plane of motion and the nozzle is inclined at 20° to that plane. The mean diameter of the blade ring circle is 3 ft. The discharge from the nozzle is 0·23 lb. per second. Assuming a friction loss of 15 per cent in the velocity of the steam relative to the blade whilst passing through the blade ring, find: (1) the instantaneous torque on the shaft when the turbine is beginning to move ; (2) the speed of the turbine so that the steam shall discharge axially, and the horse-power developed at this speed. (*U.L.*, *B.Sc.* (*Eng.*), 1925.)

5. In a De Laval turbine in which the blades of the rotor have equal inlet and outlet angles, steam is supplied dry and saturated at 150 lb. per sq. in. (abs.) and the exhaust pressure is 2 lb. per sq. in. (abs.). The nozzle makes an angle of 20° with the direction of rotation of the blades and the peripheral speed of the blades is 1,160 ft. per second. Assuming that the losses due to friction and supersaturation effects in the nozzle amount to 15 per cent, and that the relative velocity of the steam at exit from blades is 80 per cent of that at inlet, calculate the velocity of discharge of the steam from the nozzles, the inlet angle of the blades for the condition that there should be no shock at entry, and the work done per lb. of steam. (*U.L.*, *B.Sc.* (*Eng.*), 1926.)

6. In an impulse steam turbine the steam issues from the nozzle with a speed of 1,520 ft. per second. The velocity is compounded by passing the steam through a ring of moving blades, then through a ring of fixed blades, and finally through a second ring of moving blades. The tips of the moving blades at entry and exit are inclined 30° to the direction of motion of the blades, and the steam is to pass on and off the blades without shock.

Assuming that the speed of the steam relative to a blade is reduced by friction by 10 per cent whilst passing over the blade, find the speed of the moving blades so that the final discharge shall be axial, and calculate the horse-power developed in the turbine under these conditions per lb. of steam supplied per second.

What is the magnitude of the angle between the nozzle and the plane of the moving blades ? (*U.L.*, *B.Sc.* (*Eng.*), 1920.)

7. In a stage of an impulse turbine in which the velocity is compounded in two rings of moving blades separated by fixed blades the moving blades have tip angles of 30°, and the blade speed and the nozzle and fixed blade angles are

designed on the assumption that the velocity of discharge from the nozzle is 1,800 ft. per second, and the relative velocity of steam to blade is reduced by 10 per cent in passing through each of the three blade rings, and that the final discharge shall be axial.   Determine the blade velocity and find the efficiency of the stage.

What would be the approximate reduction in efficiency caused by reducing the blade speed by 10 per cent ?   (*U.L., B.Sc. (Eng.*), 1924.)

8. The first stage 'of an impulse steam turbine is supplied with steam at 200 lb. per sq. in. superheated to 250° C.   The velocity is compounded, there being two rings of moving blades separated by fixed blades.   The pressure in the wheel chamber is 100 lb. per sq. in.   Assuming an 8 per cent loss in the nozzle, find the velocity of the steam as it leaves the nozzle.

Given that the angle of the tips of the moving blades is 30°, that the blade velocity is one-fifth of that of the steam discharging from the nozzle, that the velocity of steam relative to a blade is reduced by 12 per cent during its passage through a blade ring, and that the nozzle and fixed blade angles are designed so that the steam shall pass through without shock, find the work developed per lb. of steam and the efficiency of the stage.   (*U.L., B.Sc. (Eng.*), 1923.)

9. In a reaction steam turbine the blade tips are inclined 35° and 20° to the direction of motion.   The guide blades are of the same shape as the moving blades but reversed in direction.   At a certain place in the turbine the drum diameter is 3 ft. 6 in. and the blades are 4 in. high, and at this place the steam has a pressure of 25 lb. per sq. in. and dryness 0·935.   If the speed of the turbine is 250 r.p.m. and the steam passes through the blades without shock, find the power developed in the ring of moving blades.
(*U.L., B.Sc. (Eng.*), 1923.)

10. Explain carefully what is meant by " re-heat factor " in turbine work, and indicate how this factor involves two efficiency ratios.

An impulse turbine installation is to be arranged in three casings, H.P., I.P., and L.P., and to work between pressures of 210 lb. per sq. in., with 80° C. superheat and 1·1 lb. per sq. in.   Allowing a re-heat factor of 1·075, and a loss of available heat (in receiver pipes, etc.) of 5 C.H.U. per lb., determine the heat to be allocated to each unit if the H.P. and I.P. are each to develop quarter power and the L.P. half power.   Allow for hydraulic or stage efficiencies of H.P. 0·79, I.P. 0·76, L.P. 0·72.   (*U.L., B.Sc. (Eng.*), 1921.)

11. Explain what is meant by the " re-heat factor " in steam turbine design.

An impulse steam turbine consists of eight stages and the efficiency of each stage is 70 per cent.   The steam supply is at 215 lb. per sq. in. and 250° C., and the pressure in successive stages are in geometrical progression, the condenser pressure being 1 5 lb. per sq. in.   Find the re-heat factor for the turbine and the steam consumption per horse-power-hour.

What is the thermal efficiency of the turbine ?
(*U.L., B.Sc. (Eng.*), 1925.)

12. The low-pressure turbine of a naval installation receives 30·7 lb. of steam per second from the high-pressure unit at 27 lb. per sq. in. and 0·915 dry.   This steam expands to 12·5 lb. per sq. in. in the first two stages of the L.P. turbine with an efficiency ratio of 0·69.   Into this steam at 12·5 lb. per sq. in. pressure there also passes 8·2 lb. per second of exhaust steam from the auxiliaries at 0·94 dry.   The total steam quantity then expands through the succeeding stages to an exhaust pressure of 1 3 lb. per sq. in.   If the horse-power developed in the turbine is 6,120, determine (*a*) the efficiency ratio of these later stages ; (*b*) the condition of the steam at entrance to the third stage ; (*c*) the condition of the steam at exhaust from the last stage.   Assume a leaving velocity at exhaust of 600 ft. per second.
(*U.L., B.Sc. (Eng.*), 1921.)
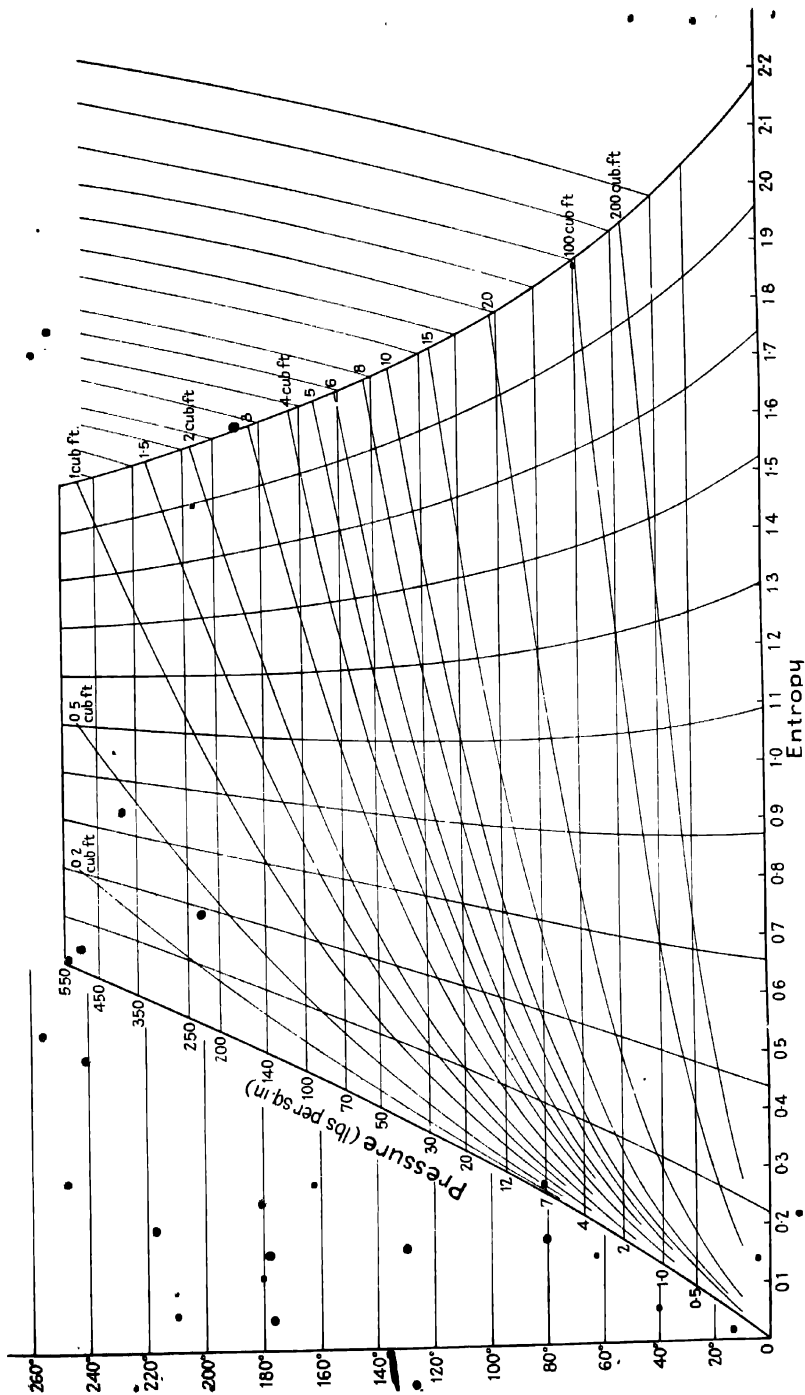
Fig. 208. Temperature-entropy Diagram for Water and Steam

APPENDIX I

TABLES OF THE PROPERTIES OF STEAM

THE values in Tables I and II are taken from *Steam Power* (by Professor W. E. Dalby, F.R.S.), by the kind permission of the author and the publishers, Messrs. Edward Arnold & Co.

The values in Tables III and IV are taken from the *Abridged Callendar Tables Centigrade Units* by the kind permission of the author and the publishers, Messrs. Edward Arnold & Co.

TABLE I
Properties of Dry Saturated Steam

Pressure lb. per sq. in. (Abs.) p	Temperature t° C.	Volume cub. ft. per lb. Steam V_{sa}	Total Heat of Water C.H.U per lb. h	Latent Heat of Steam C.H.U. per lb. L	Total Heat of Dry Steam C.H.U. per lb. H_{sa}	Internal Energy C.H.U. per lb. E	Entropy per lb.		
							Water ϕ_w	Increase Due to L L/T	Steam ϕ_{sa}
·0892	0·0	3276	0·0	594 27	594·27	564·21	0 0	2·1760	2·1760
0·1	1·59	2940	1 59	593·44	595·03	564·79	0·0057	2·1605	2·1662
0·2	11 69	1524 4	11 67	588·14	599 81	568·45	0·0417	2·0651	2·1068
0 3	17·99	1037 7	17 94	584·83	602·77	570·74	0·0635	2·0092	2·0727
0·4	22 66	790·7	22·60	582·37	604·97	572 44	0·0794	1·9688	2·0482
0·5	26·41	640 50	26 34	580 39	606·73	573·80	0 0922	1·9377	2·0299
1	38 74	333 12	38·63	573·83	612·46	578 20	0·1323	1 8401	1·9724
2	52 27	173·54	52 16	566 51	618·67	582 98	0·1747	1·7412	1·9159
3	60·83	118 56	60 70	561·83	622 53	585·95	0 2007	1·6826	1·8833
4	67 23	90 54	67 10	558·28	625·38	588 14	0 2197	1 6403	1·8600
5	72·38	73·44	72·26	555 38	627 64	589·88	0 2346	1·6076	1·8422
6	76·72	61·91	76 61	552·92	629 53	591·33	0 2472	1 5805	1·8277
7	80 49	53 59	80 39	550·76	631 15	592 58	0 2579	1·5577	1 8156
8	83·84	47·30	83·75	548·82	632·57	593 67	0 2673	1·5376	1·8049
9	86·84	42·36	86·76	547 08	633·84	594·64	0 2757	1·5199	1·7956
10	89·58	38 39	89·51	545·50	635 01	595·55	0 2833	1·5041	1·7874
12	94·44	32 37	94 40	542·61	637 01	597·08	0·2967	1·4764	1·7731
14	98·66	28 02	98 66	540·12	638·78	598 46	0 3081	1·4530	1·7611
14·689	100·00	26·79	100·00	539 30	639·30	598·85	0 3118b	1·44546	1·7573
16	102 41	24·73	102·43	537·82	640·25	599 58	0·3184	1·4322	1·7506
18	105 79	22·16	105·84	535·75	641·59	600·61	0 3274	1·4140	1·7414
20	108·87	20 08	108·95	533·87	642·82	601·56	0 3356	1·3977	1·7333
22	111·71	18 37	111·83	532 09	643 92	602·41	0 3430	1 3828	1·7258
24	114·34	16 93	114 50	530·43	644·93	603 18	0 3499	1 3690	1·7189
26	116·80	15 71	116 98	528 87	645 85	603·89	0·3563	1·3563	1·7126
28	119·11	14 66	119·32	527·42	646·74	604 58	0·3622	1 3447	1·7069
30	121 28	13 74	121·51	526·01	647 52	605 18	0 3679	1 3337	1·7016
32	123·35	12 94	123·63	524·67	648 30	605 78	0 3732	1 3234	1·6966
34	125·31	12·22	125·63	523·40	649·02	606 34	0 3782	1·3137	1·6919
36	127·17	11·59	127 52	522·17	649 69	606·85	0·3830	1·3044	1·6876
38	128·96	11 02	129·34	521·00	650·34	607·35	0·3874	1·2957	1·6831
40	130·67	10·500	131·08	519 87	650·95	607·82	0 3918	1·2874	1·6792
50	138·30	8·520	138 89	514·71	653·60	609·87	0 4109	1·2511	1·6620
60	144 79	7·184	145 54	510·22	655 76	611 53	0 4269	1·2210	1·6479
70	150·46	6 218	151·37	506·23	657·60	612·96	0 4407	1·1952	1·6359
80	155·52	5·487	156·61	502·59	659·20	614·20	0·4530	1·1726	1·6256
90	160·09	4·913	161 35	499·24	660·59	615·27	0·4620 -	1·1525	1·6165
100	164·28	4·451	165·71	496·12	661·83	616 24	0·4739	1 1343	1·6082
110	168·15	4·070	169 75	493·18	662·93	617 09	0·4831	1·1176	1·6007
120	171·75	3 751	173 52	490·40	663·92	617 85	1 1023	1 0923	1·5938
130	175·13	3·479	177 07	487·76	664·83	618 55	0·4994	1·0882	1·5876
140	178·31	3 245	180·42	485·27	665·69	619·23	0·5068	1·0750	1·5818
150	181·31	3·041	183 59	482·90	666·49	619·85	0 5138	1·0627	1·5765
160	184 16	2·862	186·61	480·61	667·22	620 42	0 5204	1·0511	1·5715
170	186·88	2·703	189 50	478·40	667·90	620 95	0·5266	1·0400	1·5666
180	189·48	2·562	192·27	476·26	668·53	621·44	0·5326	1·0294	1·5620

TABLE I
PROPERTIES OF DRY SATURATED STEAM
(*contd.*)

Pressure lb. per sq. in (Abs.) p	Temperature t° C.	Volume cub. ft per lb. Steam V_{sa}	Total Heat of Water C.H.U. per lb. h	Latent Heat of Steam C.H.U. per lb L	Total Heat of Dry Steam C.H U per lb. H_{sa}	Internal Energy C.H U per lb E	ENTROPY PER LB.		
							Water ϕ_w	Increase Due to L L/T	Steam ϕ_{sa}
190	191·97	2 435	194·94	474·19	669 13	621 91	0 5383	1 0194	1·5577
200	194·35	2·320	197·49	472·20	669·69	622 35	0 5437	1·0101	1·5538
210	196·66	2·216	199·94	470·26	670·20	622·75	0 5490	1·0012	1·5502
220	198·87	2·120	202·32	468·38	670·70	623·14	0 5540	0·9925	1·5465
230	201·02	2·034	204 64	466·55	671·19	623·52	0 5588	0·9841	1·5429
240	203·09	1·954	206·88	464·76	671·64	623·88	0·5635	0·9760	1 5395
250	205·19	1·880	209·07	463 01	672 08	624·23	0·5680	0 9682	1·5362
260	207 04	1·811	211·17	461·31	672 48	624 54	0 5724	0·9608	1·5332
270	208·93	1·748	213 23	459 65	672 88	624·86	0·5767	0 9536	1·5303
280	210 77	1 689	215·25	458 02	673 27	625·17	0 5808	0 9466	1·5274
300	214·32	1·583	219·12	454·85	673 97	625·72	0 5887	0·9332	1·5219
350	222 45	1·368	228·08	447·44	675 52	626 95	0·6067	0 9029	1·5096
400	229 75	1·206	236·21	440 63	676·84	628 02	0·6228	0 8763	1·4991

TABLE II

PROPERTIES OF DRY SATURATED STEAM

Temperature t° C.	Pressure lb. per sq in. p	Volume Cub. Ft. per Lb.		Total Heat of Water lb.-calories per lb. h	Latent Heat of Steam lb.-calories per lb L	Total Heat of Steam lb.-calories per lb. H_{sa}	Internal Energy lb.-calories per lb. E_{sa}	Entropy per Lb.		
		Dry Steam V_{sa}	Water V_w					Water ϕ_w	Increase Due to Latent Heat $\frac{L_s}{T}$	Steam ϕ_{sa}
0	0·0892	3275·9	0·01602	0·0	594·27	594·27	564·21	0·0	2·17602	2·17602
10	0·1788	1693·8	0·01603	9·98	589·03	599·01	567·85	0·03585	2·08064	2·11650
20	0·3400	922·2	0·01605	19·94	583·78	603·72	571·48	0·07046	1·99174	2·06220
30	0·6162	525·8	0·01609	29·91	578·49	608·40	575·07	0·10393	1·90855	2·01248
40	1·0703	312·4	0·01614	39·89	573·15	613·04	578·64	0·13631	1·83057	1·96688
50	1·7888	192·7	0·01621	49·88	567·75	617·63	582·17	0·16770	1·75720	1·92490
60	2·8873	122·9	0·01629	59·87	562·29	622·16	583·66	0·19815	1·68805	1·88620
70	4·5156	80·8	0·01638	69·88	556·72	626·60	589·07	0·22774	1·62265	1·85039
80	6·8627	54·6	0·01648	79·90	551·05	630·95	592·41	0·25652	1·56061	1·81713
90	10·161	37·82	0·01659	89·94	545·25	635·19	595·67	0·28454	1·50165	1·78619
100	14·689	26·79	0·01671	100·00	539·30	639·30	598·83	0·31186	1·44546	1·75732
110	20·77	19·37	0·01684	110·09	533·17	643·26	601·86	0·33853	1·39174	1·73027
120	28·808	14·271	0·01698	120·22	526·85	647·07	604·78	0·36400	1·34026	1·70485
130	39·213	10·696	0·01713	130·40	520·32	650·72	607·58	0·39011	1·29081	1·68092
140	52·482	8·143	0·01729	140·62	513·57	654·19	610·23	0·41511	1·24321	1·65832
150	69·150	6·289	0·01746	150·91	506·56	657·47	612·73	0·43963	1·19726	1·63689
160	89·801	4·923	0·01765	161·26	499·29	660·55	615·08	0·46373	1·15284	1·61657
170	115·06	3·902	0·01785	171·69	491·75	663·44	617·27	0·48743	1·10981	1·59724
180	145·39	3·128	0·01807	182·21	483·93	666·14	619·30	0·51078	1·06806	1·57884
190	182·08	2·534	0·01831	192·83	475·82	668·65	621·19	0·53381	1·02747	1·56128
200	225·24	2·074	0·01856	203·55	467·41	670·96	622·91	0·55654	0·98798	1·54452
210	275·78	1·713	0·01885	214·40	458·69	673·09	624·48	0·57904	0·94947	1·52851
220	334·38	1·428	0·01914	225·37	449·69	675·06	625·93	0·60128	0·91198	1·51326
240	478·74	1·018	0·01980	247·74	430·81	678·55	628·43	0·64517	0·83962	1·48479

TABLE III

Total Heat of Superheated Steam

Pressure Absolute lb. per sq. in.	Sat 0°	+ 10°	+ 20°	− 30°	+ 40°	− 50°	+ 60°	+ 70°	+ 80°	− 90°	+ 100°
15	639·54	644·54	649·53	654·49	659·43	664·36	669·25	674·15	679·04	683·92	688·78
20	642·83	647·89	652·91	657·91	662·90	667·85	672·79	677·72	682·63	687·52	692·42
30	647·55	652·71	657·81	662·89	667·94	672·95	677·95	682·93	687·88	692·82	697·75
40	650·96	656·19	661·37	666·50	671·61	676·69	681·73	686·76	691·76	696·73	701·69
50	653·60	658·90	664·15	669·35	674·50	679·63	684·71	689·78	694·81	699·84	704·83
60	655·78	661·14	666·44	671·69	676·89	682·07	687·20	692·29	697·37	702·42	707·45
70	657·61	663·04	668·39	673·69	678·94	684·14	689·33	694·46	699·56	704·64	709·70
80	659·20	664·67	670·07	675·43	680·73	685·97	691·18	696·35	701·49	706·59	711·68
90	660·59	666·11	671·57	676·96	682·30	687·58	692·83	698·03	703·20	708·33	713·44
100	661·82	667·39	672·88	678·33	683·70	689·02	694·30	699·53	704·73	709·90	715·03
120	663·93	669·60	675·19	680·70	686·14	691·54	696·88	702·17	707·42	712·64	717·81
140	665·70	671·46	677·13	682·71	688·23	693·68	699·08	704·42	709·73	714·98	720·20
160	667·21	673·03	678·77	684·43	690·01	695·52	700·98	706·37	711·72	717·02	722·28
180	668·53	674·43	680·24	685·96	691·60	697·17	702·66	708·12	713·51	718·85	724·16
200	669·68	675·64	681·52	687·30	693·00	698·61	704·17	709·66	715·10	720·48	725·82
250	672·07	678·20	684·23	690·15	695·97	701·73	707·38	712·98	718·51	723·99	729·41
300	673·96	680·24	686·40	692·45	698·39	704·25	710·01	715·69	721·32	726·88	732·38
350	675·52	681·94	688·22	694·38	700·44	706·38	712·24	718·01	723·72	729·36	734·93
400	676·84	683·38	689·77	696·05	702·19	708·22	714·16	720·02	725·81	731·51	737·17
450	677·97	684·62	691·12	697·49	703·72	709·84	715·88	721·81	727·65	733·44	739·14
500	678·97	685·72	692·32	698·77	705·09	711·30	717·40	723·41	729·32	735·17	740·93

Superheat in Degrees (Centigrade)

TABLE IV

ENTROPY OF SUPERHEATED STEAM

Pressure lb per sq in		Degrees of Superheat (Centigrade)									
Abs.	Gauge	− 10°	− 20°	+ 30°	+ 40°	+ 50°	+ 60°	− 70°	− 80°	+ 90°	+ 100°
15	0·31	1·7689	1·7817	1·7941	1·8062	1·8180	1·8295	1·8406	1·8515	1·8622	1·8725
20	5·31	1·7463	1·7590	1·7713	1·7832	1·7948	1·8061	1·8171	1·8279	1·8384	1·8486
30	15·31	1·7146	1·7271	1·7392	1·7509	1·7624	1·7734	1·7843	1·7949	1·8052	1·8153
40	25·31	1·6922	1·7045	1·7165	1·7282	1·7395	1·7505	1·7612	1·7716	1·7818	1·7918
50	35·31	1·6748	1·6870	1·6990	1·7105	1·7218	1·7327	1·7433	1·7537	1·7638	1·7737
60	45·31	1·6605	1·6728	1·6846	1·6961	1·7073	1·7182	1·7288	1·7390	1·7491	1·7589
70	55·31	1·6486	1·6608	1·6726	1·6841	1·6952	1·7060	1·7165	1·7268	1·7368	1·7465
80	65·31	1·6381	1·6503	1·6621	1·6736	1·6846	1·6954	1·7059	1·7161	1·7261	1·7358
90	75·31	1·6290	1·6412	1·6530	1·6644	1·6754	1·6862	1·6966	1·7068	1·7167	1·7264
100	85·31	1·6207	1·6329	1·6447	1·6560	1·6670	1·6778	1·6882	1·6984	1·7083	1·7179
120	105·31	1·6065	1·6187	1·6304	1·6418	1·6528	1·6635	1·6739	1·6840	1·6938	1·7034
140	125·31	1·5944	1·6066	1·6183	1·6297	1·6407	1·6513	1·6617	1·6717	1·6815	1·6911
160	145·31	1·5840	1·5962	1·6079	1·6194	1·6303	1·6409	1·6512	1·6613	1·6711	1·6806
180	165·31	1·5748	1·5870	1·5987	1·6101	1·6210	1·6317	1·6420	1·6520	1·6618	1·6713
200	185·31	1·5665	1·5787	1·5904	1·6018	1·6127	1·6234	1·6337	1·6437	1·6535	1·6630
250	235·31	1·5490	1·5612	1·5730	1·5843	1·5953	1·6059	1·6162	1·6262	1·6359	1·6454
300	285·31	1·5346	1·5468	1·5587	1·5700	1·5810	1·5917	1·6020	1·6120	1·6217	1·6312
350	335·31	1·5224	1·5348	1·5466	1·5580	1·5690	1·5797	·1·5900	1·6000	1·6097	1·6191
400	385·31	1·5120	1·5243	1·5362	1·5476	1·5586	1·5693	1·5796	1·5896	1·5993	1·6087
450	435·31	1·5026	1·5150	1·5269	1·5384	1·5494	1·5601	1·5704	1·5805	1·5902	1·5996
500	485·31	1·4943	1·5068	1·5187	1·5302	1·5413	1·5519	1·5623	1·5723	1·5820	1·5915

APPENDIX II

EXTRACTS FROM THE *MOLLIER STEAM TABLES*

THE values in Tables A, B, C, D, and E are taken from *The Mollier Steam Tables and Diagrams* (Third Edition) by the kind permission of the publishers, Sir Isaac Pitman & Sons, Ltd. Much more detailed figures and information will be found in the *Complete Tables*, published in med. 8vo, with two large charts, at 7s. 6d. net.

The first set of Mollier steam tables was founded upon the characteristic equation of Professor H. L. Callendar (page 67).

Owing to the tendency, in modern steam practice, to use higher pressures and high superheat, there is need for information on the properties of steam above 500 lb. per square inch.

After consideration of the available data, Dr. Mollier has devised a new characteristic equation by the addition of one term to the original equation and, by extrapolation, he has extended the range of these steam tables to the critical point.

The equation for superheated steam applicable up to 2,000 lb. per square inch is

$$\frac{pV}{T} = 1 \cdot 0731 - 0 \cdot 3204 \frac{p}{(T/100)^{13/3}} - 15 \cdot 044 \frac{p^3}{(T/100)^{15}}$$

taking coefficients $k_1 = \dfrac{32 \cdot 04}{(T/100)^{10/3}}$, and $k_2 = \dfrac{15 \cdot 044 \times 10^6}{(T/100)^{14}}$

$$V = 1 \cdot 0731 \frac{T}{p} - k_1 - k_2 \left(\frac{p}{100} \right)^2$$

where $p =$ pressure in lb. per sq. in. (abs.)

$V =$ volume of steam in cub. ft. per lb.

$T = t^\circ$ C. $+ 273 =$ absolute temperature (Centigrade)

The coefficients for the various equations, from zero to 500° C., are given in the *Complete Tables*.

Dr. H. Moss has amplified and adapted the tables to the British Centigrade system of units.

TABLE A

PROPERTIES OF SATURATED STEAM

Pressure lb./sq. in. (absolute) p	Temperature $t°$ C.	Volume of Steam cub ft /lb V_{sa}	Total Heat Water lb. cal./lb. h	Total Heat Steam lb. cal./lb. H_{sa}	Latent Heat of Steam lb. cal./lb $L = H - h$	Internal Energy of Steam lb cal./lb E	External Work of Vaporization lb. cal./lb	Entropy per lb. Water ϕ_w	Entropy per lb. Increase Due to L/T	Entropy per lb. Steam ϕ_{sa}
0.2	11.7	1526	11.7	600.4	588.7	569.0	31.38	0.0418	2.0686	2.1104
0.3	17.9	1040	17.9	603.3	585.4	571.2	32.07	0.0636	2.0122	2.0758
0.4	22.6	791.9	22.6	605.5	582.9	572.9	32.57	0.0793	1.9721	2.0514
0.6	29.5	540.1	29.5	608.6	579.1	575.3	33.32	0.1026	1.9148	2.0174
0.8	34.6	411.9	34.6	611.0	576.4	577.1	33.87	0.1193	1.8741	1.9934
1.0	38.7	334.0	38.7	612.9	574.2	578.5	34.31	0.1326	1.8423	1.9749
2.0	52.3	174.0	52.3	619.0	566.7	583.3	35.74	0.1753	1.7426	1.9179
3.0	60.9	118.8	60.9	622.8	561.9	586.2	36.64	0.2012	1.6837	1.8849
4.0	67.2	90.71	67.2	625.6	558.4	588.3	37.30	0.2200	1.6418	1.8618
5.0	72.4	73.6	72.4	627.8	555.4	590.0	37.82	0.2351	1.6086	1.8437
6	76.7	62.06	76.7	629.7	553.0	591.5	38.26	0.2476	1.5815	1.8291
8	83.9	47.41	83.9	632.7	548.8	593.7	38.96	0.2679	1.5384	1.8063
10	89.6	38.48	89.6	635.1	545.5	595.6	39.52	0.2837	1.5048	1.7885
12	94.4	32.44	94.4	637.1	542.7	597.1	39.98	0.2969	1.4772	1.7741
14	98.6	28.08	98.6	638.8	540.2	598.4	40.38	0.3083	1.4536	1.7619
16	102.4	24.79	102.4	640.4	538.0	599.7	40.74	0.3185	1.4329	1.7514
18	105.8	22.20	105.9	641.7	535.8	600.7	41.05	0.3277	1.4145	1.7422
20	108.9	20.12	109.0	642.8	533.8	601.5	41.33	0.3359	1.3981	1.7340
22	111.7	18.40	111.9	643.9	532.0	602.2	41.58	0.3433	1.3832	1.7265
24	114.3	16.96	114.6	644.9	530.3	603.0	41.81	0.3502	1.3695	1.7197
30	121.3	13.76	121.7	647.5	525.8	605.0	42.40	0.3683	1.3338	1.7021
35	126.3	11.92	126.7	649.3	522.6	606.4	42.81	0.3810	1.3090	1.6900
40	130.7	10.52	131.1	650.8	519.7	607.5	43.17	0.3922	1.2874	1.6796
50	138.4	8.531	139.0	653.4	514.4	609.5	43.75	0.4115	1.2505	1.6620
60	144.9	7.190	145.7	655.5	509.8	611.2	44.23	0.4277	1.2199	1.6476
80	155.6	5.484	156.8	658.6	501.9	613.5	44.96	0.4536	1.1711	1.6247
100	164.4	4.443	166.0	660.9	494.9	615.2	45.48	0.4747	1.1321	1.6068
120	171.8	3.738	173.7	662.8	489.1	616.7	45.88	0.4922	1.0997	1.5919

140	178·3	3·228	180·6	664·2	483·6	617·7	46·20	0·5074	1·0718	1·5792
160	184·2	2·842	186·7	665·4	478·7	618·7	46·44	0·5209	1·0471	1·5680
180	189·5	2·539	192·3	666·3	474·0	619·4	46·64	0·5331	1·0248	1·5579
200	194·3	2·295	197·5	667·0	469·5	619·9	46·79	0·5441	1·0048	1·5489
220	198·8	2·093	202·2	667·6	465·4	620·3	46·90	0·5539	0·9866	1·5405
240	203·0	1·924	206·7	668·1	461·4	620·7	46·99	0·5633	0·9695	1·5328
260	206·9	1·779	210·9	668·4	457·5	620·9	47·05	0·5721	0·9535	1·5256
300	214·1	1·547	218·7	668·8	450·1	621·1	47·11	0·5881	0·9242	1·5123
340	220·6	1·367	225·8	669·0	443·2	621·2	47·10	0·6022	0·8982	1·5004
380	226·5	1·224	232·2	668·9	436·7	621·1	47·04	0·6151	0·8745	1·4896
420	231·9	1·107	238·1	668·6	430·5	620·9	46·94	0·6268	0·8528	1·4796
460	237·0	1·010	243·8	668·2	424·4	620·5	46·81	0·6376	0·8326	1·4702
500	241·1	0·9275	249·0	667·8	418·8	620·1	46·64	0·6477	0·8138	1·4615
600	252·4	0·7679	261·0	665·9	404·9	618·6	46·12	0·6703	0·7710	1·4413
700	261·7	0·6528	271·6	663·7	392·1	616·7	45·49	0·6900	0·7334	1·4234
800	270·1	0·5656	281·3	661·0	379·7	614·5	44·77	0·7075	0·6994	1·4069
900	277·8	0·4968	290·1	658·2	368·1	612·2	43·97	0·7233	0·6684	1·3917
1000	284·8	0·4410	298·3	655·2	356·9	609·9	43·12	0·7377	0·6398	1·3773
1200	297·3	0·3574	313·3	648·4	335·1	604·4	41·27	0·7634	0·5878	1·3512
1400	308·3	0·2966	327·2	641·3	314·1	598·7	39·30	0·7868	0·5403	1·3271
1600	318·2	0·2505	340·4	633·9	293·5	592·8	37·21	0·8088	0·4960	1·3048
1800	327·2	0·2142	353·2	626·1	272·9	586·4	35·04	0·8295	0·4543	1·2838
2000	335·4	0·1850	365·9	618·3	252·4	580·2	32·77	0·8491	0·4150	1·2641
2400	350·1	0·1406	391·0	601·0	210·0	566·4	27·7	0·8877	0·3370	1·2247
2800	362·9	0·1024	421·6	576·1	154·5	546·7	20·3	0·9344	0·2432	1·1776
3000	368·6	0·0814	444·0	556·2	112·2	530·8	14·6	0·9684	0·1750	1·1434
3200	374·0	0·0497	501·1	501·1	0	484·8	0	1·0558	0	1·0558

$T = t + 273$ p (absolute) $= p$ (gauge) $+ 14.69 - 0.49(30 - Hg)$, where $Hg =$ inches of mercury vacuum

To convert to British Thermal Units per pound, multiply by 1·8   To convert to foot-pounds per pound, multiply by 1400

TABLE B
Properties of Saturated Steam

Temp t° C.	Pressure lb/sq.in (abs) p	Volume cu ft/lb		Total Heat		Latent Heat of Steam lb cal./lb L (H–h)	Internal Energy of Steam lb cal./lb E	Entropy per lb.		
		Water V_w	Steam V_sa	Water lb cal/lb h	Steam lb cal/lb H_sa			Water φ_w	Increase Due to L/T	Steam sa
0	0 088	0·016	3308	0	595·0	595 0	564·9	0	2·1800	2·1800
10	0 178	0 016	1704	10 0	599·6	589·6	568 4	0·0361	2 0839	2·1200
20	0 339	0·016	926 3	20·0	604·3	584 3	572·0	0·0708	1·9944	2·0652
30	0 616	0·016	527 5	30 0	608 9	578 9	575 5	0·1043	1·9108	2·0151
40	1·070	0·016	313 2	40 0	613·5	573 5	579·1	0 1367	1·8324	1·9691
50	1·789	0·016	193·1	50 0	618·0	568·0	582 5	0 1682	1·7586	1·9268
60	2 889	0·016	123 1	60 0	622·5	562·5	585·9	0·1986	1·6894	1·8880
70	4·519	0 017	80·92	70·0	626·8	556·8	589·2	0 2283	1·6236	1·8519
80	6 87	0·017	54·69	80 0	631 1	551·2	592 6	0·2570	1·5614	1·8184
90	10·17	0·017	37·88	90 0	635 3	545·3	595·7	0·2848	1·5024	1·7873
100	14·69	0 017	26·83	100·0	639 4	539 4	598 9	0·3121	1·4461	1·7582
120	28 80	0·018	14 28	120 3	647 0	526 7	604·7	0 3649	1·3404	1 7053
140	52·41	0·018	8·163	140·7	653 9	513·2	610 0	0 4155	1·2427	1·6583
160	89·64	0·018	4 925	161·4	659·9	498·5	614·5	0 4642	1·1514	1·6156
180	145·5	0 018	3·114	182 3	664·6	482 3	618 0	0·5112	1 0648	1·5760
200	225·4	0·019	2·043	203·5	667 7	464·2	620·4	0 5567	0·9816	1·5383
220	336·5	0·019	1·381	225 1	669 0	443 9	621 2	0·6010	0 9005	1·5015
240	485·4	0·20	0·956	247·1	668·0	420·8	620 2	0 6442	0·8204	1·4646
260	680·6	0·0205	0 6727	269 6	664 2	394 5	617 1	0 6861	0·7403	1·4267
280	930·5	0·0215	0 4786	292 7	657·3	364 6	611·5	0 7278	0·6595	1·3873
300	1246	0·0227	0·3414	316·6	646·8	330·2	603 3	0·7690	0·5763	1·3454
320	1639	0 0243	0·2429	343·0	632·5	289·5	591·6	0·8128	0·4881	1·3009
340	2119	0·0266	0·1704	373·3	613·5	240·2	576·4	0·8608	0·3918	1·2526
360	2707	0 0306	0·1115	413·0	583·4	170·4	552·4	0·9214	0·2692	1·1906
365	2873	0·0327	0·0958	428 5	570 1	141·6	542 0	0·9450	0·2220	1·1670
374	3200	0·0497	0·0497	501·1	501·1	0	484 8	1·0558	0	1·0558

TABLE C

Volume of Superheated Steam

Pressure lb/sq.in (abs) p	Saturation Temperature t_s °C	Volume, V, cu ft/lb, of Superheated Steam at Temperature—								
		$t = 100°$ C	130° C	200° C	250° C	300° C	350° C	400° C	450° C	500° C
2	52·3	199·7	226·7	253·6	280·5	307·4	334·2	361·0	387·9	414·7
4	67·2	99·67	113·3	126·7	140·2	153·6	167·1	180·5	191·0	207·4
6	76·7	66·33	75·40	84·42	93·42	102·4	111·4	120·3	129·3	138·2
8	83·9	49·63	56·46	63·26	70·02	76·76	83·49	90·22	96·93	103·6
10	89·6	39·63	45·14	50·58	56·00	61·39	66·77	72·16	77·55	82·93
12	94·4	32·97	37·56	42·13	46·65	51·16	55·65	60·13	64·61	69·09
16	102·4	—	28·11	31·54	34·95	38·33	41·71	45·08	48·44	51·80
20	108·9	—	22·44	25·20	27·92	30·64	33·35	36·05	38·75	41·44
30	121·3	—	14·87	16·75	18·59	20·41	22·22	24·03	25·83	27·62
40	130·7	—	11·08	12·51	13·90	15·28	16·64	18·00	19·35	20·70
50	138·4	—	8·81	9·97	11·09	12·20	13·30	14·39	15·47	16·55
60	144·9	—		8·278	9·227	10·16	11·07	11·98	12·89	13·79
80	155·6	—		6·159	6·885	7·590	8·284	8·970	9·653	10·33
90	160·1	—		5·456	6·108	6·737	7·356	7·969	8·576	9·182
100	164·4	—		4·890	5·483	6·053	6·614	7·167	7·714	8·259
120	171·8	—		4·041	4·546	5·028	5·499	5·963	6·422	6·878
140	178·3	—		3·434	3·877	4·296	4·703	5·103	5·498	5·890
160	184·2	—		2·978	3·375	3·747	4·106	4·458	4·804	5·148
150	189·5	—		2·622	2·986	3·321	3·643	3·957	4·266	4·573
200	194·3	—			2·672	2·978	3·270	3·554	3·835	4·114
300	214·1	—			1·730	1·951	2·156	2·351	2·542	2·731
400	229·2	—			1·252	1·436	1·597	1·749	1·896	2·039
500	241·7	—			0·9606	1·125	1·262	1·388	1·507	1·623
1000	284·8	—				0·4831	0·5852	0·6625	0·7306	0·794
2000	335·4	—					0·2170	0·2900	0·3384	0·3776
3200	374·0	—						0·1306	0·1841	0·2185

$t - t_s$ = degrees of superheat.

535

TABLE D
Total Heat of Superheated Steam

Pressure lb./sq. in. (abs.) p	Saturation Temperature t_s C.	Saturation Total Heat lb cal/lb. H_{sa}	Total Heat, H, in lb. cal./lb., of Superheated Steam at Temp							
			t 150 C.	200° C.	250° C.	300° C.	350° C.	400° C.	450° C.	500° C.
15	100·6	639·5	663 7	687·8	711·6	735·4	759·0	782 6	806·2	829·8
20	108 9	642·8	663·1	687 4	711·4	735·2	758 9	782 5	806·1	829·7
30	121·3	647·5	661·9	686 6	710·8	734 7	758·6	782·3	805·9	829·5
40	130 7	650 8	660 7	685 8	710 2	734 3	758·2	782·0	805 7	829·3
50	138·4	653·4	659 5	685 0	709 6	733 9	757·9	781 8	805·5	829·1
60	144·9	655 5	658 3	684 1	709·0	733 5	757 6	781 5	805·3	829·0
70	150 5	657 2	657·0	683 3	708 5	733·0	757 2	781·3	805·1	828·9
80	155·6	658 6	– ·	682 4	707 9	732 6	756 9	781·0	804·9	828·8
90	160 1	659 9		681 6	707·3	732·2	756 6	780·8	804·7	828·6
100	164·4	660 9		680 7	706·7	731 8	756·3	780·5	804·5	828·4
120	171·8	662·8		678 8	705 5	730 9	755 6	780·0	804·1	828·1
140	178·3	664 2		677·0	704·2	730 0	755 0	779 5	805·7	827·8
160	184 2	665·4	—	675·0	703·0	729·1	754·3	779·0	803·4	827·5
180	189·5	666·3		672 8	701·7	728 3	753 7	778·5	803·0	827·2
200	194·3	667 0	– ·	– ··	700 4	727·4	753·0	778 0	802·6	826·9
250	205·0	668·2		—	697 1	725·1	751 3	776 7	801·6	826·1
300	214·1	668·8		– ·	693 4	722 7	749·6	775·5	800·0	825·3
350	222·1	669 0	– ··		689·4	720·3	748 0	774 2	799·6	824 5
400	229 2	668·8			685·0	717 8	746·3	772·9	798·6	823·7
450	235·8	668·3			680 3	715 2	744 5	771·6	797·6	822·9
500	241 7	667·8			675 3	712·5	742 7	770·3	796·6	822·1
600	252·4	665 9	—	—	—	706 2	738·9	767·7	794·6	820·6
800	270·1	661·0	–	– ··	692 2	730 7	762·1	790·5	817·3
1000	284·8	655 2	–		—	674 7	721 6	756 2	786·2	814 0
2000	335·4	618 3	—	·—			648·5	717·6	761·6	796·5
3200	374 0	501·1	– ·					638·9	720·3	770 7

To convert to foot-pounds per pound, multiply by 1400
To convert to British Thermal Units per pound, multiply by 1·8.
$t - t_s$ degrees of superheat

TABLE E
ENTROPY OF SUPERHEATED STEAM

Pressure lb./sq. in. (abs) p	Saturation Temp. $t_s°$ C.	Entropy of Superheated Steam at Temperature—							
		$t = 150°$C	200° C.	250° C.	300° C	350° C.	400° C.	450° C	500° C
15	100·6	1·8169	1·8712	1 9191	1·9624	2·0019	2·0384	2 0722	2·1037
20	108 9	1·7839	1·8386	1·8868	1 9302	1·9699	2·0064	2·0402	2·0718
30	121·3	1·7370	1 7926	1 8412	1 8850	1 9248	1 9614	1·9953	2·0269
40	130·7	1·7030	1·7595	1·8087	1·8527	1 8927	1 9294	1 9634	1 9950
50	138·4	1·6762	1 7335	1·7832	1·8275	1·8677	1 9046	1·9386	1·9703
60	144·9	1·6539	1·7121	1·7622	1·8068	1·8472	1·8842	1·9183	1·9500
70	150·5	-	1·6937	1·7444	1·7892	1 8298	1·8668	1·9011	1·9329
80	155 6	-	1·6775	1·7287	1·7739	1·8146	1·8518	1·8861	1·9180
90	160·1	- -	1·6631	1·7148	1·7604	1·8012	1·8385	1·8729	1·9048
100	164·4		1·6502	1·7024	1·7482	1 7892	1·8266	1·8610	1·8930
120	171·8	-	1 6270	1 6805	1·7269	1·7683	1·8060	1 8405	1·8727
140	178·3		1·6068	1·6617	1 7087	1 7505	1·7884	1·8231	1·8554
160	184·2		1·5886	1·6451	1·6928	1·7350	1 7731	1·8079	1·8403
180	189 5		1 5720	1·6302	1·6786	1 7212	1·7595	1·7945	1·8270
200	194 3			1·6166	1·6658	1 7088	1·7473	1·7825	1·8150
250	205·0	—		1·5868	1·6381	1·6821	1·7213	1·7568	1·7896
300	214·1			1·5611	1 6146	1 6599	1·7997	1 7357	1·7687
350	222·1			1 5379	1·5943	1 6408	1·6812	1·7177	1·7509
400	229 2			1 5162	1·5761	1·6239	1·6650	1·7019	1·7354
450	235·8			1·4957	1 5594	1·6086	1·6505	1·6878	1·7216
500	241·7			1·4761	1·5440	1 5947	1·6375	1·6751	1·7092
600	252·4				1 5151	1·5698	1·6140	1·6528	1·6875
800	270·1				1 4634	1 5274	1·5758	1·6166	1·6526
1000	284·8				1·4122	1·4908	1 5444	1·5874	1·6247
1200	297·3					1·4565	1 5165	1·5625	1·6012
1600	318·2					1·3882	1·4671	1·5201	1·5626
2000	335·4					1·3137	1·4204	1·4836	1·5303
2400	350·1						1·3730	1·4497	1·5019
2800	362 9						1·3224	1·4169	1·4759
3200	374·0						1·2666	1·3837	1·4513

Entropy is numerically the same in all temperature systems.

ANSWERS TO EXAMPLES

EXAMPLES II

1. I.H.P., 6·17.
2. (a) I.H.P., 6·09 ; B.H.P., 5 ; (b) 1·095 H.P. ; (c) 0·82, or 82 per cent.
3. I.H.P., 18·8 ; 78·2 per cent.
4. (a) 5·25 ; (b) 80·77 per cent ; (c) 31·66 per cent ; (d) (1) 25·57 per cent ; (2) 6·09 per cent ; (3) 30·09 per cent.
5. I.H.P., 42·86 ; B.H.P., 30·39 ; mechanical loss, 12·47 H.P. ; mechanical efficiency, 70·9 per cent ; oil per I.H.P. hour, 0·453 lb. ; per B.H.P. hour, 0·6384 lb.

BALANCE SHEET.   HEAT FLOW PER HOUR

	B.Th.U. per hour	Per Cent
Heat supplied in oil fuel	349,200	100
Indicated work	109,079	31·24

Including	B.Th.U.	Per cent
Useful work on brake	77,343	22·15
Mechanical loss	31,736	9·09

	B.Th.U. per hour	Per Cent
Heat to jacket cooling water	101,500	29·07
Heat in exhaust gases and radiation (by difference) .	138,621	39·69
Total	349,200	100·00

6. (a) 157·3, and 38·94 lb. per sq. in. ; (b) 12,423 ft.-lb. ; (c) 69 lb. per sq. in. and 41·4 I.H.P. ; (d) 35 per cent.
7. (a) 0·2831 or 28·31 per cent ; (b) 87·1 per cent ; (c) 7285 C.H.U. or 46·8 per cent.
8. (a) I.H.P., 50·88 ; B.H.P., 30·46 ; (b) oil 0·288 lb. and 0·481 lb. ; (c) 2158 B.Th.U. ; (d) 1365 B.Th.U. ; (e) by difference, 629 B.Th.U.
9. (a) 36·4 per cent ; (b) 30·95 per cent ; (c) 85·03 per cent.
10. (a) 2·62 ; (b) 17·53 ; (c) 0·85 or 85 per cent.
11. Diameter $9\frac{3}{8}$ in. × stroke $14\frac{1}{10}$ in.

EXAMPLES III

1. 5 cub. ft.
2. (a) 139·9 lb. per sq. in., 276·2° C. ; (b) 73·5 lb. per sq. in.
3. 43·49 lb. per sq. in. ; 59,445 ft.-lb.
4. 417·8° F. (214·2° C.)
5. 50,825 ft.-lb.
6. (a) 46,026 ft.-lb. ; (b) 43,062 ft.-lb., and 412·6° F.
7. (1) 84 lb. per sq. in. ; (2) 36,845 ft.-lb. ; and (3) 26·32 C.H.U.

8. (a) 354 lb. per sq. in.; 359·24° C.; (b) 89,300 ft.-lb.; (c) rejected, 15·94 C.H.U. or 28·69 B.Th.U.

9. (a) − 88·8° F.; (b) 151,866 ft.-lb.; (c) heat received by air, 42 C.H.U., or 75·6 B.Th.U.

10. 42,560 ft.-lb. and 15·2 C.H.U.

11. 1824 ft.-lb.; 0·1326 cub.-ft.

12. 19,880 ft.-lb., and 14·2 C.H.U.

13. (a) 414° F., 46,548 ft.-lb.; (b) 84·252 B.Th.U., 19,000 ft.-lb.; (e) 50,000 ft.-lb. (Here, 46,548 ÷ 19,000 = 65,548 ft.-lb. = 84·252 × 778)

14. $C_p = 0·24526$; $C_v = 0·1732$; and $\gamma = 1·416$.

EXAMPLES IV

1. 0·037.

2. 47·05 C.H.U., and gain 0·094.

3. (a) 68,466 ft.-lb.; 48·9 C.H.U.; entropy loss 0·179; (b) 71,480 ft.-lb.; 25·53 C.H.U.; entropy loss 0·074.

4. 0·5, or 50 per cent.

5. 28·54 lb. per sq. in.

6. (a) 0·5345, or 53·45 per cent; (b) 0·4489, or 44·89 per cent.

EXAMPLES V

1. (a) − 110° F.; (b) 31,670 ft.-lb.; 39·78 lb. per sq. in.; (c) 3·405 cub. ft., 13·62 cub. ft.

2. 8·44 B.H.P.

3. (a) 64,860 ft.-lb. (approx.); (b) 49,660; (c) 407·6° F.

4. 76·7 B.H.P.

5. (a) Air H.P., 54·6; B.H.P., 63·5.

6. Diameter, 8·232 in.; stroke, 30 in

7. (a) 71,850 ft.-lb. and 25·66 C.H.U.   (b) 68,060 ft.-lb. and 32·4 C.H.U.

8. Air I.H.P., 69·74; stroke, 12 in.   diameter, low pressure cylinder, 12·11 in., high pressure cylinder, 4·22 in.

9. 93·91 lb. per sq. in.; air I.H.P., 2 33.

EXAMPLES VI

1. (a) 22·9 I.H.P.; (b) 38·2 I.H.P.

2. 172·5 B.Th.U. per lb. of ice; 4·13 I.H.P.

3. (a) 16,727 ft.-lb.; (b) 36·81 B.Th.U.; (c) 1·712.

4. (a) 32,060 C.H.U.; (b) 49,164 C.H.U. per hour; (c) 0·907.

5. (a) 8·77; (b) 8·29; (c) 8·28.

EXAMPLES VII

1. 10·736 lb., and excess air 7·032 lb.

2. Air, 0·957 cub.-ft.; dry products, CO_2, 14·65; O_2, 4·04; N_2, 81·31 per cent.

3. Air, 1·2823 cub. ft.; 7·25 per cent; dry products: CO_2, 13·3; O_2, 5·5; N_2, 81·2 per cent.

4. 15·43 : 1.

5. (a) 8·4 per cent; (b) 17·865 lb.; (c) 18·83 lb.

6. (a) 14·53 per cent, 8·82 per cent ; (b) 1254 C.H.U. ; (c) 444 C.H.U.

7. Air, 17·9 lb. ; flue gas, 18·856 lb. per pound of coal.

8. (a) 10,833 and 10,097 C.H.U. per lb. ; (b) 19,500 and 18,175 B.Th.U. per lb. ; (c) 170,625, and 159,030 B.Th.U. per gallon.

9. 294 and 258·9 C.H.U. per cub. ft. at N.T.P.

10. (a) 82·9, and (b) 30 per cent ; (c) 0·73 lb. ; (d) 24·9 per cent.•

11. (a) 1·37 cub. ft. ; (b) 77 per cent ; (c) 0·832 lb. ; (d) 29·65 per cent ; (e) 22·84 per cent.

12. 70 per cent ; CO, 34·6 ; N₂, 65·4 per cent ; 119 B.Th.U. per cubic foot.

13. (a) 0·726 lb. ; (b) I.H.P., 117 ; (c) 87·26 per cent ; (d) 27·55 per cent ; (e) 24 per cent.

EXAMPLES VIII

1. 0·463, or 46·3 per cent ; 20·98 cub. ft.

2. (a) 264·6 lb. ft. ; (b) B.H.P., 60·45 ; (c) indicated 30·6, and brake 24·46 per cent ; (d) 0·598, or 60 per cent (approx.).

3. Mechanical efficiency, 69·81 per cent ; indicated thermal efficiency 27·8 per cent ; efficiency ratio or relative efficiency, 0·583, or 58·3 per cent.

HEAT BALANCE FOR THE ENGINE PER MINUTE

	C.H.U. per Min.	Per Cent
Heat in gas supply	746·7	100
Indicated work	207·7	27·8
Including { Work on brake . 145 — 19·4 } { Mechanical loss . 62·7 = 8·4 } (C.H.U. Per cent)		
Heat taken by cooling water	352·3	47·2
Heat in exhaust and loss by radiation (by difference)	186·7	25·0
Total heat	746·7	100·0

4. (a) 83 per cent ; (b) 33·2 per cent.

HEAT BALANCE FOR THE ENGINE PER MINUTE

	B.Th.U. per Min.	Per Cent
Heat supplied in fuel	3155·6	100
Indicated work	1047·7	33·2
Including { Work on brake . 869·6 = 27·56 } { Mechanical loss . 178·2 = 5·64 } (B.Th.U. Per cent)		
Heat to jacket cooling water	784·5	24·9
Heat in exhaust gases	1230·7	39·0
Radiation loss, etc. (by difference)	92·7	2·9
Total	3155·6	100·0

5. (a) I.H.P., 27·76 ; (b) B.H.P., 24·36 ; (c) 87·75 per cent, thermal efficiency, 27·24 per cent ; (d) ideal air cycle 0·51, or 51 per cent ; efficiency ratio, 53·4 per cent.

(e) HEAT BALANCE SHEET FOR THE ENGINE

	C.H.U. per Min.	Per Cent
Heat supplied by gas	2401·7	100
Indicated work	654·3	27·24
Including { Useful work on brake 574·2 = 23·91 } { Mechanical loss 80·1 = 3·33 } (C.H.U. Per cent) 654 3 27·24		
Heat lost to cooling jacket water	786·6	32·75
Heat loss in exhaust and radiation (by difference)	960·3	40·01
Total	2401·7	100·00

6. (a) 12,423 ft.-lb. ; (b) 41·4 ; (c) 35 5 ; (d) 69·4 per cent.

7. 98 lb. per sq. in.

The oil engine which gives a mean pressure of 114 lb. per sq. in. does not receive all the heat at constant pressure, but part at constant volume, as shown by the actual indicator diagrams. Besides, after compression, the air blast, with the fuel at much higher pressure than that in the engine cylinder, increases the weight and energy of the charge, which raises the pressure during expansion much above that of the air cycle wherein the working charge is supposed to receive *heat only at constant pressure*.

8. 0·607 or 60·7 per cent.

9. I.H.P., 17·34 ; B.H.P., 14·45 ; mechanical efficiency, 0·8334 ; thermal efficiency, 21·96 per cent.

	B.Th.U. per min.	Per Cent
Heat supplied to engine	1861·5	100
Heat converted into indicated work	408·7	21·96
Including { Useful work on brake 340·6 = 18·3 } { Mechanical loss 68·1 = 3·66 } (B.Th.U. Per cent)		
Heat to cooling water	972·6	52·24
Heat to exhaust and radiation (by difference)	480·2	25·80
Total	1861·5	100·00

10. Diameter, 7·371 in. × stroke 11·056 in., or $7\frac{3}{8} \times 11\frac{1}{16}$ in.

11. Diameter, 9·73 in.

EXAMPLES IX

1. (a) 618·4 C.H.U., (b) 747·4 C.H.U., (c) 544·5 C.H.U., (d) 668·2 C.H U.
2. 6,539 C.H.U., 670 C.H.U.
3. 657·1 C.H.U., 611·4 C.H.U., 143° C.
4. 9·456 lb.
5. (a) dryness 0·423, (b) 20·1° C. superheat
6. 293·5° C., 7 in. and 10⅜ in.
7. 333·3° C., 222·4° C.
8. 0·988, 0·936.
9. 1·094, C.H.U.

EXAMPLES X

1. (a) 90,200 ft. lb., 12·8 per cent ; (b) 95,450 ft. lb., 12·2 per cent.
2. 19·9 lb.
3. Rankine efficiency, per cent : 13·3, 13·45, 13·77, 14·31.
   Actual efficiency, per cent : 6·0, 6·49, 7·23, 8·3.
4. Rankine efficiency, 25·5 per cent.
   Boiler   „   80   „
   Mechanical „   85   „
   Efficiency ratio, 65   „
   Overall efficiency, 11·27   „
5. Dryness fraction, 0·89, 542 H.P.
6. 13 per cent.
7. 0·866.
8. Diameter 16·75 in., stroke 30 in.
9. 72·1.
10. 0·063 lb.
11. Dryness fraction, 0·883 ; 1·84 C.H.U.
12. Diameter of H P., 13·25 in., L.P . 26·75 in. ; stroke, 36 in.
13. Diameter of H.P., 15·25 in., L.P., 26·25 in.
14. Curve.

EXAMPLES XI

1. (a) Dryness 0·79 ; 3,915 ft. sec. ; (b) 0·828, 4,000 ft. sec.
2. 4,493 ft. sec., 3,730 ft. sec.
3. 0·159 in., 0·187 in.
4. 0·366 in., 0·456 in.
5. 14·88 lb., 13·56 lb.
6. Dryness 0·972, 0·0734 lb. sec., 10·7 per cent.
7. 0·378 in., 0·468 in.
8. 0·0835 lb. sec.

INDEX

THE MÖLLIER
STEAM TABLES
AND DIAGRAMS
EXTENDED TO THE CRITICAL PRESSURE

BY

DR. RICHARD MOLLIER

Professor at the Technical High School, Dresden

ENGLISH EDITION

ADAPTED AND AMPLIFIED FROM THE THIRD GERMAN EDITION BY

H. MOSS, D.Sc., A.R.C.S., D.I.C.

CONTENTS

Use of the Tables—Use of the Diagrams—Equations for Super-heated Steam—Coefficients for Steam Equations—Conversion of Units—Interpolation—Properties of Saturated Steam—Volume of Superheated Steam—Total Heat of Superheated Steam—Entropy of Superheated Steam.

DR. MOLLIER has evolved a form of equation which not only covers the range of pressure in any commercial plant, but which also requires little extrapolation to carry it through the critical pressure, for experimental plants have been projected for the utilization of steam up to the critical point.

In adapting the tables for English publication, the pressures have been translated into pounds per square inch, but the Centigrade system of temperature has been retained in agreement with many textbooks and with university requirements.

Price **7/6** net